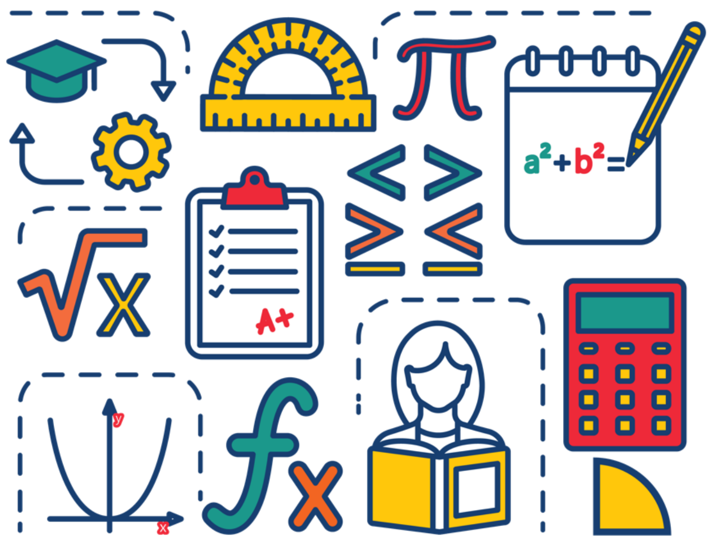
Virginia Department of Education

2023 Mathematics Standards of Learning

Grade 6 Instructional Guide



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The contents of this Instructional Guide were informed by the U.S. Department of Education's Institute of Education Sciences (IES), *What Works Clearinghouse*, as a central, trusted source of scientific evidence for what works in education. Sample questions reflect applicable and aligned content from the Virginia Department of Education's published assessment items, Mathematics Item Maps, and National Association of Educational Progress (NAEP) assessment questions.

Introduction

The Mathematics Instructional Guide, a companion document to the 2023 Mathematics *Standards of Learning*, amplifies the Standards of Learning by defining the core knowledge and skills in practice, supporting teachers and their instruction, and serving to transition classroom instruction from the 2016 *Mathematics Standards of Learning* to the newly adopted 2023 Mathematics *Standards of Learning.* Instructional supports are accessible in #GoOpenVA and support the decisions local school divisions must make concerning local curriculum development and how best to help students meet the goals of the standards. The local curriculum should include a variety of information sources, readings, learning experiences, and forms of assessment selected at the local level to create a rigorous instructional program.

For a complete list of the changes by standard, the 2023 Virginia Mathematics *Standards of Learning – Overview of Revisions* is available and delineates in greater specificity the changes for each grade level and course.

The Instructional Guide is divided into three sections: Understanding the Standard, Skills in Practice, and Concepts and Connections aligned to the Standard. The purpose of each is explained below.

Understanding the Standard

This section includes mathematics understandings and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, or examples regarding information sources that support the content. They describe what students should know (core knowledge) as a result of the instruction specific to the course/grade level and include evidence-based practices to approaching the Standard. There are also possible misconceptions and common student errors for each standard to help teachers plan their instruction.

Skills in Practice

This section outlines supporting questions and skills that are specifically linked to the standard. They frame student inquiry, promote critical thinking, and assist in learning transfer. Curriculum writers and teachers should use them to plan instruction to deepen understanding of the broader unit and course objectives. This is not meant to be an exhaustive list of student expectations.

Concepts and Connections

This section outlines concepts that transcend grade levels and thread through the K through 12 mathematics program as appropriate at each level. Concept connections reflect connections to prior grade-level concepts as content and practices build within the discipline as well as potential connections across disciplines.

Number and Number Sense

In K-12 mathematics, numbers and number sense form the foundation building blocks for mathematics understanding. Students develop a foundational understanding of numbers, their properties, and the relationships between them as they learn to compare and order numbers, understand place value, and perform numerical operations. Number sense includes an understanding of patterns and the relationships between numbers and being able to apply this knowledge to real world problems. Throughout K-12, students build on their number sense to develop a deeper understanding of mathematical concepts and reasoning.

In Grade 6, there are multiple representations of numbers and relationships among numbers that provide meaning and structure that allow students to make sense of the world around them. At this grade level, students express equivalency; compare and order numbers written as fractions, mixed numbers, decimals, and percents; represent, compare, and order integers; and recognize and represent patterns with exponents and perfect squares.

6.NS.1 The student will reason and use multiple strategies to express equivalency, compare, and order numbers written as fractions, mixed numbers, decimals, and percents.

Students will demonstrate the following Knowledge and Skills:

a) Estimate and determine the percent represented by a given model (e.g., number line, picture, verbal description), including percents greater than 100% and less than 1%.*

b) Represent and determine equivalencies among decimals (through the thousandths place) and percents incorporating the use of number lines, and concrete and pictorial models.*

c) Represent and determine equivalencies among fractions (proper or improper) and mixed numbers that have denominators that are 12 or less or factors of 100 and percents incorporating the use of number lines, and concrete and pictorial models.*

d) Represent and determine equivalencies among decimals, percents, fractions (proper or improper), and mixed numbers that have denominators that are 12 or less or factors of 100 incorporating the use of number lines, and concrete and pictorial models.*

e) Use multiple strategies (e.g., benchmarks, number line, equivalency) to compare and order no more than four positive rational numbers expressed as fractions (proper or improper), mixed numbers, decimals, and percents (decimals through thousandths, fractions with denominators of 12 or less or factors of 100) with and without models. Justify solutions orally, in writing or with a model. Ordering may be in ascending or descending order.*

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

Understanding the Standard

- Fractions, decimals, and percents are three different ways to express the same number. Any number that can be written as a fraction can be expressed as a terminating or repeating decimal or a percent.
- Percent means “per 100” or how many “out of 100”; percent is another name for hundredths.
- A number followed by a percent symbol (%) is equivalent to a fraction with that number as the numerator and with 100 as the denominator (e.g., $30\% = \frac{30}{100}$; $139\% = \frac{139}{100}$).
- Percents can be expressed as fractions or decimals (e.g., $38\% = \frac{38}{100} = 0.38$; $139\% = \frac{139}{100} = 1.39$).
- Percents are used to solve contextual problems including sales, tax, and discounts.
- When estimating a percent, students should consider benchmarks of 0%, 25%, 50%, 75%, and 100%.
- For percents less than 1, focus on benchmarks that are less than 1% (like 0.5% or $\frac{3}{4}$%).
- Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base 10 blocks, fraction circles, number lines, colored counters, cubes, decimal squares, shaded figures, shaded grids, or calculators).
- Some fractions can be rewritten as equivalent fractions with denominators of powers of 10 and can be represented as decimals or percents (e.g., $\frac{3}{5} = \frac{6}{10} = \frac{60}{100}$ = 0.60 = 60%). Fractions, decimals, and percents can be represented by using an area model, a set model, or a measurement model. For example, the fraction $\frac{1}{3}$ is shown below using each of the three models.

- The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of positive rational numbers are: 0.275, $\frac{1}{4}$, 82, 75%, $\frac{22}{5}$, $4.\overline{59}$.
- Students are not expected to know the names of the subsets of the real numbers until Grade 8.
- Proper fractions, improper fractions, and mixed numbers are terms used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$).

- Strategies using 0, $\frac{1}{2}$, and 1 as benchmarks can be used to compare fractions and decimals. For example: Which is greater: $\frac{4}{7}$ or 0.4? $\frac{4}{7}$ is greater than $\frac{1}{2}$ because 4, the numerator, represents more than half of 7, the denominator. 0.4 is less than $\frac{1}{2}$ because 0.4 is less than 0.5 which is equivalent to $\frac{1}{2}$. Therefore, $\frac{4}{7} > 0.4$.
- When comparing two fractions close to 1, the distance from 1 can be used as the benchmark. For example: Which is greater, $\frac{6}{7}$ or $\frac{8}{9}$? $\frac{6}{7}$ is $\frac{1}{7}$ away from 1 whole. $\frac{8}{9}$ is $\frac{1}{9}$ away from 1 whole. Since, $\frac{1}{9} < \frac{1}{7}$, then $\frac{6}{7}$ is a greater distance away from 1 whole than $\frac{8}{9}$. Therefore, $\frac{6}{7} < \frac{8}{9}$.
- Some fractions have a decimal representation that is a terminating decimal, which means it has a finite number of digits (e.g., $\frac{1}{8} = 0.125$). Other fractions have a decimal representation that does not terminate but continues to repeat (e.g., $\frac{2}{9}$ = 0.222…) The repeating decimal can be written with ellipses (three dots) as in 0.222… or denoted with a bar above the digits that repeat as in $0.\overline{2}$.
- Students may justify their reasoning by using benchmarks, number lines, equivalency, pictures, etc.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Connections: Number lines are a way to visualize parts of a whole. Number lines can be used to represent, compare, and order fractions as well as determine equivalencies among them. Number lines are helpful in teaching benchmark fractions. Prior to Grade 6, elementary students used simple fraction number lines with halves, thirds, fourths, eighths, or tenths as well as mixed numbers. Number lines are beneficial to use when teaching decimal place value and benchmarking with hundredths and thousandths. However, a common difficulty students may have when using number lines is labeling blank number lines and counting the spaces or tick marks. To support students' use of number lines –

- Use painter's tape to make extra-large number lines on the wall or floor or make them interactive and have students use sticky notes to label the tick marks or show different points on the line.

- Have students make a human number line. Give students pieces of paper with fractions, decimals, and percents on them, and have students arrange themselves in order. This reinforces comparing and equivalencies as well.

Mathematical Reasoning: Encourage students to justify their reasoning orally, in writing, or with a model. When doing so, students are to apply the appropriate vocabulary such as ascending and descending. The following questions may elicit students' understanding –

- What strategies are used to compare fractions and mixed numbers?
- How are fractions, decimals, and percents alike and different?

- How can fractions, decimals, and percents be represented in various ways?
- Why is it necessary to have multiple forms of numbers?
- When ordering fractions, decimals, and percents, detail a strategy used and discuss how you justified your solution.

Mathematical Representations:

- Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base 10 blocks, fraction circles, number lines, colored counters, cubes, decimal squares, shaded figures, shaded grids, calculators). Concrete materials and pictorial representations assist students in clearing up common misconceptions related to equivalencies and the application of ordering and comparing such as –
    - believing that a fraction, decimal and percent can represent the same value.
    - thinking the fraction with the larger denominator is always the largest fraction.
    - writing the decimal form of a fraction using the numerator as the first digit and the denominator as the second digit.
    - moving the decimal in the right direction when converting between decimals and percents.

- An activity to support students' understanding of equivalencies as well as ordering and comparing fractions, decimals, and percents is as follows –
    - Create sets of playing cards for each group by writing the following fractions, decimals, and percents on index cards: 0.001, 0.005, 0.01, 0.05, 0.10, 0.15, 0.2, 0.25, 0.45, 0.5, 0.55, 0.6, 0.75, 0.9, 1, 2, 2.5, 5, 0.1%, 0.5%, 1%, 5%, 10%, 15%, 20%, 25%, 45%, 50%, 55%, 60%, 75%, 90%, 100%, 200%, 250%, 500%, $\frac{1}{100}, \frac{1}{1000}, \frac{5}{1000}, \frac{5}{100}, \frac{1}{10}, \frac{15}{100}, \frac{1}{5}, \frac{1}{4}, \frac{45}{100}, \frac{1}{2}, \frac{55}{100}, \frac{3}{5}, \frac{3}{4}, \frac{9}{10}, \frac{5}{5}, \frac{10}{5}, 2\frac{1}{2}$.
    - Put students in groups of two to four and give each group a set of the cards.
    - Have students shuffle the cards and place them face down on the table. In turn, each player draws two cards, places them face up, and orders the numbers from lowest to highest. If the two numbers are equal, the cards are placed one on top of the other. Correct ordering earns a player one point. For the next round, each player draws three cards and orders the numbers from lowest to highest. If two numbers are equal, the cards are again placed one on top of the other. In this round, correct ordering earns a player two points. In the next round, each player draws four cards and follows the same procedure as in the first two rounds. Correct ordering earns a player three points.
    - Students may continue with additional rounds, adding one more card and one more point per round. The player with the most points at the end of play wins.
    - To conclude the activity or as a reflection, give each student six blank index cards, and have each student create two sets of cards for use in the game. For each set, students must write an equivalent fraction, decimal, and percent. All the students' cards can be combined for use in playing the game again in a future lesson.

Concepts and Connections

Concepts
There are multiple representations of numbers and relationships among numbers that provide meaning and structure and allow us to make sense of the world around us.
 	
Connections: In Grade 6, students will use multiple strategies to compare and order integers (6.NS.2). Prior to Grade 6, students used reasoning and justification to identify and represent equivalency between fractions (with denominators that are thirds, eighths, and factors of 100) and decimals; and compared and ordered sets of fractions (proper, improper, and/or mixed numbers having denominators of 12 or less) and decimals (through thousandths) (5.NS.1). Using these foundational understandings, students will reason and use multiple strategies to express equivalency, compare, and order numbers written as fractions, mixed numbers, decimals, and percents (6.NS.1). In the subsequent grade level, Grade 7 students will reason and use multiple strategies to compare and order rational numbers (7.NS.2).
 	
• *Within the grade level/course*:
    ○ 6.NS.2 – The student will reason and use multiple strategies to represent, compare, and order integers.
• *Vertical Progression*:
    ○ 5.NS.1 – The student will use reasoning and justification to identify and represent equivalency between fractions (with denominators that are thirds, eighths, and factors of 100) and decimals; and compare and order sets of fractions (proper, improper, and/or mixed numbers having denominators of 12 or less) and decimals (through thousandths).
    ○ 7.NS.2 – The student will reason and use multiple strategies to compare and order rational numbers.

<u>Textbooks and HQIM for Consideration</u> • A list of approved textbooks will be posted on the VDOE website.

6.NS.2 The student will reason and use multiple strategies to represent, compare, and order integers.

Students will demonstrate the following Knowledge and Skills:

a) Represent integers (e.g., number lines, concrete materials, pictorial models), including models derived from contextual situations, and identify an integer represented by a point on a number line.

b) Compare and order integers using a number line.

c) Compare integers, using mathematical symbols (<, >, =).

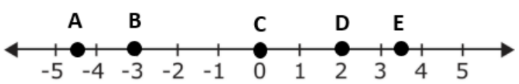
d) Identify and describe the absolute value of an integer as the distance from zero on the number line.

Understanding the Standard
● The set of integers includes the set of whole numbers and their opposites {…-2, -1, 0, 1, 2, …}. Zero has no opposite and is an integer that is neither positive nor negative. ● The opposite of a positive number is negative, and the opposite of a negative number is positive. ● Positive integers are greater than zero. ● Negative integers are less than zero. ● A negative integer is always less than a positive integer. ● On a conventional number line, a smaller number is always located to the left of a larger number (e.g., -7 lies to the left of -3, thus -7 < -3; 5 lies to the left of 8, thus 5 < 8). ● When comparing two negative integers using a number line, the negative integer that is closer to zero is greater. ● Integers are used in contextual situations, such as temperature (above/below zero degrees), deposits/withdrawals in a checking account, golf (above/below par), timelines, football yardage, positive and negative electrical charges, and altitude (above/below sea level). ● Integers should be explored by modeling on number lines, both horizontal and vertical, and using manipulatives, such as two-color counters, drawings, or algebra tiles. ● The absolute value of a number is the distance of a number from zero on the number line regardless of direction. Absolute value is represented using the symbol | | (e.g., $\lvert -6 \rvert = 6 \text{ and } \lvert 6 \rvert = 6$). Absolute value is always positive. ● The absolute value of zero is zero. ● An integer and its opposite are the same distance from zero on a number line. Thus, they have the same absolute value. For example: The opposite of 3 is -3, and $\lvert -3 \rvert = 3$ and $\lvert 3 \rvert = 3$.

Skills in Practice
While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Connections:

- Number lines are a way to visualize integers (whole numbers and their opposites) and the distance from zero on the number line (absolute value). For example, when given the following number line, there are several misconceptions that students may experience –



  One common misconception for some students is not understanding that integers are whole numbers and their opposites. Some students may select the points that represent fraction/decimal values. These students recognize that integers differ from whole numbers, but do not recognize they represent a "whole," and do not include "parts" of numbers. Other students may not select zero as an integer, thinking it is not included as part of this set. Students with these misconceptions may benefit from a visual representation in the form of a diagram of the sets of numbers. In addition, a discussion of whole numbers and what they represent as well as a discussion of the meaning of "opposite" would provide more clarity for some students. It is important for students to realize that sets of numbers in mathematics overlap, and that numbers can belong to more than one set.

- Use painter's tape to make extra-large number lines on the wall or floor or make them interactive and have students use sticky notes to label the tick marks or show different points on the line.

- Have students make a human number line. Give students pieces of paper with integers on them and they can arrange themselves in order. This reinforces comparing integers as well. Some students believe that absolute value means to take the opposite of the number instead of understanding it represents the distance away from zero. While engaging in this activity, ask students to determine the absolute value of -4 and 4. Students should state that these numbers are the same distance away from zero. Extensions of this activity include –

  - Have two students stand back-to-back at zero. What is the distance between each student? Have each student walk 3 steps in opposite directions. What is the distance between the two students now? What expression could be used to model the steps, $|-3 - 3|$ or $|3 - (-3)|$? Why is the distance not zero? How could this activity relate to absolute value?

  - Select two more students and have them stand at zero and face the same direction. Have one student walk 5 steps and one student walk 3 steps both in a positive direction. What is the distance between the two students now? What expression could be used to model the steps, $|5 - 3|$ or $|3 - 5|$? How could this activity relate to absolute value?

- Select two more students and have them stand at zero and face the same direction. Have one student walk 2 steps and one student walk 7 steps, both in a negative direction. What is the distance between the two students now? What expression could be used to model the steps, $|-2 - (-7)|$ or $|-7 - (-2)|$? How could this activity relate to absolute value?

- Provide students with the following scenario: Henry and Jenny were comparing two integers. Henry said, “My integer is greater than your integer.” Jenny said, “That may be true, but the absolute value of my integer is greater than your integer.” Locate Henry’s and Jenny’s integers on a number line and explain your reasoning. Ask students to then compare their answers with each other. Ask students to report out their observations as well as variations in responses.

Mathematical Reasoning: Encourage students to justify their reasoning and understanding of integers orally, in writing, or with a model. When doing so, students are to use the appropriate vocabulary such as positive, negative, integer, absolute value, order, and compare. The following questions may elicit students’ understanding of concepts related to this standard –

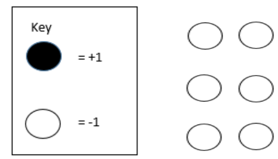
- In your own words, describe the meaning of an integer and give practical situations of integers in the real world.
- What are some examples of integers in the real world?
- Why do we need negative numbers? Give specific examples.
- What is the importance of the zero when comparing integers?
- Explain the meaning of zero in one of the following situations: temperature, elevation, sea level, or money.
- What does it mean to be an opposite number?
- Give students a number and have them state the opposite, and explain different situations where both numbers would be used.
- Which number is greater, -13 or -10? Explain your reasoning.
- Can the absolute value be a negative number? Why or why not? Explain your reasoning.
- Is the opposite of a number the same as the absolute value of a number? Why or why not? Explain your reasoning.
- Is the absolute value of zero equal to zero? Why or why not? Explain your reasoning.
- Is $-|3|$ equivalent to $|-3|$? Why or why not? Explain your reasoning.

Mathematical Representations: Integers should be explored by modeling on number lines, both horizontal and vertical, and using manipulatives, such as two-color counters, drawings, or algebra tiles. Examples with common misconceptions follow –

- Consider the following situation: The deep end of the pool is 6 feet below ground. What integer does this situation represent? Create a model to explain your response.

A common error for some students is to create a model showing a positive integer rather than the negative integer from the situation. Students may struggle with creating a model where the outcome is a negative. These students may benefit from connecting these situations to a vertical number line that provides a visual reference for negative numbers.

- Use the key below to determine what integer is represented. Explain your thinking.



A common misconception is thinking that counters always represent positive integers. Some students struggle with the idea that an object (counter) can represent a negative number.

Students with this misconception may benefit from modeling practical situations using counters to represent positive and negative numbers. As students record their thinking, they should use a key that represents the counters used in the model. Another possible strategy is to model a positive integer using two-color counters and then use the counters to model the integer that is the opposite (negative) of that integer. Students could then connect these integer models to number line models. Making connections between different representations may help students deepen their understanding of integer representations.

Concepts and Connections

Concepts
There are multiple representations of numbers and relationships among numbers that provide meaning and structure and allow us to make sense of the world around us.

Connections: In Grade 6, students will reason and use multiple strategies to express equivalency, compare, and order numbers written as fractions, mixed numbers, decimals, and percents (6.NS.1). Prior to Grade 6, students used reasoning and justification to identify and represent equivalency between fractions (with denominators that are thirds, eighths, and factors of 100) and decimals; and compared and ordered sets of fractions (proper, improper, and/or mixed numbers having denominators of 12 or less) and decimals (through thousandths) (5.NS.1). Using these foundational understandings, students will use multiple strategies to compare and order integers (6.NS.2). In the subsequent grade level, Grade 7 students will reason and use multiple strategies to compare and order rational numbers (7.NS.2).

- *Within the grade level/course*:
    - 6.NS.1 – The student will reason and use multiple strategies to express equivalency, compare, and order numbers written as fractions, mixed numbers, decimals, and percents.

- 6.CE.2 – The student will estimate, demonstrate, solve, and justify solutions to problems using operations with integers, including those in context.
- *Vertical Progression*:
    - 5.NS.1 – The student will use reasoning and justification to identify and represent equivalency between fractions (with denominators that are thirds, eighths, and factors of 100) and decimals; and compare and order sets of fractions (proper, improper, and/or mixed numbers having denominators of 12 or less) and decimals (through thousandths).
    - 7.NS.2 – The student will reason and use multiple strategies to compare and order rational numbers.

Textbooks and HQIM for Consideration

- A list of approved textbooks and instructional materials will be posted on the VDOE website.

6.NS.3 The student will recognize and represent patterns with whole number exponents and perfect squares.

Students will demonstrate the following Knowledge and Skills:

a) Recognize and represent patterns with bases and exponents that are whole numbers.

b) Recognize and represent patterns of perfect squares not to exceed 20^2, by using concrete and pictorial models.

c) Justify if a number between 0 and 400 is a perfect square through modeling or mathematical reasoning.

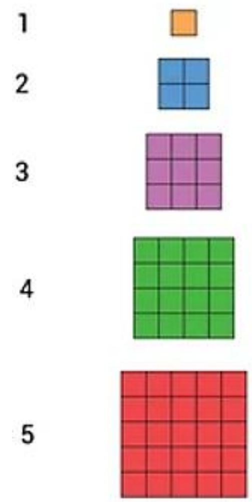
d) Recognize and represent powers of 10 with whole number exponents by examining patterns in place value.

Understanding the Standard
● The symbol $\cdot$ can be used in Grade 6 in place of "×" to indicate multiplication. ● In exponential notation, the base is the number that is multiplied, and the exponent represents the number of times the base is used as a factor. For example, in 8^3, 8 is the base and 3 is the exponent (e.g., $8^3 = 8 \cdot 8 \cdot 8$). ● Any real number other than zero raised to the zero power is 1. Patterns can be used to foster this understanding for students. See the example below. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$ ÷ 3 $3^3 = 3 \cdot 3 \cdot 3 = 27$ ÷ 3 $3^2 = 3 \cdot 3 = 9$ ÷ 3 $3^1 = 3 = 3$ ÷ 3 $\boxed{3^0 = 1}$ ● Zero raised to the zero power (0^0) is undefined according to some calculators. Other calculators will return a value of 1 when 0 is raised to the 0 power. There is debate among mathematicians surrounding this value. Students should not be expected to provide a direct value for this quantity (0^0). ● An integer that can be expressed as the square of another integer is called a perfect square (e.g., $36 = 6 \cdot 6 = 6^2$). Zero (a whole number) is a perfect square. ● Perfect squares may be represented geometrically as the areas of squares whose side lengths are whole numbers (e.g., $1 \cdot 1$, $2 \cdot 2$, $3 \cdot 3$). This can be modeled with grid paper, tiles, geoboards, and virtual manipulatives. ● The examination of patterns in place value of the powers of 10 in Grade 6 leads to the development of scientific notation in Grade 7.

Skills in Practice
While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Representations:

- Perfect squares may be represented geometrically as the areas of squares whose side lengths are whole numbers (e.g., $1 \cdot 1$, $2 \cdot 2$, $3 \cdot 3$). This can be modeled with grid paper, tiles, geoboards, and virtual manipulatives. For example –



- Allow students to derive meaning through the construction of squares using concrete manipulatives and pictorial representations to observe the perfect squares. By extension, students can determine whether a perfect square exists using concrete manipulatives or grid paper with examples such as 62, 81, 99, 100, or 144 (where these numbers would represent areas of the possible squares). Students can explore the structure of the tiles and discover similarities and differences between numbers such as 62 and 81.

Mathematical Reasoning:

- Have students create squares using tiles or draw representations of perfect squares on grid. Questions to elicit understanding of these representations may include –
    - How can the area model be used to relate perfect squares to multiplication?
    - Using square tiles or grid paper, model how you know the numbers 6 and 12 are *not* perfect squares. Explain and justify your reasoning.
    - Using square tiles or grid paper, model how you know the numbers 16 and 49 are perfect squares. Explain and justify your reasoning.
    - Is zero to the zero power (0^0) a perfect square?
    - Why is any real number other than zero raised to the zero power equivalent to 1?

- Give students the following scenario: Derrick stated, “The number 225 is not a perfect square because it is an odd number.” Sarah stated, “You are incorrect. The number 225 is a perfect square. There can be even and odd numbers that are perfect squares.” Explain who is correct in this scenario and justify your reasoning using a concrete representation, pictorial representation, examples, and/or counterexamples.

Mathematical Connections:

- A square root of a number is a number which, when multiplied by itself, produces the given number.

  - Students will confuse the square root of a number and divide it by two instead of the square root. They might know the square root of small numbers like 25 or 16 but they think the square root of 10 is 5. When given the area of a square and students are asked to find the side length, students may divide by 4 instead of finding the square root of the area.

  - To develop students’ understanding of this concept, provide them with values such as 0, 50, 125, 200, and 361 and ask them to determine whether the values are perfect squares. Encourage students to justify their responses. A common misconception students may make from the given values is assuming that even numbers are perfect squares because they can be divided by 2 to get a whole number. For example, the values of 50 and 200, when divided by 2, provide students with whole numbers, however they are not perfect squares.

- The examination of patterns in place value of the powers of 10 in Grade 6 leads to the development of scientific notation in Grade 7. For example –

  - Suppose you are given 10^2, 10^3, and 10^4. Provide the next three terms in the pattern and explain how you arrived at your answer.

  - Use a place value chart to represent 10^5.

Ten Millions	Millions	Hundred Thousands	Ten thousands	Thousands	Hundreds	Tens	Ones

A common student error is writing the number in standard form by writing 10 and then using the exponent to determine the number of zeroes written after the 10. In this example, students may write 1,000,000 because they place 5 zeroes after the initial 10. This may result from a procedural focus on using the exponent to determine the number of zeroes rather than a conceptual focus on base ten understanding. Students may understand the place value of numbers, but may not be able to relate it to powers of 10.

Students may benefit from using concrete materials like base ten blocks to build conceptual understanding by building 10^2 as 10 groups of 10, then building 10 groups of 10^2 (100) to show 10^3, etc. Then students can record the patterns as number sentences as they explore powers of 10,

beginning with thinking of 10^2 as 10 × 10 = 100, then 10^3 as 10 × 10 × 10, etc. As students build these patterns, they begin to see the connections between powers of 10 and place value.
Concepts and Connections

Concepts
There are multiple representations of numbers and relationships among numbers that provide meaning and structure and allow us to make sense of the world around us.
 	
Connections: Grade 6 begins the development of the recognition and representation of patterns with whole number exponents and perfect squares. In the subsequent grade level, Grade 7 students will recognize and describe the relationship between square roots and perfect squares.
 	
• *Within the grade level/course*:
  ○ There are no horizontal connections as this is the students' first formal exposure to whole number exponents and perfect squares.
• *Vertical Progression*:
  ○ There are no formal standards that address patterns with whole number exponents and perfect square in previous grade levels.
  ○ 7.NS.3 – The student will recognize and describe the relationship between square roots and perfect squares.

Textbooks and HQIM for Consideration • A list of approved textbooks and instructional materials will be posted on the VDOE website.

Computation and Estimation

In K-12 mathematics, computation and estimation are integral to developing mathematical proficiency. Computation refers to the process of performing numerical operations, such as addition, subtraction, multiplication, and division. It involves applying strategies and algorithms to solve arithmetic problems accurately and efficiently. Estimation is the process of obtaining a reasonable or close approximation of a result without performing an exact calculation. Estimation is particularly useful when an exact answer is not required, especially in real-world situations, and when assessing the reasonableness of an answer. Computation and estimation are used to support problem solving, critical thinking, and mathematical reasoning by providing students with a range of approaches to solve mathematics problems quickly and accurately.

In Grade 6, students learn how estimation and the operations (addition, subtraction, multiplication, and division) allow them to model, represent, and solve different types of problems with rational numbers. At this grade level, students represent and solve problems using operations with fractions, mixed numbers, and integers.

It is critical at this grade level that while students build their conceptual understanding through concrete and pictorial representations, that students also apply standard algorithms when performing operations as specified within the parameters of each standard of this strand. At this grade level, students become more efficient with problem solving strategies. Fluent use of standard algorithms not only depends on automatic retrieval (automaticity learned during the elementary grades), but also reinforces them. Automaticity further grounds students' entry points and the structures needed to solve contextual problems and execute mathematical procedures with rational numbers.

6.CE.1 The student will estimate, demonstrate, solve, and justify solutions to problems using operations with fractions and mixed numbers, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Demonstrate/model multiplication and division of fractions (proper or improper) and mixed numbers using multiple representations.*

b) Multiply and divide fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.*

c) Investigate and explain the effect of multiplying or dividing a fraction, whole number, or mixed number by a number between zero and one.*

d) Estimate, determine, and justify the solution to single-step and multistep problems in context that involve addition and subtraction with fractions (proper or improper) and mixed numbers, with and without regrouping, that include like and unlike denominators of 12 or less. Answers are expressed in simplest form.

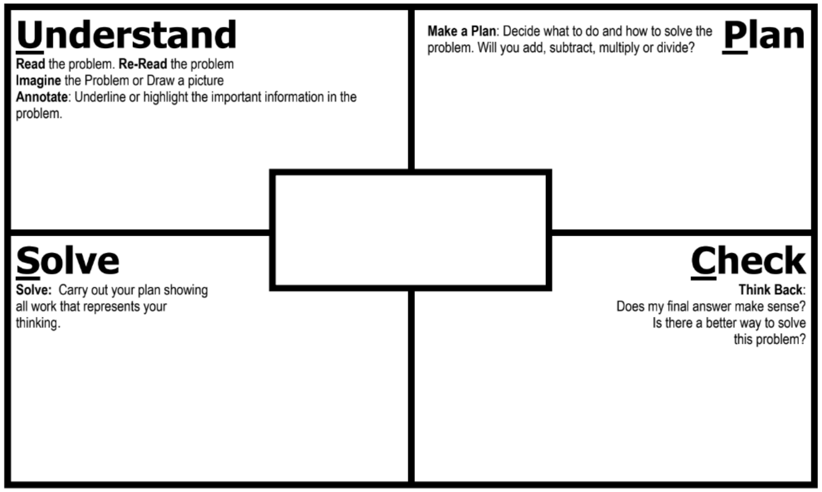
e) Estimate, determine, and justify the solution to single-step and multistep problems in context that involve multiplication and division with fractions (proper or improper) and mixed numbers that include denominators of 12 or less. Answers are expressed in simplest form.

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

Understanding the Standard
● A fraction can be expressed in simplest form (simplest equivalent fraction) by dividing the numerator and denominator by their greatest common factor. ● When the numerator and denominator have no common factors other than 1, then the fraction is in simplest form. ● Addition and subtraction are inverse operations. Multiplication and division are inverse operations. ● Models for representing multiplication and division of fractions may include arrays, paper folding, repeated addition, repeated subtraction, fraction strips, fraction rods, pattern blocks, or area models. ● It is helpful to use estimation to develop computational strategies and determine the reasonableness of a solution. For example: $2\frac{7}{8} \cdot \frac{3}{4}$ is about $\frac{3}{4}$ of 3, so the answer is between 2 and 3. ● When multiplying a whole number by a fraction such as $3 \cdot \frac{1}{2}$, the meaning is the same as with multiplication of whole numbers: 3 groups the size of $\frac{1}{2}$ of the whole. ● When multiplying a fraction by a whole number such as $\frac{1}{3} \cdot 6$, we are trying to determine a part of the whole: $\frac{1}{3}$ of six wholes. ● When multiplying a fraction by a fraction such as $\frac{1}{2} \cdot \frac{3}{4}$, the problem is asking for part of a part: one-half of $\frac{3}{4}$. ● It is helpful to use benchmark fractions or decimals to explore the effect of multiplying or dividing a fraction, whole number, or mixed number by a number between zero and one. Students should understand that multiplying by a number between zero and one will decrease the value of the original number and dividing by a number between zero and one will increase the value of the original number. ● Solving multistep problems in the context of contextual situations enhances proficiency with estimation strategies. ● Students may justify their reasoning by using estimation strategies, models, benchmarks, etc.
Skills in Practice
While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Problem Solving: Problem solving requires both an ability to correctly define a problem and find a solution to it. Model effective practices that demonstrate for students how to arrive at their solutions through estimation, exactness, and justification. To unpack contextual problems, guide students through the problem-solving process by taking care to annotate *essential vocabulary* (not “key words”) related to applied operations.
For example –

- Understand the problem by reading and then re-reading the problem; imagining the problem or drawing a picture; underlining or highlighting the important information and essential vocabulary within the problem.
- Create a plan by deciding what to do and how to solve the problem using the applicable operation(s).
- Carry out the plan by showing all work to represent thinking (e.g., using symbols, words, concrete manipulatives, pictorial representations).
- Check the solution by determining if the solution answers the question asked in the contextual problem. During this phase of problem solving, it is essential that students are exposed to strategies that allow them to determine the reasonableness of their responses and solutions; and to support multiple mathematically sound and accurate ways of arriving at a solution.
- Graphic organizers such as the four-square model (Polya method) can help students to organize their thinking as they solve contextual problems –



- As students solve contextual problems, ask questions such as –
  - Are there multiple ways to solve a single problem?
  - How do you know that you have provided a reasonable answer?
  - What role does estimation play in solving contextual problems?

Mathematical Communication: Recall multiple problem types learned at the elementary grades and apply to contextual problems as students advance in their understanding of more complex contextual problems and structures. Teach students a solution method for solving each problem type. Introduce a solution method using a worked-out example. Talk through the problem-solving process and connect the relevant problem information to the worked-out example. Say out loud the decisions that were made to solve the problem at each step. Then demonstrate how to apply the solution method by solving a similar problem with students using that method. Discuss each decision made and ask students guiding questions to engage them as problems are solved.

Mathematical Reasoning: Solving problems in contextual situations enhances proficiency with estimation strategies. Reasoning skills are essential for solving problems effectively and strategically. Reasoning involves analyzing information, identifying assumptions, evaluating evidence, and drawing logical conclusions. When solving problems in context, students may have difficulty understanding which operation(s) to use and may use the wrong operation(s) when solving. Students may not realize that all numbers in a contextual problem are not always needed. To help students derive meaning of which operation to apply, ask students how thinking about the meaning of addition can help when solving word problems. Give students a chance to explain individually and share their thoughts with each other. Repeat this process for subtraction, multiplication, and division. *Have students explain the action of the word to move away from a reliance on "key words."* For example –

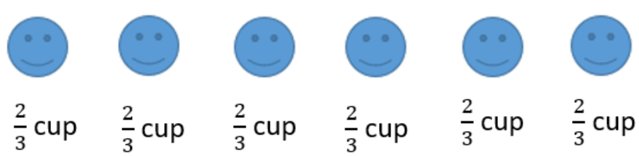
- Addition:
    - Finding the total quantity of separate quantities
    - Combining two or more quantities
- Subtraction:
    - Finding how much more or how much less
    - Finding how much further
    - Finding the difference between two quantities
    - Determining a quantity when taking one amount from another
- Multiplication:
    - Finding the quantity needed for *x* number of people or *x* number of something
    - Having equal groups and finding the total of all groups
    - Finding a part (fraction) of a whole number
    - Taking a part of a part (fraction of a fraction)
- Division:
    - Dividing an item (or quantity) into equal sized pieces
    - Dividing a quantity into equal groups
    - Using an equal amount of something over time
    - Determining how many fractional groups can be made from a quantity

Mathematical Connections: Students must connect fluency of facts, concepts, procedures, and mathematical language through mathematical reasoning to effectively solve contextual problems. Fluency with rational numbers at this grade level will help with improving logical reasoning skills, which will then lend themselves to solving contextual problems. In the following example, first, students must use their previous understanding of computation with whole numbers and fractions to arrive an exact solution. Next, students must apply their understanding of mathematical language to determine which operation(s) to use, and with this information, design a plan to solve the solution. Lastly, students must determine the reasonableness of their solution by providing their reasoning. Common misconceptions are given below when either fluency of facts, concepts, procedures, or misinterpretation of mathematical language can derail students' provision of a correct response –

- Marcus has $3\frac{1}{6}$ feet of string and he buys a piece that is $2\frac{7}{12}$ feet long. He is making necklaces that each use $1\frac{1}{2}$ feet of string. How many complete necklaces can Marcus make with both pieces of string?

  When there are multistep word problems, students often have a hard time knowing which operations to use and when to use them. A hands-on activity may help students understand the context of the problem. Breaking students into small groups and giving them $3\frac{1}{6}$ ft. and $2\frac{7}{12}$ ft. (total of $5\frac{9}{12}$ = $5\frac{3}{4}$ feet) of string is a good activity to reinforce this skill. Students can combine the string and then see how much string they have all together. They can then create "necklaces" that are each $1\frac{1}{2}$ feet long. Cutting string repeatedly into equal parts will reinforce the need to use division or repeated subtraction. Allowing students to come up with the correct answer with the activity, and then completing the computation will show them if there is a discrepancy. If students combine the string, the final quotient is $3\frac{5}{6}$ necklaces. Some students may round that to 4 necklaces. Students who think about the question holistically and do not think that the string would not be tied together when making necklaces may see that they can make two necklaces from the first piece of string and only one necklace from the second piece of string. Students can be reminded about the word *complete* so that they understand that the partial section of string cannot count toward a complete necklace. Relating this concept to real life often clears up misconceptions.

- Tiffany is having 5 friends over for a birthday party. Each serving of ice cream is $\frac{2}{3}$ of a cup. She would like to double that amount for each serving and have enough ice cream for her and her friends. How much ice cream will she need?



  One common mistake is that students skip over words in the question that are related to operations, such as double, triple, half, etc. It may be helpful to encourage students to draw a picture or model what is happening in the contextual situation. There are six children, and each child gets $\frac{2}{3}$ of a cup of ice cream (see picture above). This may be a good time to have rich conversations about when the doubling should happen. Ask students, *"Will you get the same answer if you double the $\frac{2}{3}$ first and then multiply by 6, or double the 6 and then multiply by $\frac{2}{3}$, or multiply the 6 by $\frac{2}{3}$, and then double the answer?"* Having students share and discuss different strategies also promotes greater understanding.

Concepts and Connections

Concepts
Estimation and the operations of addition, subtraction, multiplication, and division, allow us to model, represent, and solve different types of problems with rational numbers.
 	
Connections: In Grade 6, students will estimate, demonstrate, solve, and justify solutions using operations with integers, including those in context (6.CE.2). Prior to Grade 6, students estimated, represented, solved, and justified solutions to single-step and multistep problems, including those in context, using addition and subtraction of whole numbers and fractions with like and unlike denominators (with and without models), and solved single-step contextual problems involving multiplication and a proper fraction using models (5.CE.1, 5.CE.2). Further, students estimated, represented, solved, and justified solutions to single-step and multistep problems, including those in context, using addition, subtraction, multiplication, and division with decimal numbers (5.CE.3). Using these foundational understandings, students will estimate, demonstrate, solve, and justify solutions to problems using operations with fractions and mixed numbers, including those in context (6.CE.1). In the subsequent grade level, Grade 7 students will estimate, solve, and justify solutions to multistep contextual problems involving operations with rational numbers (7.CE.1) and solve problems, including those in context, involving proportional relationships (7.CE.2).
 	
• *Within the grade level/course*:
    ○ 6.CE.2 – The student will estimate, demonstrate, solve, and justify solutions to problems using operations with integers, including those in context.
• *Vertical Progression*:
    ○ 5.CE.1 – The student will estimate, represent, solve, and justify solutions to single-step and multistep contextual problems using addition, subtraction, multiplication, and division with whole numbers.
    ○ 5.CE.2 – The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction of fractions with like and unlike denominators (with and without models), and solve single-step contextual problems involving multiplication of a whole number and a proper fraction, with models.
    ○ 5.CE.3 – The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition, subtraction, multiplication, and division with decimal numbers.
    ○ 7.CE.1 – The student will estimate, solve, and justify solutions to multistep contextual problems involving operations with rational numbers.
    ○ 7.CE.2 – The student will solve problems, including those in context, involving proportional relationships.

<u>Textbooks and HQIM for Consideration</u> • A list of approved textbooks and instructional materials will be posted on the VDOE website.

6.CE.2 The student will estimate, demonstrate, solve, and justify solutions to problems using operations with integers, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Demonstrate/model addition, subtraction, multiplication, and division of integers using pictorial representations or concrete manipulatives.*

b) Add, subtract, multiply, and divide two integers.*

c) Simplify an expression that contains absolute value bars | | and an operation with two integers (e.g., $-|5 - 8|$ or $\frac{|-12|}{8}$) and represent the result on a number line.

d) Estimate, determine, and justify the solution to one- and two-step contextual problems, involving addition, subtraction, multiplication, and division with integers.

*** On the state assessment, items measuring this objective are assessed without the use of a calculator.**

Understanding the Standard
● The set of integers is the set of whole numbers and their opposites {…-2, -1, 0, 1, 2, …}. Zero has no opposite and is neither positive nor negative. ● Integers are used in contextual situations, such as temperature (above/below zero degrees), deposits/withdrawals in a checking account, golf (above/below par), timelines, football yardage, positive and negative electrical charges, and altitude (above/below sea level). ● Concrete experiences in formulating rules for adding, subtracting, multiplying, and dividing integers should be explored by examining patterns using calculators, a number line, and manipulatives, such as two-color counters, drawings, or algebra tiles. ● Sums, differences, products, and quotients of integers are either positive, negative, undefined or zero. This may be demonstrated using patterns and models. ● When determining the sum of: ○ a positive integer and a positive integer, the sum will be positive (e.g., 7 + 3 = 10). ○ a positive integer and a negative integer, the sum may be positive or negative (e.g., 7 + -1 = 6; 3 + -5 = -2). ○ a negative integer and a negative integer, the sum will be negative (e.g., -3 + (-4) = -7). ● When determining the difference of: ○ a positive integer and a positive integer, the difference may be positive or negative (e.g., 7 – 3 = 4; 2 – 5 = -3). ○ a positive integer and a negative integer, the difference will be positive (e.g., 7 – (-1) = 8). ○ a negative integer and a positive integer, the difference will be negative (e.g., -3 – 1) = -4). ○ a negative integer and a negative integer, the difference may be positive or negative (e.g., -1 – (-2) = 1; -6 – (-3) = -3).

- When determining the product of:
  - a positive integer and a positive integer, the product will be positive (e.g., $7 \cdot 2 = 14$).
  - a positive integer and a negative integer, the product will be negative (e.g., $6 \cdot (-3) = -18$).
  - a negative integer and a negative integer, the product will be positive (e.g., $-5 \cdot (-4) = 20$).
- When determining the quotient of:
  - a positive integer and a positive integer, the quotient will be positive (e.g., $14 \div 2 = 7$).
  - a positive integer and a negative integer, the quotient will be negative (e.g., $18 \div (-3) = -6$).
  - a negative integer and a positive integer, the quotient will be negative (e.g., $-15 \div 3 = -5$).
  - a negative integer and a negative integer, the quotient will be positive (e.g., $-20 \div (-2) = 10$).
- Students may justify their reasoning by using estimation strategies, models, benchmarks, etc.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Problem Solving: As students estimate, determine, and justify the solution to one- and two-step contextual problems, involving addition, subtraction, multiplication, and division with integers, it is important that multiple ways of solving problems are modeled to include graphic organizers, step-by-step processes, or modeling through concrete or pictorial representations and then make the transition to standard algorithms with each operation. Further, examining the reasonableness of the solution is a critical element to problem solving as students should go back to the contextual problem and determine whether their solution answered the question posed. As teachers provide support to students by helping them develop their problem-solving skills, consider the following –

- Ask students to discuss the following question: What do effective problem solvers do, and what do they do when they get stuck?
- How does your strategy for solving a problem affect the solution that you get?
- How do you most effectively communicate your mathematical ideas so that others can understand?
- How does the nature of the problem help you to determine the most appropriate way to solve it?
- Does creating a plan help you to become a better problem solver? Why or why not?
- Were there any other methods that you used to arrive at your answer?
- Is your answer reasonable? How do you know?
- When using concrete or pictorial representations, what is the most effective model for this problem, and what can we learn from it?

Mathematical Reasoning:

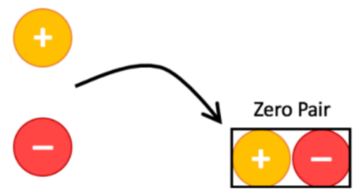
- Common misconceptions when completing integer operations include –
    - **Adding Integers:** Students may ignore the signs and just add the integers, or they may believe that adding two negative integers results in a positive answer.
    - **Subtracting Integers:** Students may ignore the signs and just subtract the integers. Students may think that subtracting two negative integers results in a negative answer. Students may think that subtraction is commutative and start with the "largest" number first instead of subtracting in the order given.
    - **Multiplying Integers:** Students may ignore the signs and just multiply the integers. Students may think that multiplying two negative integers results in a negative answer. Students may think that if the signs of the factors are different and the larger factor is positive, the answer is positive. Students may think that if the signs of the factors are different and the larger factor is negative, the answer is negative.
    - **Dividing Integers:** Students may ignore the signs and just divide the integers. Students may think that dividing two negative integers results in a negative answer. Students may think that if the signs different and the larger integer is positive, the answer is positive. Students may think that if the signs are different and the larger integer is negative, the answer is negative.

- Questions to elicit understanding of integer operations may include –
    - How do you add two integers if they have the same sign?
    - How do you add two integers if they have different signs?
    - How do you subtract integers?
    - How are the rules for multiplying and dividing integers different from the rules for adding and subtracting?
    - When multiplying or dividing integers, how do you know what the sign of your answer will be?
    - When you multiply three positive integers, what sign does the product have? Is this the same if you multiply three negative integers? Give an example to prove your answer.
    - Explain how to determine the sign of the product if the multiplication problem contains more than two factors.

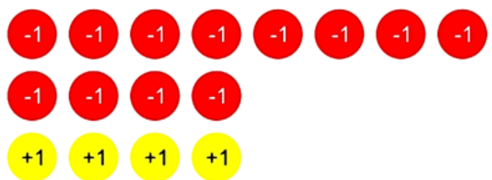
Mathematical Representations:

- Concrete experiences in formulating rules for adding, subtracting, multiplying, and dividing integers should be explored by examining patterns using calculators, number lines, and manipulatives (e.g., two-color counters, drawings, algebra tiles). Dynamic software can also be used to examine these patterns.
    - **When using two-color counters, students must understand the concept of zero pairs:** When modeling integers, one color can represent a positive number and another color can represent a negative number. In this instance and the throughout the following examples, a yellow

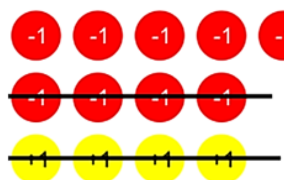
counter will represent a positive integer and a red counter will represent a negative integer. A zero pair is the pair of the positive and negative form of the same number.



- **When modeling addition with two-color counters:** For example, add -12 + 4.
  - Model the addends using counters. If the addend is positive, use yellow counters. If the addend is negative, use red counters.



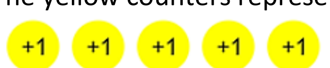
  - Equal amounts of red and yellow counters create zero pairs and cancel each other out.



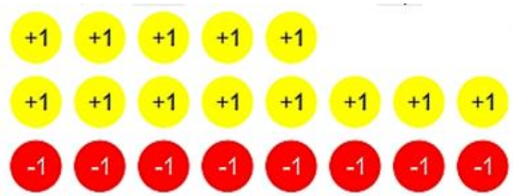
  - Determine the amount and color of the remaining counters and write the corresponding integer: -8

- **When modeling subtraction with two-color counters:** For example, subtract 5 – (-8).
  - If the minuend is positive, use yellow counters. If the minuend is negative, use red counters. Remove as many counters as indicated by the subtrahend. If the colored counters of the subtrahend are not present, create zero pairs and then subtract.

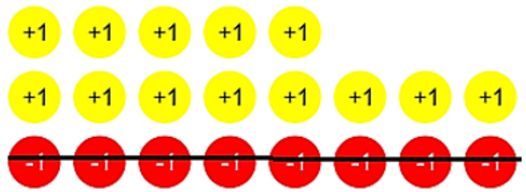
    The yellow counters represent positive 1. Use yellow counters to represent 5.



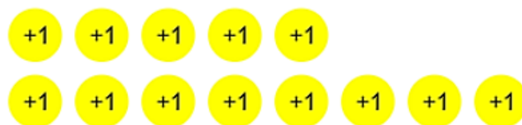
Remove 8 red counters. However, there are no red counters. Add 8 zero pairs (this is “adding the opposite”).



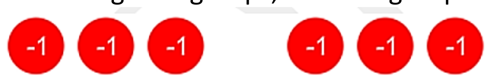
Remove 8 red counters.



Count the remaining counters to find the solution. Because there are thirteen positive yellow counters left, the answer is 13.



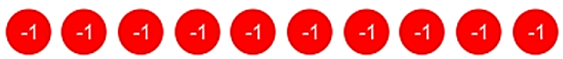
- **When modeling multiplication with two-color counters:**
  - **Multiplying two positive integers (Case 1):** Create as many groups of the second number using yellow counters as indicated by the first number.
  - **Multiplying two negative integers (Case 2)**: Create as many groups of the second number using red counters as indicated by the first number. Inverse the final answer as negative groups are the opposite of positive groups.
  - **Multiplying one positive and one negative integer (Case 3):** If the first number is positive and second is negative, create as many groups of the second number using red chips as indicated by the first number. If the first number is negative and second is positive, create as many groups of the second number using yellow chips as indicated by the first number. Inverse the final answer as negative groups are the opposite of positive groups.
  - **Example (Case 2): Multiplying two negative integers:** For example, multiply -2 × -3.
    Create as many groups of the second number using red chips as indicated by the first number. Create -2 groups of -3. As we cannot create negative groups, create 2 groups of -3.



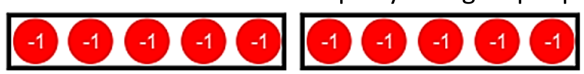
Inverse the final answer as negative groups are the opposite of positive groups. The result is 6.

- **When modeling division with two-color counters:**
  - **Dividing two positive integers (Case 1):** Use yellow integer counters to represent as many counters as indicated by the first number. Then, divide the set of counters equally into groups as specified by the second number.
  - **Dividing two negative integers (Case 2):** Use red integer counters to represent as many counters as indicated by the first number. Divide the set of counters equally into groups as specified by the second number. Inverse the groups created as negative groups are the opposite of positive groups.
  - **Dividing one positive and one negative integer (Case 3):** *If the dividend is positive and the divisor is negative:* Use yellow integer counters to represent as many counters as indicated by the first number. Divide the set of counters equally into groups specified by the second number. Inverse the groups created as negative groups are the opposite of positive groups. *If the dividend is positive and the divisor is positive:* Use red integer counters to represent as many counters as indicated by the first number. Divide the set of counters equally into groups specified by the second number.
    - For example, divide: -10 ÷ 2

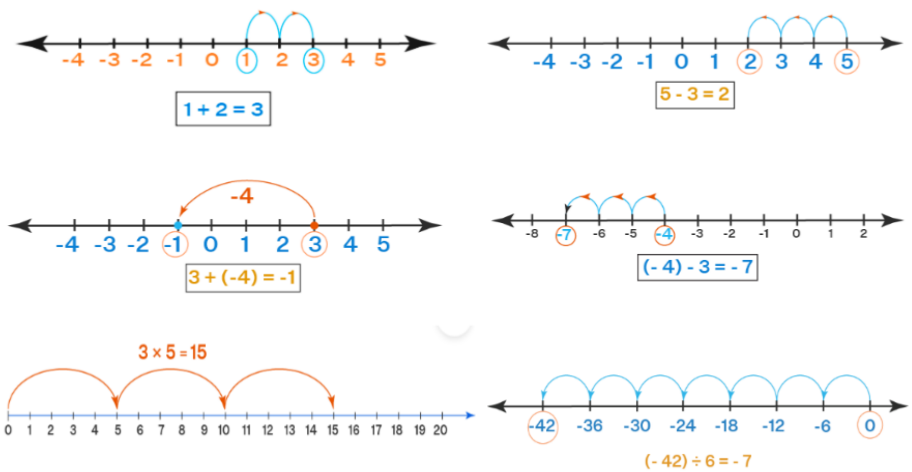
      Use red integer counters to represent as many counters as indicated by the first number.

      Divide the set of counters equally into groups specified by the second number. The result is -5.

- **When completing operations on a number line:** Key points to remember when completing operations on a number line – move to the right side, if the number is positive; and move to the left side, if the number is negative. Some examples of integer operations using number lines are pictured below.



- Ask students to create four multiple integer operations problems on an index card. One of the problems must use a concrete representation, the second a pictorial representation, the third a number line, and lastly, an algorithm. Allow the students to engage in a "give one, get one" exchange to solve one another's problems. Then, allow the students to present at least one of their problems to the class for whole group discussion.

- Use a number line to determine the sum or difference of two integers. Create a large number line on the floor with tape. Students can walk the number line to solve the problem. Ask students how their class number line would differ when multiplying and dividing integers and how they could physically model these two operations using their class number line.

Reference 6.CE.1 Skills in Practice for guidance related to* ***Mathematical Problem Solving, Mathematical Communication, Mathematical Reasoning**, and* ***Mathematical Connections*** *when solving contextual problems.*

Concepts and Connections
Concepts Estimation and the operations of addition, subtraction, multiplication, and division, allow us to model, represent, and solve different types of problems with rational numbers. **Connections**: In Grade 6, students will estimate, demonstrate, solve, and justify solutions to problems using operations with fractions and mixed numbers, including those in context (6.CE.1). Prior to Grade 6, students estimated, represented, solved, and justified solutions to single-step and multistep problems, including those in context, using addition and subtraction of whole numbers and fractions with like and unlike denominators (with and without models), and solved single-step contextual problems involving multiplication and a proper fraction, with models (5.CE.1, 5.CE.2). Further, students estimated, represented,

solved, and justified solutions to single-step and multistep problems, including those in context, using addition, subtraction, multiplication, and division with decimal numbers (5.CE.3). Using these foundational understandings, students will estimate, demonstrate, solve, and justify solutions using operations with integers, including those in context (6.CE.2). In the subsequent grade level, Grade 7 students will estimate, solve, and justify solutions to multistep contextual problems involving operations with rational numbers (7.CE.1) and solve problems, including those in context, involving proportional relationships (7.CE.2).

- *Within the grade level/course*:
  - 6.NS.2 – The student will reason and use multiple strategies to represent, compare, and order integers.
  - 6.CE.1 – The student will estimate, demonstrate, solve, and justify solutions to problems using operations with fractions and mixed numbers, including those in context.
- *Vertical Progression*:
  - 5.CE.1 – The student will estimate, represent, solve, and justify solutions to single-step and multistep contextual problems using addition, subtraction, multiplication, and division with whole numbers.
  - 5.CE.2 – The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition and subtraction of fractions with like and unlike denominators (with and without models), and solve single-step contextual problems involving multiplication of a whole number and a proper fraction, with models.
  - 5.CE.3 – The student will estimate, represent, solve, and justify solutions to single-step and multistep problems, including those in context, using addition, subtraction, multiplication, and division with decimal numbers.
  - 7.CE.1 – The student will estimate, solve, and justify solutions to multistep contextual problems involving operations with rational numbers.
  - 7.CE.2 – The student will solve problems, including those in context, involving proportional relationships.

<u>Textbooks and HQIM for Consideration</u>

- A list of approved textbooks and instructional materials will be posted on the VDOE website.

Measurement and Geometry

In K-12 mathematics, measurement and geometry allow students to gain understanding of the attributes of shapes as well as develop an understanding of standard units of measurement. Measurement and geometry are imperative as students develop a spatial understanding of the world around them. The standards in the measurement and geometry strand aim to develop students' special reasoning, visualization, and ability to apply geometric and measurement concepts in real-world situations.

In Grade 6, students analyze and describe geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes. At this grade level, students solve problems involving area and circumference of circles and solve problems involving the area and perimeter of triangles and parallelograms. In addition, students describe characteristics of the coordinate plane, graph ordered pairs, and determine congruence of segments, angles, and polygons.

6.MG.1 The student will identify the characteristics of circles and solve problems, including those in context, involving circumference and area.

Students will demonstrate the following Knowledge and Skills:

a) Identify and describe chord, diameter, radius, circumference, and area of a circle.
b) Investigate and describe the relationship between:
   i) diameter and radius;
   ii) radius and circumference; and
   iii) diameter and circumference.
c) Develop an approximation for pi (3.14) by gathering data and comparing the circumference to the diameter of various circles, using concrete manipulatives or technological models.
d) Develop the formula for circumference using the relationship between diameter, radius, and pi.
e) Solve problems, including those in context, involving circumference and area of a circle when given the length of the diameter or radius.

Understanding the Standard
● A chord is a line segment connecting any two points on a circle. A chord may or may not go through the center of a circle. The diameter is the longest chord of a circle. ● A diameter is a chord that goes through the center of a circle. The length of the diameter of a circle is twice the length of the radius. ● A radius is a line segment connecting the center of a circle to any point on the circle. Two radii end to end form a diameter of a circle.

- Perimeter is the path or distance around any plane figure. The perimeter of a circle is called the circumference.
- The circumference of a circle is about three times the measure of its diameter.
- The value of pi (π) is the ratio of the circumference of a circle to its diameter. Thus, the circumference of a circle is proportional to its diameter.
- The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle.
- The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve.
- The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle.
- When determining area and circumference of a circle, the calculation may vary depending upon the approximation for pi that is used. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi (π) button on a calculator.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Problem Solving:

- Stress that there are four steps to solving contextual problems related to circumference and area: determining which application should be used (when not explicitly stated to find the circumference and area), writing the formula, substituting the values, and solving including proper units.

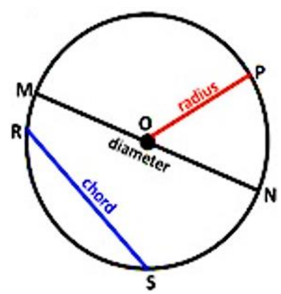
- The *Middle School Mathematics Formula Sheet* should be used in tandem with this standard to help students use the appropriate formula when solving stand-alone problems or problems in context.

- Questions to further elicit students' understanding of this standard are –
    - What is the term for the distance around a circle?
    - How do you determine the circumference and area of a circle?
    - What is the relationship between the diameter and the radius of a circle?
    - What is the relationship between the circumference of a circle and its diameter or radius?
    - How can the approximation for pi (π) be derived?

Mathematical Connections: Students must identify and describe the concepts of diameter, radius, chord, and circumference of a circle. Through this standard, students learn that circumference is the distance around, or "perimeter" of, a circle; an approximation for circumference is about three times the diameter of a circle; and an approximation for circumference is about six times the radius of a circle. Students learn to investigate and describe the relationship between (a) diameter and radius; (b) diameter and chord; (c) radius and circumference; and (d) diameter and circumference. Through these understandings, students must derive pi (π), and connect and apply their understanding of circumference and area of a circle by using the following formulas –

- $C = 2\pi r$ or $C = \pi d$, where C is the circumference, d is the diameter, and r is the radius of the circle.

- $A = \pi r^2$, where A is the area and r is the radius of the circle.



- To help students develop connections between circumference, area, radius, diameter, and pi (π) –
    - Open a discussion by asking students for the definitions of *circumference, chord, diameter,* and *radius*. Tell students that they will now investigate how the circumference of a circle compares to the circle's diameter (and radius).
    - Give each student cut-out circles of various sizes, a ruler, and a 3-foot length of yarn. Direct students to use the yarn to measure the distance around each circle, cutting the exact length of yarn needed for each circle. Then, have students use the ruler to measure the length of each piece of yarn. Emphasize that this is the circumference of each circle.
    - Have students fold each circle at some point, but *not* in half. Share with students that the line created is a chord, because the two endpoints both lie on the circle**.** Next, have students fold each circle in half, crease it, unfold it, and draw a line along the crease. Direct students to use their rulers to measure the length of this line across the center of each circle. Emphasize that this is the diameter of each circle and that a diameter is also a chord.
    - Have students divide the diameter of each circle in half. Emphasize that this is the radius of each circle.
    - Have students divide Length of Yarn by Length of Line for each circle. Advise students that they will determine if there is a relationship between the length of yarn (circumference) and the length of line (diameter)—the ratio of the circumference of a circle to its diameter. Ask

them what they observe about the circumference divided by the diameter of each circle. They should notice that each ratio is the whole number 3 followed by different numbers in the decimal places. Point out that they have discovered that the circumference of a circle is a little more than three times larger than the diameter of the same circle.

- Display the formula for circumference, $C = 2\pi r$, and explain each aspect of it as follows:
  - C = circumference (length of yarn)
  - π or pi = the ratio of the circumference of a circle to its diameter (ratio of length of yarn to length of line, or length of yarn divided by length of line)
  - $2r$ = radius multiplied by 2, which is the diameter (length of line)
- Have students use the Desmos calculator to find the exact circumference of each circle by substituting the known values into the formula and performing the indicated operation. Discuss with students how their results could differ slightly when using either the π button located on the calculator to arrive at their solutions or using the approximations for pi (3.14 or $\frac{22}{7}$).
- Next, have students use unit squares to fill in each circle without going beyond the edges. This will enable them to estimate the area of each circle. Considering that a square does not accommodate rounded edges, point out to students that they will have to estimate the amount of some of the squares being used. Share with students that the area of a closed curve is the number of non-overlapping square units required to fill the regions enclosed by the curve. Overlay graph paper over the circles, use small linking cubes, or dynamic geometry software to simulate square units.
- After students have completed their estimates of the area of each circle, introduce the formula for the area of a circle, $A = \pi r^2$, where r is the radius of the circle. Have students use the Desmos calculator to find the exact area of each circle by substituting the known values into the formula and performing the indicated operation. Discuss with students how their results could differ slightly when using either the π button located on the calculator to arrive at their solutions or using the approximations for pi (3.14 or $\frac{22}{7}$).

Mathematical Representations: Because students are expected to understand the components of a circle as well as connect the relationships between them, providing students with real-world applications to represent these relationships is key to grounding their understanding. For example –

- Given the proportional relationship between circumference and diameter, explain why $\frac{C}{d} = \pi$. Would the results of this proportional relationship be the same as $\frac{C}{2r} = \pi$? Explain why or why not.

- Have students to bring in circular objects and measure the distance around (circumference) and across (diameter) each object. Engage in a discussion regarding which object has the largest circumference and area (or smallest circumference and area). Allow students to work in small groups to derive pi (π), and to compare the circumference to the diameter. Allow students to exchange their items to verify each other's work.

- Ask students to create lengths of radii such that the area will be larger than the circumference or area will be smaller than the circumference.

Concepts and Connections

Concepts
Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.

Connections: In Grade 6, students will use reasoning to solve problems involving area and perimeter of triangles and parallelograms, including those in context (6.MG.2). Prior to Grade 6, students used multiple representations to solve problems, including those in context, involving perimeter, area, and volume (5.MG.2). Using these foundational understandings, students will identify the characteristics of circles and solve problems, including those in context, involving circumference and area (6.MG.1). In the subsequent grade level, Grade 7 students will investigate and determine the volume formula for right cylinders and the surface area formulas for rectangular prisms and right cylinders and apply the formulas in context (7.MG.1).

- *Within the grade level/course*:
  - 6.MG.2 – The student will reason mathematically to solve problems, including those in context, that involve the area and perimeter of triangles and parallelograms.
- *Vertical Progression*:
  - 5.MG.2 – The student will use multiple representations to solve problems, including those in context, involving perimeter, area, and volume.
  - 7.MG.1 – The student will investigate and determine the volume formula for right cylinders and the surface area formulas for rectangular prisms and right cylinders and apply the formulas in context.

<u>Textbooks and HQIM for Consideration</u>
- A list of approved textbooks and instructional materials will be posted on the VDOE website.

6.MG.2 The student will reason mathematically to solve problems, including those in context, that involve the area and perimeter of triangles and parallelograms.

Students will demonstrate the following Knowledge and Skills:

a) Develop the formula for determining the area of parallelograms and triangles using pictorial representations and concrete manipulatives (e.g., two-dimensional diagrams, grid paper).

b) Solve problems, including those in context, involving the perimeter and area of triangles and parallelograms.

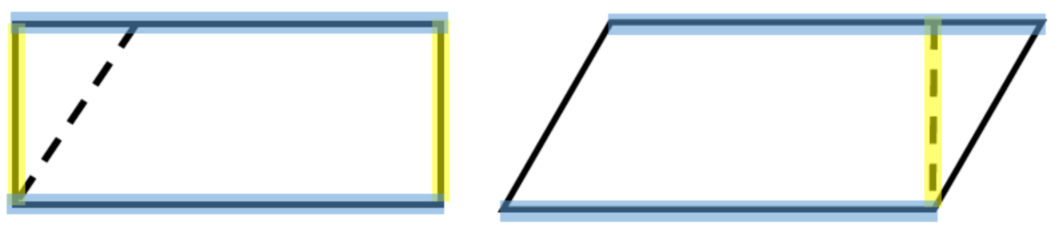
Understanding the Standard
• Experiences in developing the formulas for area and perimeter using manipulatives such as tiles, one-inch cubes, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and their use. • If a parallelogram is subdivided into two congruent right triangles and one rectangle, one of the right triangles from the original parallelogram can be repositioned to form a rectangle. The original parallelogram is now in the shape of a rectangle in which the area can be determined using the formula $A = bh$. • Any rectangle can be subdivided into two congruent right triangles by drawing a diagonal. Since the two resulting triangles are congruent, each triangle is exactly half of the area of the original rectangle. Hence, the area of a triangle can be determined using the formula $A = \frac{1}{2}bh$. • Perimeter is the path or distance around any plane figure. • The perimeter of a square whose side measures s can be determined by multiplying 4 by s ($P = 4s$), and its area can be determined by squaring the length of one side ($A = s^2$). • The perimeter of a rectangle can be determined by computing the sum of twice the length and twice the width ($P = 2l + 2w$, or $P = 2(l + w)$), and its area can be determined by computing the product of the length and the width ($A = lw$). • The perimeter of a triangle can be determined by computing the sum of the side lengths ($P = a + b + c$), and its area can be determined by computing one-half of the product of the base and the height ($A = \frac{1}{2}bh$).
Skills in Practice
While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented. **Mathematical Problem Solving:** • Stress that there are four steps to solving contextual problems related to perimeter and area: determining which application should be used (when not explicitly stated to find the perimeter and area), writing the formula, substituting the values, and solving including proper units.

- The *Middle School Mathematics Formula Sheet* should be used in tandem with this standard to help students use the appropriate formula when solving stand-alone problems or problems in context.

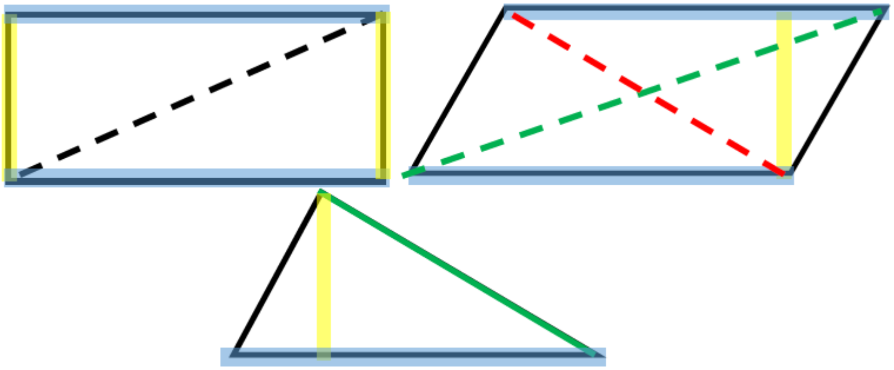
- Questions to further elicit students' understanding of this standard are –
  - How do you determine the perimeter and area of a triangle?
  - How do you determine the perimeter and area of a parallelogram?
  - Is there a difference between $A = lw$ or $A = bh$?
  - Describe the minimum amount of information needed to find the perimeter (or area) of a rectangle (or triangle or parallelogram).

Mathematical Connections: Students must develop the formula for determining the area of parallelograms and triangles using pictorial representations and concrete manipulatives. For example, start with a rectangle to derive the formulas for parallelograms and triangles –

- Give students three 3 x 5 index cards, scissors, tape, and two markers of different colors. Students will highlight the lengths and widths of each index card.
  - Rectangle (Index Card 1): Students will determine that the area of any rectangle is $A = lw$ or $A = bh$, where l is the length, w is the width; or b is the base and h is the height.
  - Parallelogram (Index Card 2): Students will determine that the area of any parallelogram is $A = bh$, where b is the base and h is the height by manipulating the rectangle into a parallelogram. To do this, draw a straight line from any point on the top edge, to a bottom vertex. Cut along this line (students should cut off a triangle). Have the students slide the triangle to the opposite side and then tape it in place.



  - Triangle (Index Card 3): Students will determine that the area of any triangle is determined by computing one-half of the product of the base and the height using $A = \frac{1}{2}bh$, where b is the base and h is the height. Have students take out the third index card. Ask them what the area is of it, and then have them use a marker or highlighter to mark the lengths (both) and the widths (both). Have them color along the edges of the card. Using their rulers, have the students draw in a diagonal of the figure (see pictures below). If students use the rectangle, then they will create two right triangles. If they use the parallelogram, then they will create either acute or obtuse triangles depending on whether they cut on the green diagonal, or the red one. *They should not cut along both diagonal*s.



- Questions to further elicit students' understanding and developing connections are –
  - How are the areas of a triangle and rectangle related? Explain and provide your reasoning.
  - How are the areas of rectangles and parallelograms related?
  - What is the relationship between the area of a triangle and the area of a rectangle?
  - What is the relationship between the area of a rectangle and a parallelogram?

Concepts and Connections

Concepts
Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.

Connections: In Grade 6, students will identify the characteristics of circles and solve problems, including those in context, involving circumference and area (6.MG.1). Prior to Grade 6, students used multiple representations to solve problems, including those in context, involving perimeter, area, and volume (5.MG.2). Using these foundational understandings, students will use reasoning to solve problems involving area and perimeter of triangles and parallelograms, including those in context (6.MG.2). In the subsequent grade level, Grade 7 students will investigate and determine the volume formula for right cylinders and the surface area formulas for rectangular prisms and right cylinders and apply the formulas in context (7.MG.1) and compare and contrast quadrilaterals based on their properties and determine unknown side lengths and angle measures of quadrilaterals (7.MG.3).

- *Within the grade level/course*:
  - 6.MG.1 – The student will identify the characteristics of circles and solve problems, including those in context, involving circumference and area.
- *Vertical Progression*:

- 5.MG.2 – The student will use multiple representations to solve problems, including those in context, involving perimeter, area, and volume.
- 7.MG.1 – The student will investigate and determine the volume formula for right cylinders and the surface area formulas for rectangular prisms and right cylinders and apply the formulas in context.
- 7.MG.3 – The student will compare and contrast quadrilaterals based on their properties and determine unknown side lengths and angle measures of quadrilaterals.

<u>Textbooks and HQIM for Consideration</u>

- A list of approved textbooks and instructional materials will be posted on the VDOE website.

6.MG.3 The student will describe the characteristics of the coordinate plane and graph ordered pairs.

Students will demonstrate the following Knowledge and Skills:

a) Identify and label the axes, origin, and quadrants of a coordinate plane.

b) Identify and describe the location (quadrant or the axis) of a point given as an ordered pair. Ordered pairs will be limited to coordinates expressed as integers.

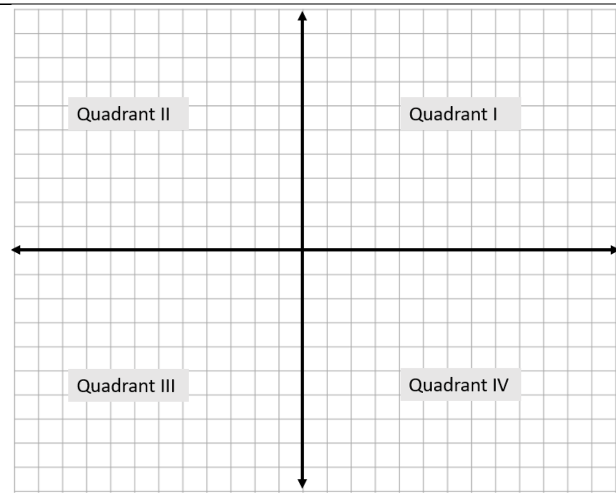
c) Graph ordered pairs in the four quadrants and on the axes of a coordinate plane. Ordered pairs will be limited to coordinates expressed as integers.

d) Identify ordered pairs represented by points in the four quadrants and on the axes of the coordinate plane. Ordered pairs will be limited to coordinates expressed as integers.

e) Relate the coordinates of a point to the distance from each axis and relate the coordinates of a single point to another point on the same horizontal or vertical line. Ordered pairs will be limited to coordinates expressed as integers.

f) Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate. Ordered pairs will be limited to coordinates expressed as integers. Apply these techniques in the context of solving contextual and mathematical problems.

Understanding the Standard
• In a coordinate plane, the coordinates of a point are typically represented by the ordered pair (*x, y*), where *x* is the first coordinate and *y* is the second coordinate. • Any given point is defined by only one ordered pair in the coordinate plane. • The grid lines on a coordinate plane are perpendicular. • The axes of the coordinate plane are the two intersecting perpendicular lines that divide the coordinate plane into four quadrants. The *x*-axis is the horizontal axis, and the *y*-axis is the vertical axis. • The quadrants of a coordinate plane are the four regions created by the two intersecting perpendicular lines (*x*- and *y*-axes). Quadrants are named in counterclockwise order. The signs on the ordered pairs for quadrant I are (+,+); for quadrant II (–,+); for quadrant III (–, –); and for quadrant IV (+,–).



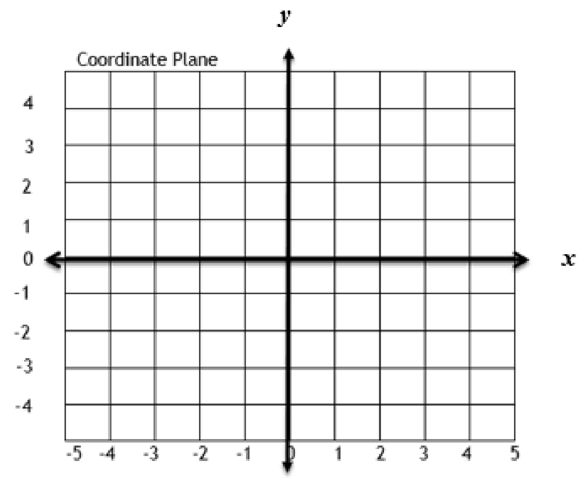
- In a coordinate plane, the origin is the point at the intersection of the *x*-axis and *y*-axis; the coordinates of this point are (0, 0).
- For all points on the *x*-axis, the *y*-coordinate is 0. For all points on the *y*-axis, the *x*-coordinate is 0.
- The coordinates may be used to name a point (e.g., the point (2, 7)). It is not necessary to say, "The point whose coordinates are (2, 7)." The first coordinate tells the location or distance of the point to the left or right of the *y*-axis and the second coordinate tells the location or distance of the point above or below the *x*-axis. For example, (2, 7) is two units to the right of the *y*-axis and seven units above the *x*-axis.
- Coordinates of points having the same *x*-coordinate are located on the same vertical line. For example, (2, 4) and (2, -3) are both two units to the right of the *y*-axis and are vertically seven units from each other.
- Coordinates of points having the same *y*-coordinate are located on the same horizontal line. For example, (-4, -2) and (2, -2) are both two units below the *x*-axis and are horizontally six units from each other.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Reasoning: As students draw polygons in the coordinate plane given coordinates for the vertices and use coordinates to determine the length of a side joining points with the same first coordinate or the same second coordinate, it is important to consider misconceptions students may have in determining the distance between coordinates. When looking at the length of a line segment between given points, students may add the *x*- or *y*-coordinates to determine the distance rather than considering the absolute value of the distance of a point from the *x*- or *y*-axis and each other. For example –

Parker needs to draw a model of his backyard on the coordinate plane.
Use the ordered pairs to draw the model of his backyard: A(-2, 3), B(4, 0), C(4, -3), and D(-2, 0). What are the lengths of $\overline{AD}$ and $\overline{DB}$?



Teachers may wish to encourage students to trace the length from point A to point D with a colored pencil and count the units as they are tracing. Students should associate this length with how many units each point is away from the axis.

Mathematical Representations: Students must practice identifying and labeling the axes, origin, and quadrants of a coordinate plane. One common error in labeling the quadrants is that students may go clockwise around the coordinate plane to label the quadrants as I, II, III, and IV rather than counterclockwise. To help students, have them create various models of the coordinate plane with labels to make the vocabulary more meaningful. For example –

- Label the classroom as a giant coordinate plane by using tape to mark the axes and vocabulary cards to note the key terms. Label the four corners of your room as the four quadrants and have students play four corners to review which quadrant is which and later which quadrant a given ordered pair belongs to.

- Consistent review of vocabulary, including horizontal vs. vertical in relation to the axes, will help to keep students aware of terms. For example –

  Describe the direction of the *x*-axis and *y*-axis. Which is the horizontal axis, and which is the vertical axis?

  Students often confuse the *x*- and *y*-axes and the terms horizontal and vertical to describe them. Making a large model of the coordinate plane in the classroom and labeling the axes as both *x*- and *y*-, as well as horizontal and vertical, can provide a powerful visual to help students distinguish between and identify the axes correctly. After working with a larger model, having students create their own models with geoboards and/or grid paper with labels will provide a concrete example that students may refer to as needed. Additionally, referring to objects in the class and asking students to describe features of the objects as horizontal or vertical will provide real world practice (e.g., Is the bottom edge of a bulletin board running horizontal or vertical?).

- As students work through this standard, the following questions may be helpful in addressing misconceptions –
  - How can you determine the quadrant in which an ordered pair should be placed without plotting the point?
  - What is the same about the four quadrants? How are the four quadrants different from each other?
  - How do you graph a particular point in a coordinate plane?
  - How do you identify the ordered pair of a particular point in a coordinate plane? How do you know you are correct?
  - Where is the origin located on a coordinate plane?
  - How can you find the distance between points on the same horizontal or vertical line?
  - Can any given point be represented by more than one ordered pair?
  - In naming a point in the coordinate plane, does the order of the two coordinates matter?
  - How can you determine whether a polygon on a coordinate plane is a regular polygon using the ordered pairs?

Concepts and Connections

Concepts
Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.

Connections: In sixth grade, students add, subtract, multiply, and divide two integers. Beginning in Grade 6, students describe the characteristics of the coordinate plane and graph ordered pairs. Students use integers to plot points in all four quadrants and complete computation with integers as they determine the lengths of line segments in the coordinate plane. In the subsequent grade level, Grade 7 students will apply dilations of polygons in the coordinate plane (7.MG.4).

- *Within the grade level/course*:
  - 6.CE.2b – Add, subtract, multiply, and divide two integers.
- *Vertical Progression*:
  - There are no formal standards that address describing the characteristics of the coordinate plane and graphing ordered pairs in previous grade levels.
  - 7.MG.4 – The student will apply dilations of polygons in the coordinate plane.

<u>Textbooks and HQIM for Consideration</u>

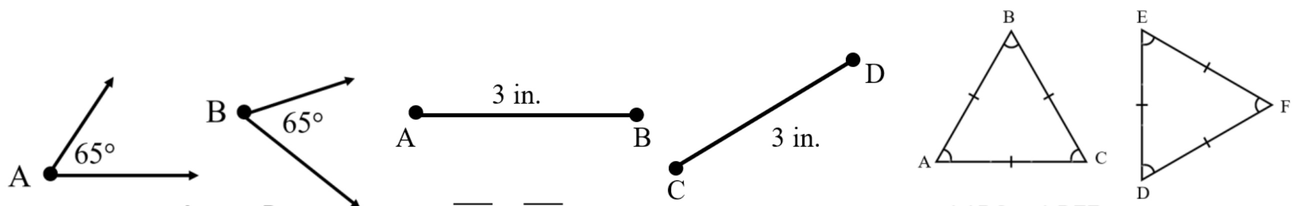
- A list of approved textbooks and instructional materials will be posted on the VDOE website.

6.MG.4 The student will determine congruence of segments, angles, and polygons.

Students will demonstrate the following Knowledge and Skills:

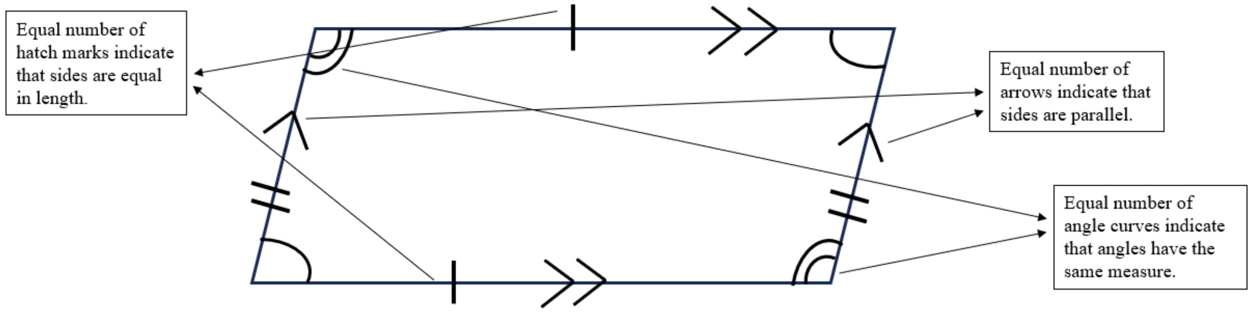
a) Identify regular polygons.
b) Draw lines of symmetry to divide regular polygons into two congruent parts.
c) Determine the congruence of segments, angles, and polygons given their properties.
d) Determine whether polygons are congruent or noncongruent according to the measures of their sides and angles.

Understanding the Standard
• The symbol for congruency is ≅.
• Congruent figures have the same size and the same shape. Angles are congruent if they have the same measure. Line segments are congruent if they have the same length. Polygons are congruent if they have an equal number of sides, and all the corresponding sides and angles are congruent.



$\angle A \cong \angle B$ $\overline{AB} \cong \overline{CD}$ $\Delta ABC \cong \Delta DEF$

- A polygon is a closed plane figure composed of at least three line segments that do not cross.
- A regular polygon has congruent sides and congruent interior angles. The number of lines of symmetry of a regular polygon is equal to the number of sides of the polygon.
- A line of symmetry divides a figure into two congruent parts, each of which is the mirror image of the other. Lines of symmetry are not limited to horizontal and vertical lines.
- Noncongruent figures may have the same shape but not the same size.
- Students should be familiar with geometric markings in figures to indicate congruence of sides and angles and to indicate parallel sides. An equal number of hatch (hash) marks indicate that those sides are equal in length. An equal number of arrows indicate that those sides are parallel. An equal number of angle curves indicate that those angles have the same measure. See the diagram below.



- The determination of the congruence or noncongruence of two figures can be accomplished by placing one figure on top of the other or by comparing the measurements of all corresponding sides and angles.
- Construction of congruent angles, line segments, and polygons helps students understand congruency.

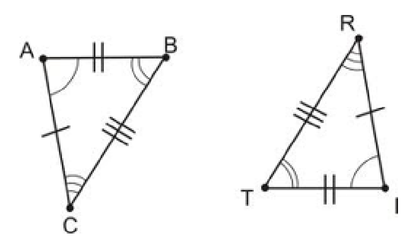
Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Reasoning: When determining congruence or noncongruence of two figures, students need practice comparing figures or components of figures and place them on top of the other or compare the measurements of all corresponding sides and angles. An example has been provided with common misconceptions –

Look at the triangles below and complete the statements. Are the two triangles congruent? If so, explain your reasoning.

$\angle C \cong \angle$ ______
$\overline{AB} \cong$ ________
$\angle T \cong \angle$ ______
$\overline{RT} \cong$ ________



Some students may have difficulty visualizing the corresponding congruent sides and angles when shapes are rotated or reflected. This may indicate that a student struggles with spatial relationships and potentially with geometric markings that indicate congruency. Teachers may wish to utilize the word wall cards and/or co-create anchor charts with the students to help solidify this information. Exploring congruent polygons with manipulatives and being able to place them on top of each other may assist students in understanding the characteristics that make them congruent. Students could also be encouraged to color code corresponding sides and angles to assist with determining congruency.

Mathematical Representations:

- **Determining congruence:** When determining congruence of segments, angles, and polygons, students may have a difficult time understanding that the shape and the angles must be the same to be congruent. Further, students may confuse congruent with similar. It is important for students to fully understand that line segments and other shapes may be congruent even if they look different because they are oriented differently. If students need reinforcement with this concept, have them practice reorienting pairs of congruent line segments and pairs of congruent figures. As a strategy –

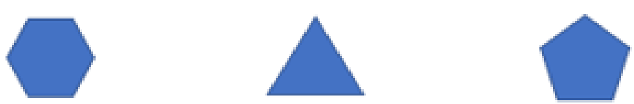
    - Distribute patty paper or tracing paper and permanent markers. Have students trace on the paper one of the polygons in each pair, using a permanent marker, and compare the pair of polygons by placing the traced polygon on top of the other polygon in the pair. If they are an exact match in size and shape, then the two polygons are congruent; if the two polygons differ in size and/or shape, then the two polygons are non-congruent.

    - As congruent and non-congruent figures are defined –

        - Discuss the geometric markings for congruency (≅) and similarity (~). For example, figure ABC is congruent to figure DEF (ABC≅DEF) or figure LMN is similar to figure PQR (LMN ~ PQR).

        - Discuss the geometric markings on figures that indicate congruence of length (hash marks), angle measure (arcs) and parallel sides (arrows).

- **Drawing lines of symmetry to divide regular polygons into two congruent parts:** Students must understand that lines of symmetry must be either be horizontal or vertical; and visualize rotations of polygons to determine lines of symmetry. For example, if given the following polygons, ask students to draw all lines of symmetry –



Student struggling with drawing the lines of symmetry may need hands on experiences with paper folding to help them conceptualize the two congruent parts created by the line of symmetry. Many times, students see the polygon in only one way, giving them a limited number of lines of symmetry that are visible. For example, they draw a line from the top of the triangle to the bottom, not realizing that there are two additional lines if they turn the triangle. Hands on exploration helps students visualize the multiple lines of symmetry. Once students complete the exploration with a few regular polygons, they usually discover that the number of lines of symmetry for regular polygons is the same as the number of sides and angles.

Concepts and Connections

Concepts
Analyzing and describing geometric objects, the relationships and structures among them, or the space that they occupy can be used to classify, quantify, measure, or count one or more attributes.
 	
Content Connections: Prior to Grade 6, in the elementary grades (particularly Grade 2), students identified, described, and created plane figures (including circles, triangles, squares, and rectangles) that had at least one line of symmetry and explained its relationship with congruency (2.MG.3). Using these foundational understandings, students will determine congruence of segments, angles, and polygons (6.MG.3). In the subsequent grade level, Grade 7 students will solve problems and justify relationships of similarity using proportional reasoning (7.MG.2).
 	
• *Within the grade level/course*:
  ○ There are no horizontal connections.
• *Vertical Progression*:
  ○ 2.MG.3 – The student will identify, describe, and create plane figures (including circles, triangles, squares, and rectangles) that have at least one line of symmetry and explain its relationship with congruency.
  ○ 7.MG.2 – The student will solve problems and justify relationships of similarity using proportional reasoning.

<u>Textbooks and HQIM for Consideration</u> • A list of approved textbooks and instructional materials will be posted on the VDOE website.

Probability and Statistics

In K-12 mathematics, probability and statistics introduce students to the concepts of uncertainty and data analysis. Probability involves understanding the likelihood of events occurring, often using concepts such as experiments, outcomes, and the use of fractions and percentages. The formal study or probability begins in Grade 4. Statistics focuses on collecting, organizing, and interpreting data and includes a basic understanding of graphs and charts. Probability and statistics help students analyze and make sense of real-world data and are fundamental for developing critical thinking skills and making informed decisions using data.

In Grade 6, students will understand that the world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena. At this grade level, students use statistical investigation to determine experimental and theoretical probability and apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graphs. Further, students represent the mean as a balance point and describe how statistical measures are affected when a data value is added, removed, or changed.

6.PS.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graphs.

Students will demonstrate the following Knowledge and Skills:

a) Formulate questions that require the collection or acquisition of data with a focus on circle graphs.

b) Determine the data needed to answer a formulated question and collect the data (or acquire existing data) using various methods (e.g., observations, measurement, surveys, experiments).

c) Determine the factors that will ensure that the data collected is a sample that is representative of a larger population.

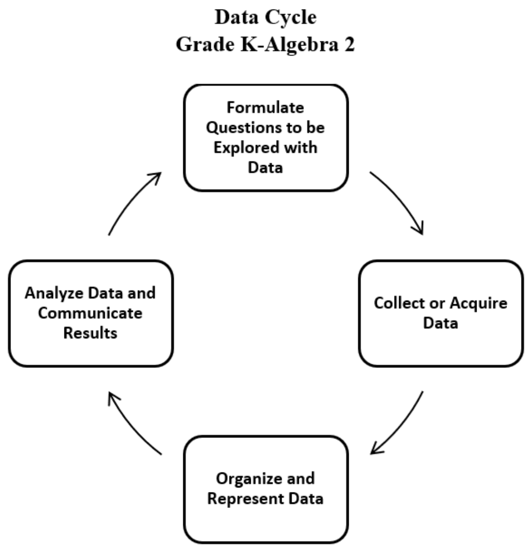
d) Organize and represent data using circle graphs, with and without the use of technology tools. The number of data values should be limited to allow for comparisons that have denominators of 12 or less or those that are factors of 100 (e.g., in a class of 20 students, 7 choose apples as a favorite fruit, so the comparison is 7 out of 20, $\frac{7}{20}$, or 35%).

e) Analyze data represented in a circle graph by making observations and drawing conclusions.

f) Compare data represented in a circle graph with the same data represented in other graphs, including but not limited to bar graphs, pictographs, and line plots (dot plots), and justify which graphical representation best represents the data.

Understanding the Standard

- Students should explore the entire data cycle with a question and set of data that has been collected or acquired. Student reflection should occur throughout the data cycle. The data cycle includes the following steps: formulating questions to be explored with data, collecting or acquiring data, organizing and representing data, and analyzing and communicating results.

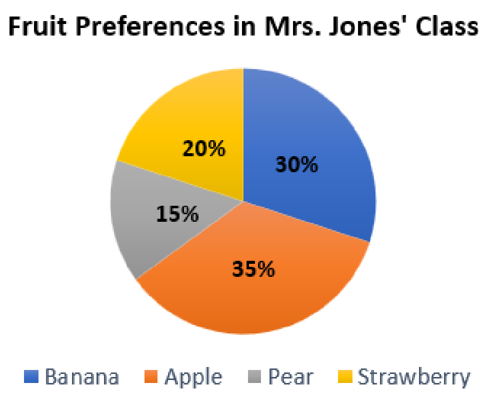


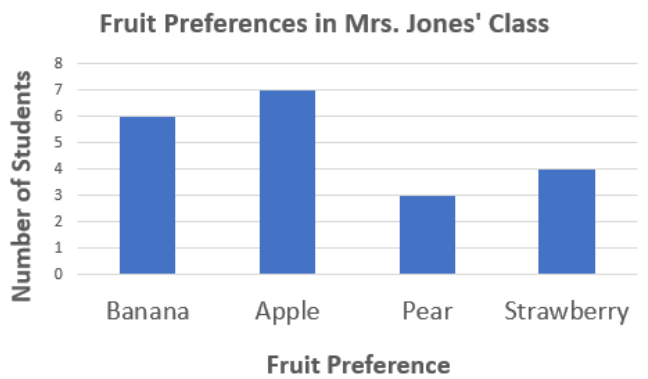
- There are many methods to collect data for any problem situation. These may include experiments, surveys, observations, or other data-gathering strategies. The data can be organized, displayed, analyzed, and interpreted to solve the problem.
- The teacher can provide data sets to students in addition to students engaging in their own data collection or acquisition.
- A population is the entire set of individuals or items from which data is drawn for a statistical study.
- A sample is a data set obtained from a subset that represents an entire population. This data set can be used to make reasonable assumptions about the whole population.
- Sampling is the process of selecting a suitable sample, or representative part of a population, for the purpose of determining characteristics of the whole population. A cursory overview of sampling is intended for Grade 6.
- An example of a population would be the entire student body at a school, whereas a sample might be selecting a subset of students from each grade level. A sample may or may not be representative of the population as a whole. Questions to consider when determining whether an identified sample is representative of the population include:
    - What is the target population of the formulated question?

- Who or what is the subject or context of the formulated question?

- Examples of questions to consider in building good samples:
    - What is the context of the data to be collected?
    - Who is the audience?
    - What amount of data should be collected?
- A circle graph is used for categorical and discrete numerical data. Circle graphs are used for data showing a relationship of the parts to the whole.
    - Example: The favorite fruit of 20 students in Mrs. Jones' class was recorded in a table. Compare the same data displayed in both a circle graph and a bar graph.

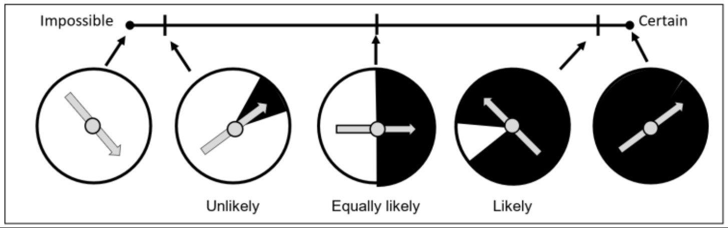
Fruit Preference	# of students
banana	6
apple	7
pear	3
strawberry	4





- Circle graphs can represent percent or frequency.
- Circle graphs are not effective for representing data with large numbers of categories.
- Teachers should be reasonable about the selection of data values. The number of data values can affect how a circle graph is constructed (e.g., 10 out of 25 would be 40%, but 7 out of 9 would be $77.\overline{7}\%$, making the construction of a circle graph more complex). Students should have experience constructing circle graphs, but a focus should be placed on the analysis of circle graphs.
- Students are not expected to construct circle graphs by multiplying the percentage of data in a category by 360° in order to determine the central angle measure. Limiting comparisons to fraction parameters noted in the standard will assist students in constructing circle graphs.

- Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data.
- Circle graphs must include a title, percent or number labels for data categories, and a key. A key is essential to explain how to read the graph. A title is essential to explain what the graph represents.
- Circle graphs can be created in programs such as Excel or Google spreadsheets. Some programs refer to circle graphs as pie charts.
- In previous grades, students had experience with pictographs, bar graphs, line graphs, line plots and stem-and-leaf plots. In Grade 6, students are not expected to construct these graphs.
  - A pictograph is used to show categorical data. Pictographs are used to show frequency and compare categories.
  - A bar graph is used for categorical data and is used to show comparisons between categories.
  - A line graph is used to show how numerical data changes over time.
  - A line plot provides an ordered display of all values in a data set and shows the frequency of data on a number line. Line plots are used to show the spread of the data, to include clusters (groups of data points) and gaps (large spaces between data points), and quickly identify the range, mode, and any extreme data values.
  - A stem-and-leaf plot uses columns to display a summary of discrete numerical data while maintaining the individual data points. A stem-and-leaf plot displays data to show its shape and distribution.
- Different situations call for different types of graphs (e.g., visual representations). The way data are displayed is often dependent upon what question is being investigated and what someone is trying to communicate.
- Comparing different types of representations (e.g., charts, graphs, line plots) provides students with opportunities to learn how different graphs can show different aspects of the same data. Following the construction of representations, discussions around what information each representation provides or does not provide should occur.
- The information displayed in different graphs may be examined to determine how data are or are not related, differences between characteristics (comparisons), trends that suggest what new data might be like (predictions), and/or questions such as “What could happen if…” (inferences).
- Connections can be made with probability and drawing conclusions from a circle graph.
- In general, if all outcomes of an event are equally likely, the probability of an event occurring is equal to the ratio of desired outcomes to the total number of possible outcomes in the sample space.
- The probability of an event occurring can be represented as a ratio or equivalent fraction, decimal, or percent.
- Based on the data in the circle graph, the likelihood of an event can be determined as impossible, unlikely, equally likely, likely, and certain.



Skills in Practice
While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Communication: There are many good questions students can ask about circle graphs. Not all questions can be answered with the types of data displays that are the focus at this grade.

- As students are formulating questions, they should consider –
  - What type of data can be collected for this question?
  - Would the data collected make sense to display in a circle graph?
  - Is the data easily categorized – *What is your favorite flavor of ice cream? Choose from the given list. Which video game is your favorite?*
  - OR is the data discrete numerical data – *How many siblings do you have? How many hours per day do you spend on a screen?*

- Circle graphs require a clear "whole."
  - Do I know what the "whole" is for this situation?
  - Can my question be answered with percentages?

- Data displays are intended to provide a simplified display of the data. (It is challenging to digest the raw data.) However, most data displays will lose details.
  - Does a circle graph address your question? The labels are key to communication.
  - What title and labels are necessary to clearly communicate?
  - Do I need to include totals, or just percentages?

Mathematical Reasoning: Students can find real-life examples of circle graphs in newspapers, magazines, or online sources. They should analyze the data represented in the circle graphs and write a short paragraph explaining the insights they gained from the graphs. At this grade, it would be appropriate to formulate a hypothesis about the relationship between two variables based on the information presented in a circle graph and design an experiment to test it. This will require students to draw conclusions about a given circle graph and explain how the data supports their conclusions. Further, students should assess the effectiveness of using a circle graph to represent a specific set of data and justify their evaluation.

Mathematical Representations: Students should be given opportunities to look at the same data presented in different graphs and determine which is the most effective representation. Students could also consider which question types would be best represented by different types of graphs. Students should return to their question to see if their data answers the question and if not, consider at which point in the cycle they would want to modify their process to gather better data or represent the data in a more meaningful way.

Concepts and Connections

Concepts
The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.
 	
Connections: In sixth grade, students will represent the mean as a balance point and determine the effect on statistical measures when a data point is added, removed, or changed (6.PS.2) which may have implications impacting the how data may be displayed. Prior to Grade 6, students applied the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line plots (dot plots) and stem-and-leaf plots (5.PS.1). Using these foundational understandings, students will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graph (6.PS.1). In the subsequent grade level, Grade 7 students will apply the data cycle with a focus on histograms (7.PS.1).
 	
• *Within the grade level/course*:
  ○ 6.PS.2 – The student will represent the mean as a balance point and determine the effect on statistical measures when a data point is added, removed, or changed.
• *Vertical Progression*:
  ○ 5.PS.1 – The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line plots (dot plots) and stem-and-leaf plots.
  ○ 7.PS.1 – The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on histograms.

Textbooks and HQIM for Consideration • A list of approved textbooks and instructional materials will be posted on the VDOE website.

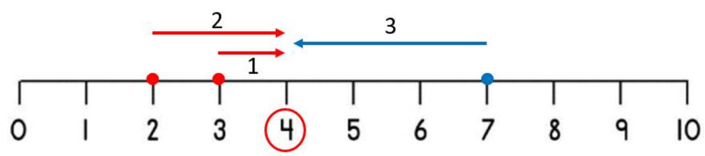
6.PS.2 The student will represent the mean as a balance point and determine the effect on statistical measures when a data point is added, removed, or changed.

Students will demonstrate the following Knowledge and Skills:

a) Represent the mean of a set of data graphically as the balance point represented in a line plot (dot plot).

b) Determine the effect on measures of center when a single value of a data set is added, removed, or changed.

c) Observe patterns in data to identify outliers and determine their effect on mean, median, mode, or range.

Understanding the Standard
● Categorical data can be sorted into groups or categories while numerical data are values or observations that can be measured. For example, types of fish caught would be categorical data while weights of fish caught would be numerical data. ● Measures of center are types of averages for a data set. They represent numbers that describe a data set. Mean, median, and mode are measures of center that are useful for describing different situations. ● Mean may be appropriate for sets of data where there are no values much higher or lower than those in the rest of the data set. ● Median may be appropriate when data sets have some values that are much higher or lower than most of the other values in the data set. The median is the middle value of a data set in ranked order. If there are an odd number of pieces of data, the median is the middle value. If there are an even number of pieces of data, the median is the numerical average of the two middle values. ● Mode may be appropriate when the set of data has some identical values, when data is categorical, or when the data reflect the most popular option. The mode is the piece of data that occurs most frequently. If no value occurs more often than any other, there is no mode. If there are multiple values that occur most often, each of these values is a mode. When there are exactly two modes, the data set is bimodal. ● Mean can be defined as the point on a number line where the data distribution is balanced. This requires that the sum of the distances from the mean of all the points above the mean is equal to the sum of the distances from the mean of all the data points below the mean. This is the concept of mean as the balance point. ○ Example: Given the data set: 2, 3, 7, the mean value of 4 can be represented on a number line as the balance point:  ● The mean can also be found by calculating the numerical average of the data set. ● In Grade 5 mathematics, students had experiences defining the mean as fair share. ● Defining mean as the balance point is a prerequisite for understanding standard deviation, which is addressed in high school level mathematics.

- An outlier can be identified by sorting the data in ascending order. A data value that is an abnormal distance relative to the other values in the data set is an outlier. It represents a value that "lies outside" (is much smaller or larger than) most of the other values in a set of data. Outliers have a greater effect on the mean and range of a data set but have less of an effect on the median or mode.
- In Grade 6, students are not expected to mathematically determine outliers. Instead, at this level, they are expected to visually determine outliers when provided a representation of a data set.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Reasoning: Although students at this grade level are not expected to mathematically determine outliers, they are expected to visually determine outliers when given a data set. Students must understand that an outlier lies outside (much smaller or larger than) most of the values in the data set. Outliers impact the mean and range of the data set, but have less of an effect on the median or mode. Students should engage in problems that have outliers, and determine the effect on measures of center when a single value of a data set is added, removed, or changed. For example –

Amber has 5 tests in science this semester. Her scores on these tests are 93, 99, 83, 100, and 65.

- **What are the mean, median, and mode of Amber's science test scores?** Some students may obtain an incorrect mean, median, or mode. Some students may not realize that it is possible to have no mode for a data set or will represent "no mode" by standing that the mode is 0; or, may not put the numbers in ascending order to calculate median. These students may need additional review on how to calculate these measures of center and what they mean in terms of representing a data set.

- **Amber retook the test on which she received a 65 and earned an 85. How did changing this one test score affect the mean, median, and mode?** In changing the data point from 65 to 85 students may accidentally add the 85 rather than replacing the 65. The use of manipulatives to model a situation like this can aid students as they will physically remove one data point and add another. The students should indicate that the mean became higher, while the median became lower. The teacher may wish to engage students in discussions about why this happened. *Does this always happen? What would happen if a different data point was changed?*

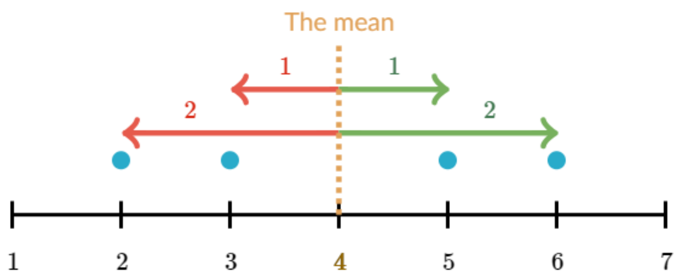
- **Which measure of center do you think Amber would want the teacher to use for her report card grade? Explain your thinking.:** A student's explanation will give the teacher insight into understanding and/or misconceptions. In this situation, Amber would want the teacher to use the mean since it increased. Students would benefit from investigating different scenarios and their effect on the measures of center as well as which measures of center better represent a data set.

Mathematical Communication: As students engage with this standard, use the following questions to elicit student discourse as students examine data sets, interpret the data rendered, and share out their results –

- How does the mean balance the distribution of a data set?
- How does the mean summarize the center of a distribution?
- How are the mean, median, mode, and range effective in describing a data set?
- Are there limitations to the measures of center (mean, median, and mode)? If so, what?
- Is there a limitation to the measure of variability (or spread) (range)? If so, what?
- How can a line plot (dot plot) help us make sense of this interpretation of the mean as a balance point?
- Could another value, besides the mean, balance a data distribution? How can we tell?

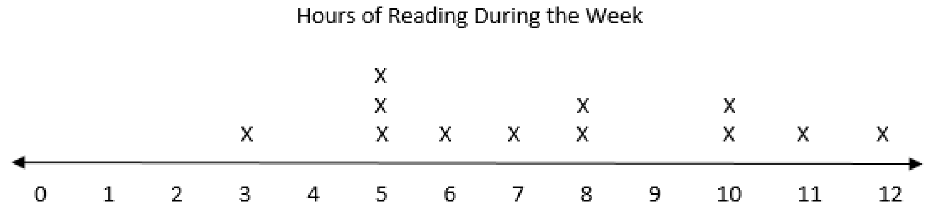
Mathematical Representations:

- It is important for students to think of the mean as the balance point. The total distance from the mean to the data points below the mean is equal to the total distance from the mean to the data points above the mean. For example, when determining the mean of {2, 3, 5, 6} (which is 4), it can be observed that the total distance from the mean to the data points below the mean is equal to the total distance from the mean to the data points above the mean because **1** + **2** = **1** + **2**. It is always true that the total distance below the mean is equal to the total distance above the mean.



- When students develop line plots, it is important for students to understand that the mean is not always a whole number – a common misconception. For example –

  Ms. Rogers made a line plot of how many hours each of her students read during the week. She organized the data into the line plot below. What is the balance point for the data? Each X represents one student.



Some students may identify the balance point as 7 or 8, not realizing that it could be a decimal. In this example, the balance point lies between two of the points on the line plot (7 and 8), and the balance point is 7.5. Provide students with experience locating a balance point in which the result is not always a whole number. Students also may find the mean using computation (algorithm) or a calculator and note the balance point using that quotient. The intent of this standard is for students to define mean as the balance point as it is a prerequisite for understanding standard deviation, which is addressed in high school level mathematics. Hands-on practice as noted above will help students conceptualize this idea of balance point.

Concepts and Connections

Concepts
The world can be investigated through posing questions and collecting, representing, analyzing, and interpreting data to describe and predict events and real-world phenomena.

Connections: In Grade 6, students will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graph (6.PS.1). Prior to Grade 6, students applied the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on line plots (dot plots) and stem-and-leaf plots (5.PS.1).
Using these foundational understandings, students will represent the mean as a balance point and determine the effect on statistical measures when a data point is added, removed, or changed (6.PS.2). Students will continue their application of measures of center and spread in Algebra 2 (A2.ST.1h).

- *Within the grade level/course*:
  - 6.PS.1 – The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graph.
- *Vertical Progression*:
  - 5.PS.2 – The student will solve contextual problems using measures of center and the range.
  - A2.ST.1h – Determine the solution to problems involving the relationship of the mean, standard deviation, and z-score of a data set represented by a smooth or normal curve.

Textbooks and HQIM for Consideration

- A list of approved textbooks and instructional materials will be posted on the VDOE website.

Patterns, Functions, and Algebra

In K-12 mathematics, the patterns, functions, and algebra strand focuses on the recognition, description, and analysis of patterns, functions, and algebraic concepts. Students develop an understanding of mathematical relationships and represent these using symbols, tables, graphs, and rules. In later grades, students use models as they solve equations and inequalities and develop an understanding of functions. This strand is designed to develop students' algebraic thinking and problem-solving skills, laying the foundation for more advanced mathematical concepts.

In Grade 6, students learn that proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic equations and inequalities can be used to represent and solve real world problems. At this grade level, students will use ratios to represent relationships between quantities; identify and represent proportional relationships between two quantities; create and solve one-step linear equations in one-variable; and represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line.

6.PFA.1 The student will use ratios to represent relationships between quantities, including those in context.

Students will demonstrate the following Knowledge and Skills:

a) Represent a relationship between two quantities using ratios.
b) Represent a relationship in context that makes a comparison by using the notations $\frac{a}{b}$, *a:b*, and *a* to *b.*
c) Represent different comparisons within the same quantity or between different quantities (e.g., part to part, part to whole, whole to whole).
d) Create a relationship in words for a given ratio expressed symbolically.
e) Create a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio.
f) Create a table of equivalent ratios to represent a proportional relationship between two quantities, when given a contextual situation.

Understanding the Standard
• A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities. Ratios are used in contextual situations when there is a need to compare quantities. • In the elementary grades, students are taught that fractions represent a part-to-whole relationship. However, fractions may also express a measurement, an operator (multiplication), a quotient, or a ratio. Examples of fraction interpretations include: ○ fractions as parts of wholes: $\frac{3}{4}$ represents three parts of a whole, where the whole is separated into four equal parts; ○ fractions as measurement: the notation $\frac{3}{4}$ can be interpreted as three one-fourths of a unit;

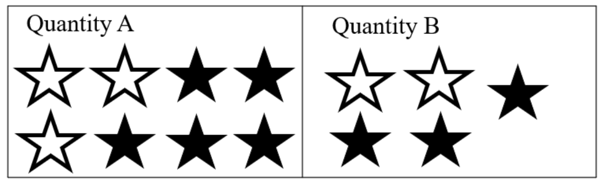
- fractions as an operator: $\frac{3}{4}$ represents a multiplier of three-fourths of the original magnitude;
- fractions as a quotient: $\frac{3}{4}$ represents the result obtained when three is divided by four; and
- fractions as a ratio: $\frac{3}{4}$ is a comparison of 3 of a quantity to the whole quantity of 4.

- A ratio may be written using a colon (a:b), the word "to" (a to b), or fraction notation $\frac{a}{b}$.
- The order of the values in a ratio is directly related to the order in which the quantities are compared. For example, in a certain class, there is a ratio of 3 girls to 4 boys (3:4).
- Another comparison that could represent the relationship between these quantities is the ratio of 4 boys to 3 girls (4:3). Both ratios give the same information about the number of girls and boys in the class, but they are distinct ratios. When you switch the order of comparison (girls to boys vs. boys to girls), there are two different ratios expressed.
- Fractions may be used when determining equivalent ratios.
  - Example: The ratio of girls to boys in a class is 3:4, this can be interpreted as:
    - number of girls = $\frac{3}{4} \cdot$ number of boys;
    - in a class with 16 boys, number of girls = $\frac{3}{4} \cdot (16) = 12$ girls.
  - Example: A similar comparison could compare the ratio of boys to girls in the class as being 4:3, which can be interpreted as:
    - number of boys = $\frac{4}{3} \cdot$ number of girls;
    - in a class with 12 girls, number of boys = $\frac{4}{3} \cdot (12) = 16$ boys.
- A ratio can compare two real-world quantities (e.g., miles per gallon, unit rate, and circumference to diameter of a circle).
- Ratios may or may not be written in simplest form.
- A ratio can represent different comparisons within the same quantity or between different quantities.

Ratio	Comparison
part-to-whole (within the same quantity)	compare part of a whole to the entire whole
part-to-part (within the same quantity)	compare part of a whole to another part of the same whole

whole-to-whole (different quantities)	compare all of one whole to all another whole
part-to-part (different quantities)	compare part of one whole to part of another whole

- Examples: Given Quantity A and Quantity B, the following comparisons could be expressed.



Ratio	Example	Ratio Notation(s)
part-to-whole (within the same quantity)	compare the number of unfilled stars to the total number of stars in Quantity A	3:8; 3 to 8; or $\frac{3}{8}$
part-to-part (within the same quantity)	compare the number of unfilled stars to the number of filled stars in Quantity A	3:5 or 3 to 5
whole-to-whole (different quantities)	compare the number of stars in Quantity A to the number of stars in Quantity B	8:5 or 8 to 5
part-to-part (different quantities)	compare the number of unfilled stars in Quantity A to the number of unfilled stars in Quantity B	3:2 or 3 to 2

- Part-to-part comparisons and whole-to-whole comparisons are ratios that are not typically represented in fraction notation except in certain contexts, such as determining whether two different ratios are equivalent.
- Equivalent ratios are created by multiplying each value in a ratio by the same constant value. For example, the ratio of 4:2 would be equivalent to the ratio 8:4, since each value in the first ratio could be multiplied by 2 to obtain the second ratio.
- Students will begin to make the connection between equivalent ratios and proportionality. A proportional relationship consists of two quantities where there exists a constant number such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.
- Proportional thinking requires students to think multiplicatively, rather than additively. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e., one quantity is the result of multiplying the other quantity by a value). See the examples below.

Additive relationship:

x	y
2 $\xrightarrow{+8}$	10
3 $\xrightarrow{+8}$	11
4 $\xrightarrow{+8}$	12
5 $\xrightarrow{+8}$	13

Multiplicative relationship:

x	y
2 $\xrightarrow{\cdot 5}$	10
3 $\xrightarrow{\cdot 5}$	15
4 $\xrightarrow{\cdot 5}$	20
5 $\xrightarrow{\cdot 5}$	25

  - In the additive relationship, *y* is the result of adding 8 to *x.*
  - In the multiplicative relationship, *y* is the result of multiplying *x* times 5.
  - The ordered pair (2, 10) is a quantity in both relationships; however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.
- It is important to use contextual situations to model proportional relationships. Context can help students to see the relationship between two quantities.
- In the elementary grades, students had experiences with tables of values (input/output tables that are additive and multiplicative). The concept of a ratio table should be connected to students' prior knowledge of representing number patterns in tables.
- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios. A constant exists that can be multiplied by the measure of one quantity to get the measure of the other quantity for every ratio pair. The same proportional relationship exists between each pair of quantities in a ratio table.

- Example: Given that the ratio of *y* to *x* in a proportional relationship is 8:4, create a ratio table that includes three additional equivalent ratios.

x	*y*
1	2
2	4
3	6
4	8
5	10

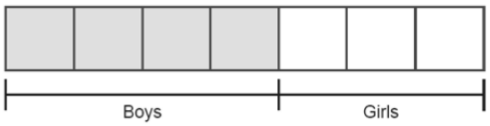
Ratio that is given

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Problem Solving: Expand students' ability to identify relevant information in word problems by presenting problem information differently. It is essential to include problems that vary the unknown quantity to help students understand the mathematical structure in each problem type. Other problems that look different may require additional steps to solve or include irrelevant numerical information or information on a chart, graph, or diagram. Problem types related to ratios are provided below –

- **Ratio problem with ratio given**: Rena bought some food at the farmer's market. For every 1 cucumber she bought, she bought 3 tomatoes. If she bought 12 tomatoes, how many cucumbers did she buy?
- **Ratio problem with a diagram (chart or graph):** In Mr. Lardin's class, there are more boys than girls. Below is a diagram representing the number of boys to girls. If there are 12 boys in the class, how many girls are there?



- **Ratio problem with irrelevant information:** Roe loves to garden. She keeps 5 flower gardens and 1 vegetable garden. In the flower garden, for every 5 daisies she plants, she also plants 1 rose. If she planted 3 roses, how many daisies did she plant?

- **Ratio problem with multiple steps:** Carolina loves to plant flowers in her garden. For every 5 daisies she plants, she also plants 1 rose. If she planted 3 roses, how many flowers did she plant altogether?

Mathematical Connections: When creating a table of equivalent ratios to represent a proportional relationship between two quantities either when given a ratio or contextually, students must make the connection that proportional thinking requires students to thinking multiplicatively, rather than additively.

- A step-by-step process to help students is to (a) identify the ratio provided in the problem; (b) create equivalent ratios by either multiplying or dividing both sides of the ratio by the same number; and (c) repeating (b) until a table is filled out with equivalent ratios by putting all the numbers on the left or top of the ratio in the left column and all numbers on the right or bottom of the ratio in the other column. For example –

  The ratio of apple trees to peach trees at Field Family Farm is 5 to 2. Create a table of equivalent ratios and determine the type of proportional relationship given.

  (a) **Identify the ratio provided in the problem:** In this problem, the ratio provided is $\frac{5 \text{ apple trees}}{2 \text{ peach trees}}$.

  (b) **To create equivalent ratios, multiply or divide both sides of the ratio by the same number:** In this problem, students will multiply. Multiply both sides of the ratio by 2. The first equivalent ratio is 10 apple trees to 4 peach trees.

$$\frac{5 \cdot 2}{2 \cdot 2} = \frac{10 \text{ apple trees}}{4 \text{ peach trees}}$$

  (c) **Repeat (b) to fill out the table with equivalent ratios.**

Apple Trees	Peach Trees
5	2
10	4
15	6
20	8
25	10

- The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative. Below is an example of an additive comparison as a misconception and the correct multiplicative comparison where students do not understand that 4 more than (additive comparison) has a different meaning than 4 times (multiplicative comparison) –

  A recipe calls for 2 cups of flour for every 1 cup of sugar. How many cups of flour are needed if a recipe is increased to 3 cups of sugar?

  Students can think 1 cup of sugar for 2 cups of flour, 2 cups of sugar for 4 cups of flour, and 3 cups of sugar for 6 cups of flour. Therefore, the amount of flour now needed is 6 cups, which maintains the same relationship as the original one given (1:2). Students who think ratios are additive would have mistakenly thought that 3 cups of sugar is an increase of 2 cups from the original 1 cup. They would then add to the original 2 cups of flour 2 more cups to get 4 cups needed flour.

Additive Comparison

1 cup of sugar	2 cups of flour
↓	↓
1 cup + 2 cups = 3 cups of sugar	2 cups + 2 cups = 4 cups of flour

1:2 ≠ 3:4

Multiplicative Comparison

1 cup of sugar	2 cups of flour
↓	↓
1 cup × 3 cups = 3 cups of sugar	2 cups × 3 cups = 6 cups of flour

1:2 = 3:6

- As students explore ratios and make connections between and among them, engaging in the following will help to elicit their understanding of this concept –
  - Create a real-world situation in which ratios have been used. Swap with a partner and have your partner solve.
  - Explain how fractions and ratios are similar and how understanding one concept can assist you in understanding the other.
  - Explain how to translate a ratio written symbolically and write it into words and vice versa.

Mathematical Representations: A ratio can represent different comparisons within the same quantity or between different quantities. A common misconception that students may exhibit is not understanding the type of relationship that exists between different quantities (whether it be part-to-part, part-to-whole, whole-to-whole, or whole-to-part). Make sure students understand that a ratio is a comparison of any two quantities and that it is used to represent a relationship within or between sets. Emphasize that the two quantities in a ratio *must be ordered in the same order as the quantities in the relationship*. A ratio may be written using a colon (a:b), the word *to* (a to b), or fraction notation $\frac{a}{b}$. Part-to-part comparisons and whole-to-whole comparisons are ratios that are not typically represented in fraction notation, except in certain contexts, such as determining whether two different ratios are equivalent. The following contextual examples are provided as well as the common misconceptions students may experience –

- **Distinguishing between a part-to-part and a part-to-whole relationship:**

  The Linwood High School Basketball Team wins 28 of the games they play. They play a total of 36 games. What is the ratio of the games they win to the games they lose? How do you know?

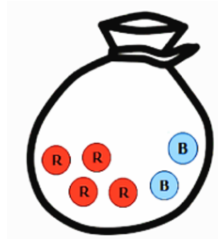
  A common error for some students is to describe a part-to-whole relationship rather than a part-to-part relationship. These students may have the misconception that all the numbers used in the ratio will be included in the problem. They may not realize that they need to use the details in the problem to determine how many games were lost. Provide students with opportunities to explore ratio relationships with counters and part-part-whole mats to model scenarios concretely and then write statements to represent them. As students model different part-part-whole combinations, ask them to use ratios to describe part-to-part relationships and part-to-whole relationships. This will help students develop the understanding that ratios can reflect different types of relationships.

- **Creating proportional relationships from context (with or without a model):**

  - **Without a model:** Create a set of items with squares and stars. The set of items should represent a 2:3 ratio for the number of squares to the number of stars. The set created should contain more than 5 items.

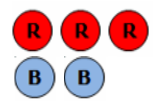
    A common error for some students is to switch the order of the ratio when representing the relationship presented. These students may have the misconception that the order of the numbers or items does not matter. Additionally, some students may be able to create a 2:3 ratio with 5 counters, but may struggle to extend it to a larger set of items. It may be helpful for these students to use concrete objects to represent relationships, focusing on matching the objects named to a ratio relationship. Then ask students how they could extend this ratio relationship to a larger set. As students become more confident, they may build several different sets of items showing the same ratio relationship.

- **With a model:** There are 4 red marbles and 2 blue marbles shown in the bag. What is the least number of red and blue marbles that can be added to the bag to create a ratio of 3 red marbles to 1 blue marble? Explain how you know.

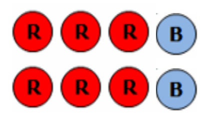


Key: R = red B = blue

In this scenario, the least number of marbles needed to create a 3 to 1, red to blue ratio, would be two red marbles and zero blue marbles. Some students may reverse the order of the ratio to blue to red, and then add items to the set to support the reversal. In this case students might add ten blue marbles and zero red marbles. Some students may also add more red and/or blue marbles than needed and disregard the “least” criteria to create a 3 to 1 (or incorrectly create a 1 to 3 ratio). It may be helpful for students to arrange concrete counters linearly to see the proportional relationship they are creating. Example concrete solution –
Given set before creating a red to blue 3 to 1 ratio:



Linear concrete arrangement to see the least amount needed (2 more red marbles) to create a red to blue, 3 to 1, ratio:



Concepts and Connections

Concepts
Proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic equations and inequalities can be used to represent and solve real world problems.

Connections: In Grade 6, students will identify and represent proportional relationships between two quantities, including those in context (unit rates are limited to positive values) (6.PFA.2). It is at this grade level that students have the first opportunity to use ratios to represent relationships between quantities, including those in context. In the subsequent grade level, Grade 7 students will investigate and analyze proportional relationships between two quantities using verbal descriptions, tables, equations in $y = mx$ form, and graphs, including problems in context (7.PFA.1).

- *Within the grade level/course*:
    - 6.PS.1 – The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graphs.
- *Vertical Progression*:
    - 7.PFA.1 – The student will investigate and analyze proportional relationships between two quantities using verbal descriptions, tables, equations in $y = mx$ form, and graphs, including problems in context.

Textbooks and HQIM for Consideration

- A list of approved textbooks and instructional materials will be posted on the VDOE website.

6.PFA.2 The student will identify and represent proportional relationships between two quantities, including those in context (unit rates are limited to positive values).

Students will demonstrate the following Knowledge and Skills:

a) Identify the unit rate of a proportional relationship represented by a table of values, a contextual situation, or a graph.

b) Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate.

c) Determine whether a proportional relationship exists between two quantities, when given a table of values, context, or graph.

d) When given a contextual situation representing a proportional relationship, find the unit rate and create a table of values or a graph.

e) Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

- A rate is a ratio that involves two different units and how they relate to each other. Relationships between two units of measure are also rates (e.g., inches per foot).
- A unit rate describes how many units of the first quantity of a ratio correspond to one unit of the second quantity.
  - Example: If it costs $10 for 5 items at a store (a ratio of 10:5 comparing cost to the number of items), then the unit rate would be $2.00/per item (a ratio of 2:1 comparing cost to number of items).

Unit Rate

# of items (x)	1	2	5	Given ratio 10
Cost in $ (y)	$2.00	$4.00	$10.00	$20.00

- Any ratio can be converted into a unit rate by writing the ratio as a fraction and then dividing the numerator and denominator each by the value of the denominator.
  - Example: It costs $8 for 16 gourmet cookies at a bake sale. What is the price per cookie (unit rate) represented by this situation?
  - $\frac{8}{16} = \frac{8 \div 16}{16 \div 16} = \frac{0.5}{1}$
  - It would cost $0.50 per cookie, which would be the unit rate.

- Examples such as $\frac{8}{16}$ and 40 to 10 are ratios but are not unit rates. However, $\frac{0.5}{1}$ and 4 to 1 are examples of unit rates.
- Example of a proportional relationship:
  - Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This ratio table represents the cost (*y*) per number of pizzas ordered (*x*).

x number of pizzas	1	2	3	4
y total cost	8	16	24	32

x8 x8

  - In this relationship, the ratio of *y* (cost in $) to *x* (number of pizzas) in each ordered pair is the same:

$$\frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4}$$

- Example of a non-proportional relationship:
  - Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.

x number of pizzas	1	2	3	4
y total cost	10	17	24	31

  - The ratios represented in the table above are not equivalent.
  - In this relationship, the ratio of *y* to *x* in each ordered pair is not the same:

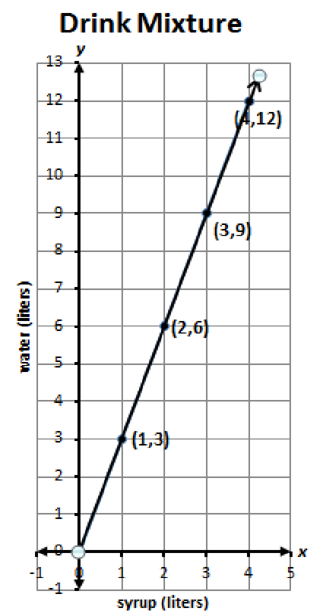
$$\frac{10}{1} \neq \frac{17}{2} \neq \frac{24}{3} \neq \frac{31}{4}$$

- Other non-proportional relationships will be studied in later mathematics courses.
- Proportional relationships involve collections of pairs of equivalent ratios that may be graphed in the coordinate plane. The graph of a proportional relationship includes ordered pairs (*x, y*) that represent pairs of values that may be represented in a ratio table.

- Proportional relationships can be expressed using verbal descriptions, tables, and graphs. When describing proportional relationships verbally, the phrases “for each,” “for every,” and “per” are used.
- Example: (verbal description) To make a drink, mix 1 liter of syrup with 3 liters of water. If *x* represents how many liters of syrup are in the mixture and *y* represents how many liters of water are in the mixture, this proportional relationship can be represented using a ratio table:

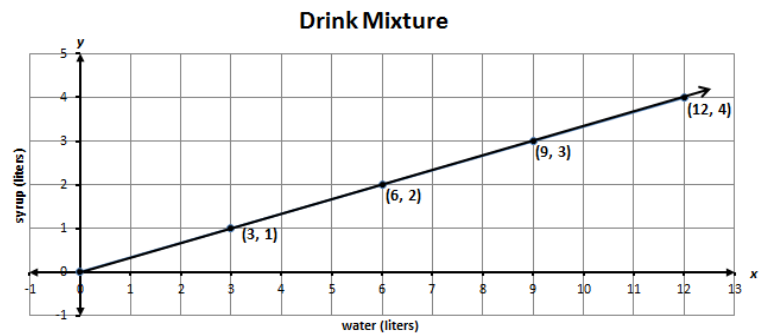
Syrup (liters) *x*	1	2	3	4
Water (liters) *y*	3	6	9	12

  - The ratio of the amount of water (*y*) to the amount of syrup (*x*) is 3:1. Additionally, the proportional relationship may be graphed using the ordered pairs in the table.



- The representation of the ratio between two quantities may depend upon the order in which the quantities are being compared. For example: In the mixture example above, we could also compare the ratio of the liters of syrup per liters of water, as shown:

Water (liters) *x*	3	6	9	12
Syrup (liters) *y*	1	2	3	4

- In this comparison, the ratio of the amount of syrup (y) to the amount of water (x) would be 1:3.
- The graph of this relationship could be represented by 

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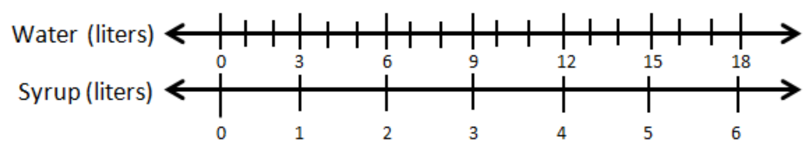
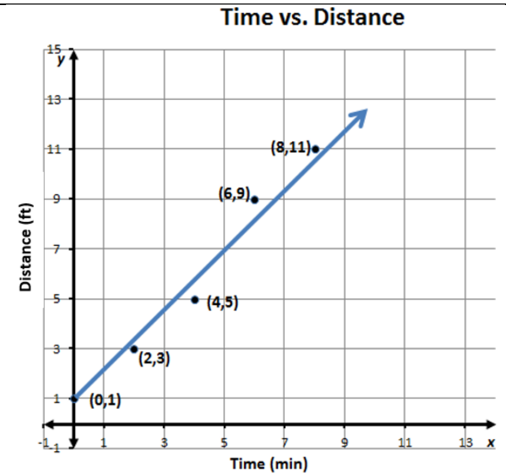
- Students should be aware of how the order in which quantities are compared affects the way in which the relationship is represented as a table of equivalent ratios or as a graph.
- Double number line diagrams can also be used to represent proportional relationships and determine pairs of equivalent ratios. See the example below.



- In this proportional relationship, there are three liters of water for each liter of syrup represented on the number lines.

- A graph representing a proportional relationship includes ordered pairs that lie in a straight line that, if extended, would pass through (0, 0). The context of the problem and the type of data represented by the graph must be considered when determining whether the points are to be connected by a straight line on the graph.
- Example of the graph of a non-proportional relationship:



- The relationship of distance (y) to time (x) is non-proportional. The ratio of y to x for each ordered pair is not equivalent. That is,

$$\frac{11}{8} \neq \frac{9}{6} \neq \frac{5}{4} \neq \frac{3}{2} \neq \frac{1}{0}$$

- The points of the graph do not lie in a straight line. Additionally, the line does not pass through the point (0, 0), thus the relationship of y to x cannot be considered proportional.

- Contextual situations that model proportional relationships can typically be represented by graphs in the first quadrant, since in most contextual situations, the values for x and y are positive. Additionally, unit rates are typically positive in contextual situations involving proportional relationships.
- A unit rate could be used to find missing values in a ratio table.
  - Example: A store advertises a price of $25 for 5 DVDs. What would be the cost to purchase 2 DVDs? 3 DVDs? 4 DVDs?

# DVDs	1	2	3	4	5
Cost	$5	?	?	?	$25

- The ratio of $25 per 5 DVDs is also equivalent to a ratio Of $5 per 1 DVD, which would be the unit rate for this relationship. This unit rate could be used to find the missing value of the ratio table above. If we multiply the number of DVDs by a constant multiplier of 5 (the unit rate) we obtain the total cost. Thus, 2 DVDs would cost $10, 3 DVDs would cost $15, and 4 DVDs would cost $20.

- At this level, students should build a conceptual understanding of proportional relationships and unit rates before moving to more abstract representations and complex computations in higher grade levels. Students are not expected to use formal calculations for slope and unit rates (e.g., slope formula) in Grade 6.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Reasoning:

- Use ratio and rate reasoning to solve real-world and mathematical problems. To do this, make tables of equivalent ratios relating quantities with whole number measurements; find missing values in the tables; and plot the pairs of values on the coordinate plane.

- Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. As examples, *"This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*

Mathematical Connections:

- By presenting representations simultaneously, students can connect how symbols represent actions or relationships embodied in physical models or diagrams. In a related way, generalization questions ask students to identify patterns and use those patterns to make conjectures or generalizations. To use generalization questions, apply the following guidelines:
    - Have students identify patterns that they notice.
    - Ask students to name representations they are familiar with that work for the new problem.
    - Have students identify a strategy that they have learned that can be used to solve the problem.

  From the introduction of a topic to the final lesson in the instructional sequence, these guiding statements provide opportunities for students to think deeply about significant ideas.

- When working to find a pattern in a table, students mistakenly look at only the pattern from row to row rather than using covariational thinking (i.e., thinking about how two quantities vary together).

- Create problems that will provide an indication of student thinking. Provide feedback regarding students' responses. Use reversibility, flexibility, and generalization to create the problems. For example, when given the following table, which of these statements is true?

A. 1m because m represents the number of math problems.
B. 2m because each math problem takes 2 minutes to solve.
C. 3m because it takes 6 minutes to solve 3 problems.
D. 10m because it takes 20 minutes to solve 10 problems.

Number of math problems	Number of minutes to complete
1	2
2	4
3	6
4	8
5	10
10	20
20	40
m	??

Mathematical Representations:

- Use activities that promote the use of multiple representations. Have students publicly share their processes or solution approaches. Encourage students to show their thinking rather than only sharing an algorithm or step-by-step process. Consider options for students to critique each other's presentations.
    - **Example 1:** Have students to work with a partner to complete a table like the following –

Number of hours worked	1	2	3	4	5	6			x
Amount of money earned	$7.50	$15.00	$22.50	$30.00			$75.00	$150.00	

    - **Example 2:** Complete the three missing parts of the given scenario using the information provided about the proportional relationship.

<table>
<tr><th>Verbal Description</th><th>Table</th></tr>
<tr><td>I use $\frac{2}{5}$ cup of sugar per 2 cups of flour.</td><td><table><tr><th>Sugar (cups)</th><th>Flour (cups)</th></tr><tr><td>$\frac{2}{5}$</td><td>2</td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></td></tr>
<tr><th>Graph</th><th>Connection</th></tr>
<tr><td></td><td>For this problem, the verbal description connects to the graph because:</td></tr>
</table>

Several misconceptions can be present as students work to connect a verbal description/ratio table to a graph. A common error some students may have is omitting the ordered pair (0,0) when describing or identifying a proportional relationship from a table or a graph. Additionally, students may struggle with choosing which variable is on the *x*-axis and which variable goes on the *y*-axis when graphing ratios. Making the connection between the ordered pairs in the table and plotting them on the graph using their labeled axes proves to be challenging when connecting the two different representations.

It is important for students to use relevant contexts in which students analyze data and represent them in various ways, with a focus on the graph. Analyzing authentic issues can help engage students in a meaningful and exciting ways, which will help them to see how these representations connect. Some examples of authentic issues might include examining rates of global warming and how it might affect decisions about our global systems. Allow for experiences where students are provided with data and decide how to represent that data in a graph. Ask students, "*How did you decide on the title of the x-axis? The y-axis? What would happen if you switched them?"* As students compare graphs that show proportional relationships, ask follow-up questions, such as, *"What do you notice about all of these graphs? What do they have in common? What is different?"* Students also may benefit from using a graphing manipulative tool, such as a coordinate pegboard (or geoboard), for a concrete experience with graphing a proportional relationship prior to moving to drawing in the coordinate points.

Concepts and Connections

Concepts
Proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic equations and inequalities can be used to represent and solve real world problems.

Connections: In Grade 6, students will use ratios to represent relationships between quantities, including those in context (6.PFA.1) and apply the data cycle with a focus on circle graphs (6.PS.1). It is at this grade level that students have the first opportunity to identify and represent proportional relationships between two quantities, including those in context (unit rates are limited to positive values). In the subsequent grade level, Grade 7 students will investigate and analyze proportional relationships between two quantities using verbal descriptions, tables, equations in $y = mx$ form, and graphs, including problems in context (7.PFA.1).

- *Within the grade level/course*:
  - 6.PS.1 – The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on circle graph.
  - 6.PFA.1 –The student will use ratios to represent relationships between quantities, including those in context.
- *Vertical Progression*:
  - 7.PFA.1 – The student will investigate and analyze proportional relationships between two quantities using verbal descriptions, tables, equations in $y = mx$ form, and graphs, including problems in context.

<u>Textbooks and HQIM for Consideration</u>
- A list of approved textbooks will be posted on the VDOE website, Spring 2024.

6.PFA.3 The student will write and solve one-step linear equations in one variable, including contextual problems that require the solution of a one-step linear equation in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Identify and develop examples of the following algebraic vocabulary: equation, variable, expression, term, and coefficient.

b) Represent and solve one-step linear equations in one variable, using a variety of concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles, weights on a balance scale).

c) Apply properties of real numbers and properties of equality to solve a one-step equation in one variable. Coefficients are limited to integers and unit fractions. Numeric terms are limited to integers.

d) Confirm solutions to one-step linear equations in one variable using a variety of concrete manipulatives and pictorial representations (e.g., colored chips, algebra tiles, weights on a balance scale).

e) Write a one-step linear equation in one variable to represent a verbal situation, including those in context.

f) Create a verbal situation in context given a one-step linear equation in one variable.

Understanding the Standard

• An algebraic equation is a mathematical statement that says two expressions are equal (e.g., $2x + 7 = 15$).
• An expression is a representation of quantity. It may contain numbers, variables, and/or operation symbols. It does not have an "equal sign (=)" (e.g., $\frac{3}{4}$, $5x$, $140 - 38.2$, $18 \cdot 21$, $5 + x$). An algebraic expression is an expression that contains at least one variable (e.g., $x - 3$). An expression cannot be solved.
• A variable is a symbol used to represent an unknown quantity.
• A term is a number, variable, product, or quotient in an expression of sums and/or differences. In the expression $7x^2 + 5x - 3$, there are three terms, $7x^2$, $5x$, and 3.
• A coefficient is the numerical factor in a term. In the term $3xy^2$, 3 is the coefficient; in the term z, 1 is the coefficient.
• A one-step linear equation may include, but not be limited to, equations such as the following:
  ○ $2x = 5$
  ○ $y - 3 = -6$
  ○ $\frac{1}{5}x = {}^{-}3$
  ○ $a - (-4) = 11$

- A variety of concrete materials such as colored chips, algebra tiles, or weights on a balance scale may be used to model solving equations in one variable.
- A verbal expression can be represented by a variable expression. Numbers are used when they are known; variables are used when the numbers are unknown. Example, the verbal expression “a number multiplied by 5” could be represented by the variable expression “*n* · 5” or “5*n*.”
- A verbal sentence is a complete word statement (e.g., “The sum of a number and two is five” could be represented by “*n* + 2 = 5”).
- The solution to an equation is a value that makes it a true statement. Many equations have one solution and are represented as a point on a number line. Solving an equation or inequality involves a process of determining which value(s) from a specified set, if any, make the equation or inequality a true statement. Substitution can be used to determine whether a given value(s) makes an equation or inequality true.
- Word choice and language are very important when representing verbal situations in context using mathematical operations, equality, and variables. When presented with an equation or context, student choice of language should reflect the situation being modeled. Identifying the mathematical operation(s) to represent the action(s) and the variable to represent the unknown quantity will help students to write equations that represent the contextual situation.
- Properties of real numbers and properties of equality can be used to solve equations, justify equation solutions, and express simplification. Students should use the following properties, where appropriate, to further develop flexibility and fluency in problem solving (limitations may exist for the values of *a*, *b*, or *c* in this standard).
  - Commutative property of addition: $a + b = b + a$
  - Commutative property of multiplication: $a \cdot b = b \cdot a$
  - Subtraction and division are neither commutative nor associative.
  - Identity property of addition (additive identity property): $a + 0 = a$ and $0 + a = a$
  - Identity property of multiplication (multiplicative identity property): $a \cdot 1 = a$ and $1 \cdot a = a$
  - The additive identity is zero (0) because any number added to zero is equal to the number. The multiplicative identity is one (1) because any number multiplied by one is equal to the number. There are no identity elements for subtraction and division.
  - Inverses are numbers that combine with other numbers and result in identity elements.
  - Inverse property of addition (additive inverse property): $a + (-a) = 0 \ and \ (-a) + a = 0$ (e.g., 5 + (-5) = 0)
  - Inverse property of multiplication (multiplicative inverse property): $a \cdot \frac{1}{a} = 1$ and $\frac{1}{a} \cdot a = 1$ (e.g., $5 \cdot \frac{1}{5} = 1$)
  - Zero has no multiplicative inverse.
  - Multiplicative property of zero: $a \cdot 0 = 0$ and $0 \cdot a = 0$
  - Division by zero is not a possible mathematical operation. It is undefined.
  - Addition property of equality: If $a = b$, then $a + c = b + c$
  - Subtraction property of equality: If $a = b$, then $a - c = b - c$
  - Multiplication property of equality: If $a = b$, then $a \cdot c = b \cdot c$

- Division property of equality: If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$
- Substitution property: If $a = b$, then *b* can be substituted for *a* in any expression, equation, or inequality

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Problem Solving:

- General algorithms exist for solving many kinds of equations. These algorithms are broadly applicable for solving a wide range of similar equations.

- Some problems or situations – circumstances that students explore concretely and immediately (for example by working with algebra tiles) and circumstances in stated problems – should be based on situations from everyday life.

- Linear equations can be solved by symbolic, graphical, or numerical methods. Students may have trouble moving from solving using the concrete materials to solving numerically. It is important to link the two methods together before releasing students to solve solely using algebraic methods.

- Students must review and apply the properties of real numbers and properties of equality to solve equations. Students should be familiar with the properties of real numbers and properties of equality. Common errors or misconceptions include –
    - **Additive inverse and identity:** Students may get the additive identity and additive inverse properties confused because they both use addition.
    - **Multiplicative inverse and identity:** Students may get the multiplicative identity and multiplicative inverse properties confused because they both use multiplication.
    - **Multiplicative property of zero:** Students may confuse the multiplicative property of zero and additive identity properties confused because they both have a zero in the expression.

Mathematical Reasoning: Equations are solved as a process of reasoning using properties of equality, which can justify each step of the process. Provide students with example like the following and when doing so, ask students to justify each step in the process and their solutions. Common misconceptions are provided below –

- Solve each equation and justify your solution: $-10 = h + 14$ $2b = 48$ $\frac{n}{5} = -10$

  A common misconception some students may have when solving $-10 = h + 14$ is to add fourteen to both sides of the equation.

A common misconception some student may have when solving $\frac{n}{5} = -10$ is to divide both sides of the equation by 5. Each of these misconceptions may indicate that the student sees an addition and division type equation, *h* + 14 and $\frac{n}{5}$, and uses the same operation to solve the equation instead of using an inverse operation.

A common misconception that some students may have when solving 2*b* = 48 is to subtract two from both sides of the equation. This may indicate that a student interprets the coefficient as a +2 but thinks subtracting two is the appropriate inverse operation to solve the equation.

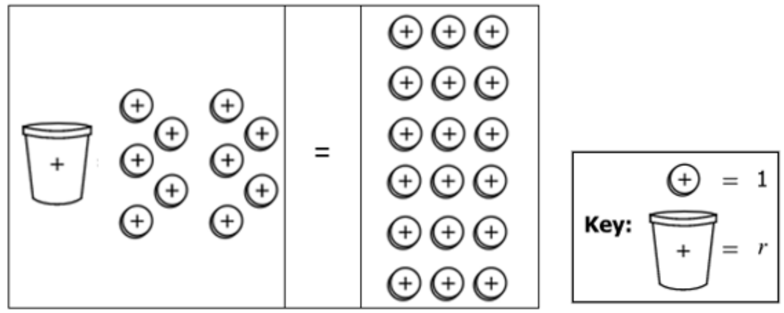
It may be beneficial to have students use models to represent the equations when applicable. It would also be helpful to have students work with balance scales in conjunction with open sentences to develop the connection between the equal sign and the expression on each side of the equal sign. In addition, as students are solving equations, include verbal descriptions that explain the meaning of the equation. Encourage students to explain their thinking and even try to determine more than one way to solve each equation. Additionally, have students write equations in more than one way in connection to the models that they are working with across this standard.

- Hillary determines that 32 is the solution to the equation $\frac{1}{4}x$ = 8. How can Hillary confirm her solution is correct?

  A common misconception some students may make is to interpret that 32 must equal the product of $\frac{1}{4}x$. This may indicate that a student interprets eight as the value of *x* and incorrectly multiplies eight times four to obtain a value of 32 in the denominator, $\left(\frac{1}{4 \cdot 8}\right) = 32$. It may be beneficial to have students write the given equation as $\frac{x}{4} = 8$. The teacher can facilitate a discussion with students to help them translate or make sense of this equation. Have students think about a context that might fit the equation. Ask students, "*How many objects would be needed to make four equal groups of eight objects? If each of eight students received the same number of crayons, how many total crayons would be needed for each student to receive four crayons?"*

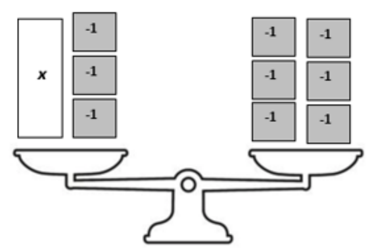
Mathematical Representations: Students must have opportunities to use concrete and pictorial representations of solving equations before proceeding to the algorithm. Remember, concrete manipulatives like algebra tiles make mathematics visual and provide the conceptual understanding that students need to successfully transition to the algorithm. As students solve equations, have them write out what is happening as they are using manipulatives to solve an equation (e.g., making sure to say [and make students say], "2 times *x* equals 6" as opposed to "2*x* = 6"). Examples with common misconceptions follow –

- **Equation Mat:** Write an equation to represent the model shown.



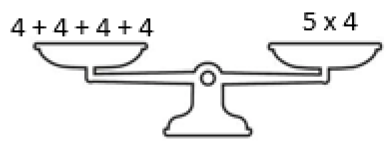
A common misconception some students may have is to interpret the left side of the equation mat as 10*r*. This may indicate a student believes the number of unit counters on the left side of the mat represents the coefficient of *r*. It may be beneficial to have students think about the shapes that are used in the model and replace each shape in the mat with the value as designated in the available key. Making a connection between like shapes in the equation mat and like terms of an equation would be helpful as well.

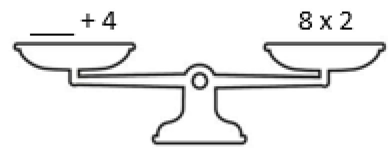
- **Multiple representations (contextual situation, model, equation, and justifying a solution):** Complete the missing parts of the grid using the information provided about the contextual problem.

Contextual Problem	**Model**
The temperature dropped three degrees. Now the thermometer says -6 degrees. What was the temperature before it dropped?	
Equation	**Solution**
	x = ___________
Explain and justify your answer:	

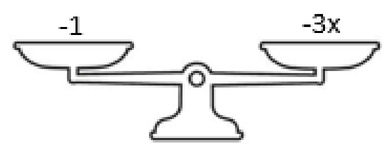
In justifying a solution for this problem, students must be able to make sense of the model and context to explain why their solution makes sense. Confirming solutions can prove to be difficult for students if they are only thinking procedurally.

If students need more support in connecting models and equations, consider using true/false sentences. Use a balance scale with a modeled equation or concrete objects to represent an equation (such as algebra tiles) and ask, "*Is this true or false? How do you know*? Students should support their reasoning by explaining their thinking. Here are some examples: *What would make this tilt? Which way would it tilt? What would make it balance?"*





Students could benefit from using balance scales when solving equations. Having a concrete tool to model if something is only performed to one side of the balance scale; it changes the balance (relationship). To maintain balance, you would also have to make a change to the other side. Consider giving scales with variables to students and asking students, *"How could we change these to keep the balance?"* For example, here we would divide both sides by -3:



Lastly, plotting the value of *x* (or any variable) on a number line might help students to see the value of the variable, keeping in mind that sometimes variables can be multiple values and sometimes only one value makes the equation true.

Concepts and Connections

Concepts
Proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic equations and inequalities can be used to represent and solve real world problems.

Connections: In Grade 6, students represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line (6.PFA.4). Prior to Grade 6, students investigate and use variables in contextual problems (5.PFA.2). Using these foundational understandings, students will write and solve one-step linear equations in one variable, including contextual problems that require the solution of a one-step linear equation in one variable (6.PFA.3). In the subsequent grade level, Grade 7 students will simplify numerical expressions, simplify and generate equivalent algebraic expressions in one variable, and evaluate algebraic expressions for given replacement values of the variables (7.PFA.2).

- *Within the grade level/course*:
  - 6.PFA.4 – The student will represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line.
- *Vertical Progression*:
  - 5.PFA.2 – The student will investigate and use variables in contextual problems.
  - 7.PFA.2 – The student will simplify numerical expressions, simplify and generate equivalent algebraic expressions in one variable, and evaluate algebraic expressions for given replacement values of the variables.

Textbooks and HQIM for Consideration

- A list of approved textbooks and instructional materials will be posted on the VDOE website.

6.PFA.4 The student will represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line.

Students will demonstrate the following Knowledge and Skills:

a) Given the graph of a linear inequality in one variable on a number line, represent the inequality in two equivalent ways (e.g., $x < -5$ or $-5 > x$) using symbols. Symbols include <, >, ≤, ≥.

b) Write a linear inequality in one variable to represent a given constraint or condition in context or given a graph on a number line.

c) Given a linear inequality in one variable, create a corresponding contextual situation or create a number line graph.

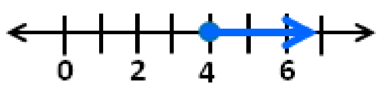
d) Use substitution or a number line graph to justify whether a given number in a specified set makes a linear inequality in one variable true.

e) Identify a numerical value(s) that is part of the solution set of a given inequality in one variable.

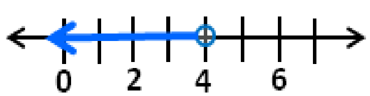
Understanding the Standard

- The solution set to an inequality is the set of all numbers that make the inequality true.
- Inequalities can represent contextual situations.
  - Example: Jaxon works at least 4 hours per week mowing lawns. Write an inequality representing this situation and graph the solution.

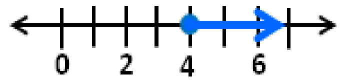
    $x \geq 4$ or $4 \leq x$

  - Students might then be asked: "Would Jaxon ever work 3 hours in a week? 6 hours?
- The variable in an inequality may represent values that are limited by the context of the problem or situation. For example, if the variable represents all children in a classroom who are taller than 36 inches, the variable will be limited to have a minimum and maximum value based on the heights of the children. Students are not expected to represent these situations with a compound inequality (e.g., $36 < x < 70$) but only recognize that the values satisfying the single inequality, $x > 36$, will be limited by the context of the situation.
- Inequalities using the < or > symbols are represented on a number line with an open circle on the number and a shaded line in the direction of the solution set.
  - Example: When graphing $x < 4$, use an open circle on the 4 to indicate that the 4 is not included in the solution set.

- Inequalities using the ≤ or ≥ symbols are represented on a number line with a closed circle on the number and a shaded line in the direction of the solution set.
    - Example: When graphing $x \geq 4$, fill in the circle on the 4 to indicate that the 4 is included in the solution set.



- It is important for students to see inequalities written with the variable before the inequality symbol and after. Example: $x > 5$ is not the same relationship as $5 > x$. However, $x > 5$ is the same relationship as $5 < x$.

Skills in Practice

While the five process goals are expected to be embedded in each standard, the Skills in Practice highlight the most prevalent process goals in relation to the content presented.

Mathematical Reasoning: There are several misconceptions that students may experience when writing a linear inequality in one variable to represent a given constraint or condition in context or given a graph on a number line. Examples with common misconceptions follow –

- Joe sent at least 100 texts yesterday. Write an inequality to represent the number of texts Joe might have sent and explain your reasoning. Then, give three possible numbers that could represent a solution to the inequality and explain why these solutions would work.

  A common error made by some students is to use the less than or equal to sign, rather than the greater than or equal to sign because they associate "at least" with less than. To assist students in understanding what "at least" means conceptually, ask them questions like: *If Joe sent at least 100 texts yesterday, could he have sent 101 texts? Could he have sent 105 texts? Could he have sent 90 texts?* Substituting possible values for the solutions will help students conceptualize "at least." Additionally, having students examine "at most" in the same fashion will also help them to understand the difference between these two phrases and the associated inequalities.

- Callie sold 22 tickets to the school talent show. She sold more than twice the number of tickets that Ava sold. Write an inequality to show how many tickets Ava might have sold. Explain your reasoning.

  Writing inequalities that match the words in story problems is a common challenge related to algebraic thinking. Here, a student might write an inequality that has "22*c*" or might not understand that Ava sold more. To help students make the connection between verbal descriptions and inequalities, use concrete materials, such as algebra tiles, balance scales, or unifix cubes. Ask questions like, "*What is the relationship between tickets sold by Callie and tickets sold by Ava? Who sold more? How do you know?"* Starting with contextual problems, students can use post-its or larger paper to add labels to the models they are creating: "*What exactly is showing us Callie's tickets? Ava's tickets? How does your representations show the relationship between Ava and Callie's tickets?"* They can also do this by substituting in numbers for the inequalities that would make sense:

"*What are the possibilities that would make it true?"* Asking questions of students that get them to think critically about how the inequality connects to the story problem is imperative for building inequality understanding.

Mathematical Representations: Students should have intentional exposure to creating an inequality from a verbal description, graphing the inequality using the appropriate symbols, and explaining their answers. A four-part graphic organizer like the one below, when given the verbal description, and having students to complete the remaining areas is an example of capturing the intention of this standard –

Verbal description: Luca has at most 10 brownies to share with his friends.	**Inequality (using symbols):**
Explain:	**Graph of the Inequality:**

- This problem is designed to uncover student understanding about comparison vocabulary. Students are most likely familiar with "less" and "more" but may have misunderstandings about how those vocabulary terms connect to the symbols. If student explanations demonstrate a misunderstanding of the comparison vocabulary, have class discussions about what it means to say, "less than" or "less than or equal to," having students debate which signs make the most sense given specific situations. Here, they can also use a pictorial representation such as a number line to prove their solutions. Ask questions like: "*If this is the inequality, would Luca be able to have 11 brownies? How do you know? Does this inequality make sense with the context of the story? How do you know? How would the context change with each that does not match?"*

- As students continue their work in writing inequalities to match verbal descriptions, have them write an example and non-example for each. Conversely, give similar inequalities and have students write the verbal expression or story context that would make sense for each. Ask, "*How are these alike? How are they different?"*

- Consider the given graphic organizer, and then replacing another section such as the graph of the inequality, and having students complete the remaining sections to draw not only representations, but also mathematical connections.

Concepts and Connections

Concepts
Proportional relationships can be described, and generalizations can be made using patterns, relations, and functions. Algebraic equations and inequalities can be used to represent and solve real world problems.
 	
Connections: In Grade 6, students will write and solve one-step linear equations in one variable, including contextual problems that require the solution of a one-step linear equation in one variable (6.PFA.3). Prior to Grade 6, students investigated and used variables in contextual problems (5.PFA.2). Using these foundational understandings, students will represent a contextual situation using a linear inequality in one variable with symbols and graphs on a number line (6.PFA.4). In the subsequent grade level, Grade 7 students will simplify numerical expressions, simplify and generate equivalent algebraic expressions in one variable, and evaluate algebraic expressions for given replacement values of the variables (7.PFA.2).
 	
• *Within the grade level/course*:
    ○ 6.PFA.3 – The student will write and solve one-step linear equations in one variable, including contextual problems that require the solution of a one-step linear equation in one variable.
• *Vertical Progression*:
    ○ 5.PFA.2 – The student will investigate and use variables in contextual problems.
    ○ 7.PFA.2 – The student will simplify numerical expressions, simplify and generate equivalent algebraic expressions in one variable, and evaluate algebraic expressions for given replacement values of the variables.

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