2023 Mathematics *Standards of Learning*
Understanding the Standards – Algebra 1

The Understanding the Standards document includes the mathematics understandings and key concepts that assist teachers in planning standards-focused instruction of the Algebra 1 2023 Mathematics *Standards of Learning*. The Understanding the Standards includes definitions, explanations, and examples regarding each mathematics standard and describes what students should know (core knowledge) as a result of the instruction specific to the course/grade level.

Expressions and Operations

A.EO.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables.

Students will demonstrate the following Knowledge and Skills:

a) Translate between verbal quantitative situations and algebraic expressions, including contextual situations.

b) Evaluate algebraic expressions which include absolute value, square roots, and cube roots for given replacement values to include rational numbers, without rationalizing the denominator.

A.EO.1 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. *Additional Content Background and Instructional Guidance:*
• Mathematical modeling involves creating algebraic representations of quantitative practical situations. • The numerical value of an expression depends upon the values of the replacement set for the variables. • Evaluating algebraic expressions and determining the value of numerical expressions can be accomplished using a range of methods, all of which adhere to the order of operations. • The operations and the magnitude of the numbers in an expression affect the choice of an appropriate computational technique (e.g., mental mathematics, estimation, calculator, paper and pencil).

A.EO.2 The student will perform operations on and factor polynomial expressions in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Determine sums and differences of polynomial expressions in one variable, using a variety of strategies, including concrete objects and their related pictorial and symbolic models.

b) Determine the product of polynomial expressions in one variable, using a variety of strategies, including concrete objects and their related pictorial and symbolic models, the application of the distributive property, and the use of area models. The factors should be limited to five or fewer terms (e.g., $(4x + 2)(3x + 5)$ represents four terms and $(x + 1)(2x^2 + x + 3)$ represents five terms).

c) Factor completely first- and second-degree polynomials in one variable with integral coefficients. After factoring out the greatest common factor (GCF), leading coefficients should have no more than four factors.

d) Determine the quotient of polynomials, using a monomial or binomial divisor, or a completely factored divisor.

e) Represent and demonstrate equality of quadratic expressions in different forms (e.g., concrete, verbal, symbolic, and graphical).

A.EO.2 The student will perform operations on and factor polynomial expressions in one variable.

Additional Content Background and Instructional Guidance:

- Operations with polynomials can be represented concretely, pictorially, and symbolically.
- Polynomial expressions can be used to define functions and model practical situations.
- Example of multiplying $(3x - 4)(x + 2)$ using the box method:

	x	2
$3x$	$3x^2$	$6x$
-4	$-4x$	-8

After combining like terms $6x$ and $-4x$, the final answer would be $3x^2 + 2x - 8$.

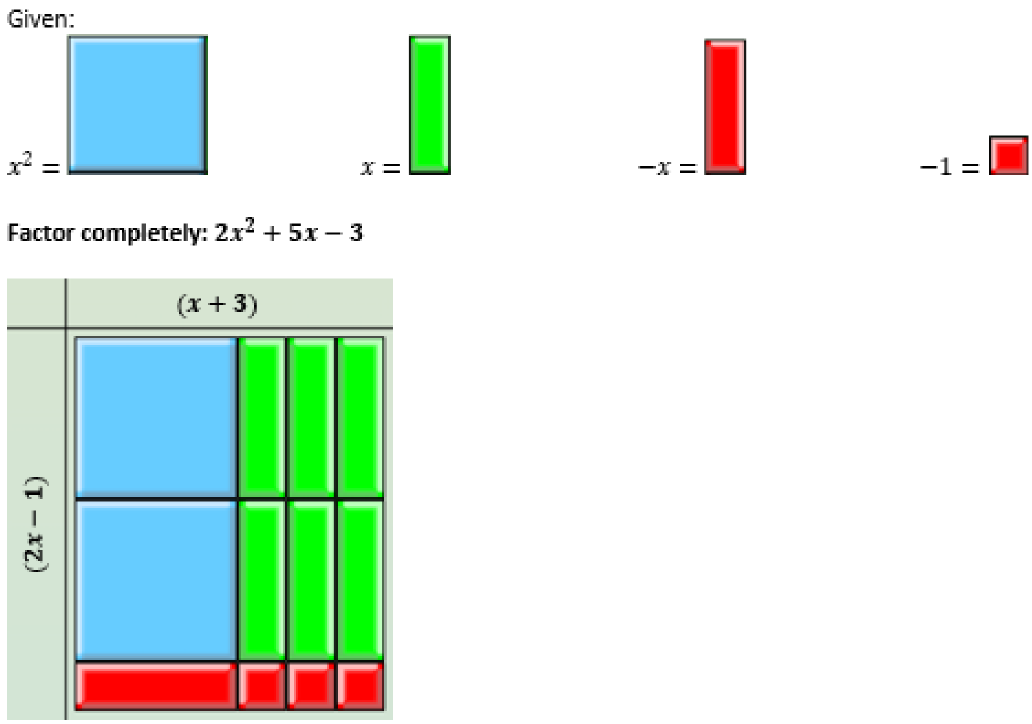
- Factoring reverses distribution and polynomial multiplication.
- Prime polynomials cannot be factored over the set of integers into two or more factors, each of lesser degree than the original polynomial.
- The factors of a number, n, include 1 and n.
- Trinomials may be factored by various methods including factoring by grouping and using models.
- Example of factoring by grouping:

$$2x^2 + 5x - 3$$
$$2x^2 + 6x - x - 3$$
$$2x(x + 3) - 1(x + 3)$$
$$(x + 3)(2x - 1)$$

- Example of factoring a quadratic trinomial with models:

A.EO.2 The student will perform operations on and factor polynomial expressions in one variable.

Additional Content Background and Instructional Guidance:



- For the division of polynomials in this standard, instruction on the use of long or synthetic division is not required, but students may benefit from experiences with these methods, which become more useful and prevalent in the study of advanced levels of algebra.

A.EO.3 The student will derive and apply the laws of exponents.

Students will demonstrate the following Knowledge and Skills:

a) Derive the laws of exponents through explorations of patterns, to include products, quotients, and powers of bases.

b) Simplify multivariable expressions and ratios of monomial expressions in which the exponents are integers, using the laws of exponents.

A.EO.3 The student will derive and apply the laws of exponents.

Additional Content Background and Instructional Guidance:

- Students should have opportunities to engage in exploration activities that will help them to generalize and derive the laws of exponents prior to be given the rules of exponents. The rules of exponents include:
- Product Rule: When multiplying two exponential expressions with the same base, add the exponents. For example, $a^m \cdot a^n = a^{m+n}$
  $a^2 \cdot a^5 = (a \cdot a)(a \cdot a \cdot a \cdot a \cdot a)$
  $a^2 \cdot a^5 = a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$
  $a^2 \cdot a^5 = a^7$
- Quotient Rule: When dividing two exponential expressions with the same base, subtract the exponents. For example, $\frac{a^m}{a^n} = a^{m-n}$
  $$\frac{a^5}{a^2} = \frac{a \cdot a \cdot a \cdot a \cdot a}{a \cdot a}$$
  $$\frac{a^5}{a^2} = \frac{a \cdot a \cdot a}{1} \cdot \frac{a \cdot a}{a \cdot a}$$
  $$\frac{a^5}{a^2} = \frac{a^3}{1} \cdot 1 = a^3$$
- Power Rule: When raising an exponential expression to a power, multiply the exponents. For example, $(a^m)^n = a^{m \cdot n}$
  $(a^2)^3 = (a^2)(a^2)(a^2)$
  $(a^2)^3 = (a \cdot a)(a \cdot a)(a \cdot a)$
  $(a^2)^3 = a \cdot a \cdot a \cdot a \cdot a \cdot a$
  $(a^2)^3 = a^6$
- Negative Exponent Rule: Any non-zero base raised to a negative exponent is equal to 1 divided by the same base raised to the opposite positive exponent. For example, $a^{-n} = \frac{1}{a^n}$

$\frac{a^2}{a^5}$	$\frac{a^2}{a^5}$
$\frac{a^2}{a^5} = \frac{a \cdot a}{a \cdot a \cdot a \cdot a \cdot a}$	$\frac{a^2}{a^5} = a^{2-5}$
$\frac{a^2}{a^5} = \frac{a \cdot a}{a \cdot a} \cdot \frac{1}{a \cdot a \cdot a}$	$\frac{a^2}{a^5} = a^{-3}$

A.EO.3 The student will derive and apply the laws of exponents.

Additional Content Background and Instructional Guidance:

$$\frac{a^2}{a^5} = 1 \cdot \frac{1}{a^3} \qquad \frac{a^2}{a^5} = \frac{1}{a^3}$$

- Zero Rule: Any non-zero base raised to the power of zero is equal to 1. For example, $a^0 = 1$

$$a^{3-3}$$

$$\frac{a \cdot a \cdot a}{a \cdot a \cdot a} \qquad \frac{a^3}{a^3} = a^0$$

$$\frac{a^3}{a^3} = 1 \qquad \frac{a^3}{a^3} = 1$$

- The laws of exponents can be applied to perform operations involving numbers written in scientific notation.

A.EO.4 The student will simplify and determine equivalent radical expressions involving square roots of whole numbers and cube roots of integers.

Students will demonstrate the following Knowledge and Skills

a) Simplify and determine equivalent radical expressions involving the square root of a whole number in simplest form.
b) Simplify and determine equivalent radical expressions involving the cube root of an integer.
c) Add, subtract, and multiply radicals, limited to numeric square and cube root expressions.
d) Generate equivalent numerical expressions and justify their equivalency for radicals using rational exponents, limited to rational exponents of $\frac{1}{2}$ and $\frac{1}{3}$ (e.g., $\sqrt{5} = 5^{\frac{1}{2}}$; $\sqrt[3]{8} = 8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2$).

A.EO.4 The student will simplify and determine equivalent radical expressions involving square roots of whole numbers and cube roots of integers. *Additional Content Background and Instructional Guidance:*
• Radical expressions in Algebra 1 can be expressed with the square root symbol ($\sqrt{}$) or the cube root symbol ($\sqrt[3]{}$). • Radical expressions in Algebra 1 can also be expressed as expressions containing rational exponents $\frac{1}{2}$ and $\frac{1}{3}$. • A square root of a number, a, is a number, y, such that $y^2 = a$. • A cube root of a number, b, is a number, y, such that $y^3 = b$. • The square root of a perfect square is an integer. • The cube root of a perfect cube is an integer. • A square root in simplest form is one in which the radicand has no perfect square factors other than one. • A cube root in simplest form is one in which the radicand has no perfect cube factors other than one. • The inverse of squaring a number is determining the square root. • The inverse of cubing a number is determining the cube root. • Any non-negative number other than a perfect square has a principal square root that lies between two consecutive whole numbers. • The cube root of a nonperfect cube lies between two consecutive integers. • The radicand should be limited to integers. • Radical expressions should be limited to numerical radicands when adding, subtracting, or multiplying. • The inverse of a rational (unit fraction) exponent describes the radical.

Equations and Inequalities

A.EI.1 The student will represent, solve, explain, and interpret the solution to multistep linear equations and inequalities in one variable and literal equations for a specified variable.

Students will demonstrate the following Knowledge and Skills:

a) Write a linear equation or inequality in one variable to represent a contextual situation.

b) Solve multistep linear equations in one variable, including those in contextual situations, by applying the properties of real numbers and/or properties of equality.

c) Solve multistep linear inequalities in one variable algebraically and graph the solution set on a number line, including those in contextual situations, by applying the properties of real numbers and/or properties of inequality.

d) Rearrange a formula or literal equation to solve for a specified variable by applying the properties of equality.

e) Determine if a linear equation in one variable has one solution, no solution, or an infinite number of solutions.

f) Verify possible solution(s) to multistep linear equations and inequalities in one variable algebraically, graphically, and with technology to justify the reasonableness of the answer(s). Explain the solution method and interpret solutions for problems given in context.

A.EI.1 The student will represent, solve, explain, and interpret the solution to multistep linear equations and inequalities in one variable and literal equations for a specified variable. *Additional Content Background and Instructional Guidance:*

• Practical problems may be interpreted, represented, and solved using linear equations and inequalities.
• The process of solving linear equations and inequalities can be modeled in a variety of ways, using concrete, pictorial, and symbolic representations.
• Properties of Real Numbers and Properties of Equality/Inequality are applied to solve equations/inequalities.
• Properties of Real Numbers:
    ○ Associative Property of Addition
    ○ Associative Property of Multiplication
    ○ Commutative Property of Addition
    ○ Commutative Property of Multiplication
    ○ Identity Property of Addition (Additive Identity)
    ○ Identity Property of Multiplication (Multiplicative Identity)
    ○ Inverse Property of Addition (Additive Inverse)
    ○ Inverse Property of Multiplication (Multiplicative Inverse)
    ○ Distributive Property
• Properties of Equality:
    ○ Multiplicative Property of Zero
    ○ Zero Product Property
    ○ Reflexive Property
    ○ Symmetric Property
    ○ Transitive Property of Equality
    ○ Addition Property of Equality

A.EI.1 The student will represent, solve, explain, and interpret the solution to multistep linear equations and inequalities in one variable and literal equations for a specified variable.

Additional Content Background and Instructional Guidance:

  - Subtraction Property of Equality
  - Multiplication Property of Equality
  - Division Property of Equality
  - Substitution
- Properties of Inequality:
  - Transitive Property of Inequality
  - Addition Property of Inequality
  - Subtraction Property of Inequality
  - Multiplication Property of Inequality
  - Division Property of Inequality
  - Substitution
- A solution to an equation/inequality is the value or set of values that can be substituted to make the equation/inequality true.
- Solutions and intervals may be expressed in different formats, including equations and inequalities, set notation, and interval notation. Examples may include:

Equation/ Inequality	Set Notation	Interval Notation
$x = 3$	$\{3\}$	$\{3\}$
$x = 3$ or $x = 5$	$\{3, 5\}$	$\{3, 5\}$
$y \geq 3$	$\{y: y \geq 3\}$	$[3, \infty)$
$-2 < x \leq 6$	$\{x: -2 < x \leq 6\}$	$(-2, 6]$
Empty (null) set ∅	{ }	{ }

- Formulas are a type of literal equation.

A.EI.2 The student will represent, solve, explain, and interpret the solution to a system of two linear equations, a linear inequality in two variables, or a system of two linear inequalities in two variables.

Students will demonstrate the following Knowledge and Skills:

a) Create a system of two linear equations in two variables to represent a contextual situation.

b) Apply the properties of real numbers and/or properties of equality to solve a system of two linear equations in two variables, algebraically and graphically.

c) Determine whether a system of two linear equations has one solution, no solution, or an infinite number of solutions.

d) Create a linear inequality in two variables to represent a contextual situation.

e) Represent the solution of a linear inequality in two variables graphically on a coordinate plane.

f) Create a system of two linear inequalities in two variables to represent a contextual situation.

g) Represent the solution set of a system of two linear inequalities in two variables, graphically on a coordinate plane.

h) Verify possible solution(s) to a system of two linear equations, a linear inequality in two variables, or a system of two linear inequalities algebraically, graphically, and with technology to justify the reasonableness of the answer(s). Explain the solution method and interpret solutions for problems given in context.

A.EI.2 The student will represent, solve, explain, and interpret the solution to a system of two linear equations, a linear inequality in two variables, or a system of two linear inequalities in two variables.

Additional Content Background and Instructional Guidance:

- Systems of two linear equations or inequalities can be used to model two practical conditions that must be satisfied simultaneously.
- A system of linear equations with exactly one solution is characterized by the graphs of two lines whose intersection is a single point, and the coordinates of this point satisfy both equations. The solution is represented as an ordered pair such as (3, 4).
- A system of two linear equations with no solution is characterized by the graphs of two parallel lines that do not intersect.
- A system of two linear equations having an infinite number of solutions is characterized by two lines that coincide (the lines appear to be the graph of one line) and the coordinates of all points on the line satisfy both equations. These lines will have the same slope and *y*-intercept.
- The graph of the solutions of a linear inequality is a half-plane bounded by the graph of its related linear equation. Points on the boundary are included unless the inequality contains only < or > (no equality condition).
- The graph of the solutions of a system of linear inequalities is the intersection of the graphs of the corresponding half-planes. Any point in the intersecting planes is a valid mathematical solution.
- There may be mathematically valid answers that do not satisfy contextual problems.

A.EI.2 The student will represent, solve, explain, and interpret the solution to a system of two linear equations, a linear inequality in two variables, or a system of two linear inequalities in two variables. *Additional Content Background and Instructional Guidance:*
• Properties of Real Numbers, Properties of Equality, and Properties of Inequality are used to solve equations/inequalities. See Understanding the Standard for SOL A.EI.1 for a list of properties.

A.EI.3 The student will represent, solve, and interpret the solution to a quadratic equation in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Solve a quadratic equation in one variable over the set of real numbers with rational or irrational solutions, including those that can be used to solve contextual problems.

b) Determine and justify if a quadratic equation in one variable has no real solutions, one real solution, or two real solutions.

c) Verify possible solution(s) to a quadratic equation in one variable algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

A.EI.3 The student will represent, solve, and interpret the solution to a quadratic equation in one variable. *Additional Content Background and Instructional Guidance:*
• A solution to an equation is the value or set of values that can be substituted to make the equation true. • Practical problems may be interpreted, represented, and solved using quadratic equations. • The process of solving quadratic equations can be modeled in a variety of ways using concrete, pictorial, and symbolic representations. • Quadratic equations in one variable may be solved algebraically by factoring and applying properties of equality or by using the quadratic formula over the set of real numbers (Algebra 1) or the set of complex numbers (Algebra 2). • The number of real solutions of a quadratic equation can be found by graphing the quadratic function and finding the number of *x*-intercepts. • The number of real solutions of a quadratic equation $y = ax^2 + bx + c$ can be found algebraically using the discriminant, $b^2 - 4ac$. • If $b^2 - 4ac > 0$, there are 2 real solutions. • If $b^2 - 4ac = 0$, there is 1 real solution. • If $b^2 - 4ac < 0$, there are 0 real solutions. • The real solutions of a quadratic equation can be found by graphing the quadratic function and finding the *x*-intercepts or zeros. • The real solutions of a quadratic equation $0 = ax^2 + bx + c$ can be found algebraically using the quadratic formula, $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$. • The real, rational solutions of a quadratic equation can also be found algebraically through factoring.

Functions

A.F.1 The student will investigate, analyze, and compare linear functions algebraically and graphically, and model linear relationships.

Students will demonstrate the following Knowledge and Skills:

a) Determine and identify the domain, range, zeros, slope, and intercepts of a linear function, presented algebraically or graphically, including the interpretation of these characteristics in contextual situations.

b) Investigate and explain how transformations to the parent function $y = x$ affect the rate of change (slope) and the y-intercept of a linear function.

c) Write equivalent algebraic forms of linear functions, including slope-intercept form, standard form, and point-slope form, and analyze and interpret the information revealed by each form.

d) Write the equation of a linear function to model a linear relationship between two quantities, including those that can represent contextual situations. Writing the equation of a linear function will include the following situations:

   i) given the graph of a line;

   ii) given two points on the line whose coordinates are integers;

   iii) given the slope and a point on the line whose coordinates are integers;

   iv) vertical lines as $x = a$; and

   v) horizontal lines as $y = c$.

e) Write the equation of a line parallel or perpendicular to a given line through a given point.

f) Graph a linear function in two variables, with and without the use of technology, including those that can represent contextual situations.

g) For any value, x, in the domain of f, determine $f(x)$, and determine x given any value $f(x)$ in the range of f, given an algebraic or graphical representation of a linear function.

h) Compare and contrast the characteristics of linear functions represented algebraically, graphically, in tables, and in contextual situations.

A.F.1 The student will investigate, analyze, and compare linear functions algebraically and graphically, and model linear relationships. *Additional Content Background and Instructional Guidance:*
• The domain of a function is the set of all possible values of the independent variable and may be restricted by the practical situation modeled by a function. • The range of a function is the set of all possible values of the dependent variable. • The slope of a line represents a constant rate of change in the dependent variable when the independent variable changes by a constant amount. • Slope can be described as a rate of change and will be positive, negative, zero, or undefined. • The x-intercept is the point at which the graph of a relation or function intersects with the x-axis. It can be expressed as a value or a coordinate. • The y-intercept is the point at which the graph of a relation or function intersects with the y-axis. It can be expressed as a value or a coordinate. • Functions describe the relationship between two variables where each input is paired with a unique output.

A.F.1 The student will investigate, analyze, and compare linear functions algebraically and graphically, and model linear relationships. *Additional Content Background and Instructional Guidance:*

- Function families consist of a parent function and all transformations of the parent function. The parent function for linear functions is $f(x) = x$.
- Transformations are limited to horizontal and vertical translations, reflections over the *x*-axis, and vertical dilations.
- For all functions, including linear functions, the transformation $f(x) + k$ translates the graph vertically by k units.
- For all functions, including linear functions, the transformation $kf(x)$ dilates the graph vertically by a factor of k. When $k < 0$, the graph reflects vertically.
- When graphing a parent function with multiple transformations, order of operations determines which transformation should be applied first.
- Changes in slope may be described by dilations, reflections, or both.
- Changes in the *y*-intercept may be described by translations.
- A line can be represented by its graph or by an equation. The equation of a line defines the relationship between two variables.
- Linear equations can be written in three forms: slope-intercept form, standard form, or point-slope form.
- Slope-intercept form, $y = mx + b$, expresses a line using the slope, m, and the y-intercept, b.
- Standard form, $Ax + By = C$, expresses a line using only integers, A, B, and C, for constants and coefficients.
- Point-slope form, $(y - y_1) = m(x - x_1)$, expresses a line using the slope, m, and any point on the line, (x_1, y_1).
- Lines in standard form can be graphed by solving for the x and y-intercepts.
- Standard form is particularly useful for solving systems of equations algebraically.
- The point-slope form can have multiple equivalent expressions as (x_1, y_1) can be any point on the line.
- The slopes of parallel lines are equal or congruent.
- The slopes of perpendicular lines are negative reciprocals (e.g., $\frac{2}{5}$ and $-\frac{5}{2}$).
- The product of the slope of two perpendicular lines is -1 unless one of the lines has an undefined slope.
- Linear equations can be graphed using a point that lies on the line and the slope of the line, *x*- and *y*-intercepts, two points that lie on the line, and/or transformations of the parent function.
- The graph of a line represents the set of points that satisfies the equation of a line.
- Each point on the graph of a linear equation in two variables is a solution of the equation.
- The *x*-coordinate of the point where the graphs of the linear equations $y = f(x)$ and $y = g(x)$ intersect is the solution of the equation $f(x) = g(x)$.
- For each *x* in the domain of *f*, *x* is a member of the input of the function *f*, *f(x)* is a member of the output of *f*, and the ordered pair *x*, *f(x)*) is a member of *f*.

A.F.2 The student will investigate, analyze, and compare characteristics of functions, including quadratic, and exponential functions, and model quadratic and exponential relationships.

Students will demonstrate the following Knowledge and Skills:

a) Determine whether a relation, represented by a set of ordered pairs, a table, a mapping, or a graph is a function; for relations that are functions, determine the domain and range.

b) Given an equation or graph, determine key characteristics of a quadratic function including x-intercepts (zeros), y-intercept, vertex (maximum or minimum), and domain and range (including when restricted by context); interpret key characteristics as related to contextual situations, where applicable.

c) Graph a quadratic function, $f(x)$, in two variables using a variety of strategies, including transformations $f(x) + k$ and $kf(x)$, where k is limited to rational values.

d) Make connections between the algebraic (standard and factored forms) and graphical representation of a quadratic function.

e) Given an equation or graph of an exponential function in the form $y = ab^x$ (where b is limited to a natural number), interpret key characteristics, including y-intercepts and domain and range; interpret key characteristics as related to contextual situations, where applicable.

f) Graph an exponential function, $f(x)$, in two variables using a variety of strategies, including transformations $f(x) + k$ and $kf(x)$, where k is limited to rational values.

g) For any value, x, in the domain of f, determine $f(x)$ of a quadratic or exponential function. Determine x given any value $f(x)$ in the range of f of a quadratic function. Explain the meaning of x and $f(x)$ in context.

h) Compare and contrast the key characteristics of linear functions ($f(x) = x$), quadratic functions ($f(x) = x^2$), and exponential functions ($f(x) = b^x$) using tables and graphs.

A.F.2 The student will investigate, analyze, and compare characteristics of functions, including quadratic, and exponential functions, and model quadratic and exponential relationships.

Additional Content Background and Instructional Guidance:

- A relation is a function if and only if each element in the domain is paired with a unique element of the range.
- Functions describe the relationship between two variables where each input is paired to a unique output.
- The domain of a function is the set of all possible values of the independent variable.
- The range of a function is the set of all possible values of the dependent variable.
- Solutions and intervals may be expressed in different formats, including equations and inequalities, set notation, and interval notation. Examples may include:

Equation/ Inequality	Set Notation	Interval Notation
$x = 3$	$\{3\}$	$\{3\}$
$x = 3$ or $x = 5$	$\{3, 5\}$	$\{3, 5\}$
$y \geq 3$	$\{y: y \geq 3\}$	$[3, \infty)$
$-2 < x \leq 6$	$\{x: -2 < x \leq 6\}$	$(-2, 6]$
Empty (null) set $\emptyset$	$\{\ \}$	$\{\ \}$

A.F.2 The student will investigate, analyze, and compare characteristics of functions, including quadratic, and exponential functions, and model quadratic and exponential relationships.

Additional Content Background and Instructional Guidance:

- For each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair $(x, f(x))$ is a member of f.
- A value, x, in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$.
- Given a quadratic function $f(x)$, the following statements are equivalent for any real number, k, such that $f(k) = 0$:
  - k is a zero of the function $f(x)$, located at $(k, 0)$;
  - $(x – k)$ is a factor of $f(x)$;
  - k is a solution or root of the equation $f(x) = 0$; and
  - the point $(k, 0)$ is an x-intercept for the graph of $y = f(x)$.
- The x-intercept is the point at which the graph of a relation or function intersects with the x-axis. It can be expressed as a value or a coordinate.
- The y-intercept is the point at which the graph of a relation or function intersects with the y-axis. It can be expressed as a value or a coordinate.
- The y-intercept of a quadratic function is the constant term when a function is written in standard form.
- The vertex of a quadratic function can be found graphically as well as algebraically.
- Given the quadratic function $f(x) = ax^2 + bx + c$, the x-coordinate of the vertex can be found by solving $\frac{-b}{2a}$. The y-coordinate can be found by substituting the produced x-coordinate back into the function.
- The domain of a function may be restricted by the practical situation modeled by a function.
- Function families consist of a parent function and all transformations of the parent function.
  - The parent function for quadratics is $f(x) = x^2$.
  - The parent function for exponential functions is $f(x) = b^x$ where b is a natural number.
- Transformations are limited to horizontal and vertical translations, reflections over the x-axis, and vertical dilations.
- For all functions, including quadratic and exponential functions, the transformation $f(x) + k$ translates the graph vertically by k units.
- For all functions, including quadratic and exponential functions, the transformation $kf(x)$ dilates the graph vertically by a factor of k. When $k < 0$, the graph reflects vertically.

Statistics

A.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear and quadratic functions.

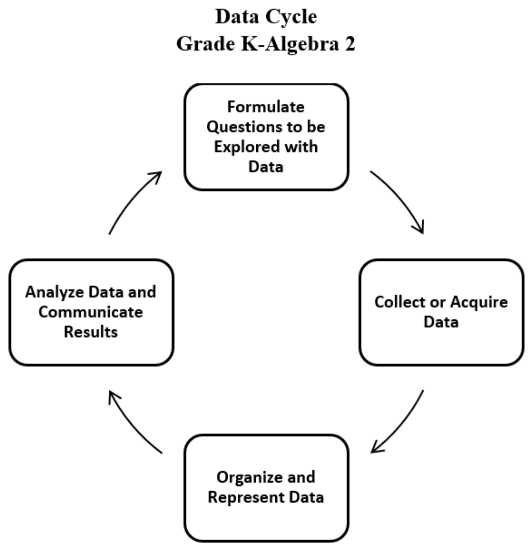
Students will demonstrate the following Knowledge and Skills:

a) Formulate investigative questions that require the collection or acquisition of bivariate data.
b) Determine what variables could be used to explain a given contextual problem or situation or answer investigative questions.
c) Determine an appropriate method to collect a representative sample, which could include a simple random sample, to answer an investigative question.
d) Given a table of ordered pairs or a scatterplot representing no more than 30 data points, use available technology to determine whether a linear or quadratic function would represent the relationship, and if so, determine the equation of the curve of best fit.
e) Use linear and quadratic regression methods available through technology to write a linear or quadratic function that represents the data where appropriate and describe the strengths and weaknesses of the model.
f) Use a linear model to predict outcomes and evaluate the strength and validity of these predictions, including through the use of technology.
g) Investigate and explain the meaning of the rate of change (slope) and y-intercept (constant term) of a linear model in context.
h) Analyze relationships between two quantitative variables revealed in a scatterplot.
i) Make conclusions based on the analysis of a set of bivariate data and communicate the results.

A.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear and quadratic functions. *Additional Content Background and Instructional Guidance:*
• Students should explore the entire data cycle with a question and set of data that has been collected or acquired. Student reflection should occur throughout the data cycle. The data cycle includes the following steps: formulating questions to be explored with data, collecting or acquiring data, organizing and representing data, and analyzing and communicating results.

A.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear and quadratic functions.

Additional Content Background and Instructional Guidance:



- Statistical questions should not be worded in a way that leads an individual to a particular answer.
- Many problems can be solved by using a mathematical model as an interpretation of a practical situation. The solution must then refer to the original practical situation.
- Randomization is a process that often results in a representative sample.
- A simple random sample is one type of random sampling method that gives every member of the population an equal chance of being selected (e.g., pulling names out of a hat).
- Non-random sampling methods such as convenience sampling may result in a sample that is not representative of the population.
- Collecting data involves conducting an experiment, surveying or polling individuals, making observations or measurements, etc. Acquiring data involves gathering data that already exists.
- Data and scatterplots may indicate patterns that can be modeled with an algebraic equation.
- Determining the curve of best fit for a relationship among a set of data points is a tool for the algebraic analysis of data. In Algebra 1, curves of best fit are limited to linear or quadratic functions.
- By examining patterns in data (shape), given different representations like a table or scatterplot, specific bivariate relationships can be determined.
- Bivariate data is data that is collected from two different variables and compared against each other.
- Knowledge of transformational graphing using parent functions can be used to verify a mathematical model from a scatterplot that approximates the data.
- Categorical variables can be added to a scatterplot using color or different symbols.
- Graphing utilities can be used to collect, organize, represent, and generate an equation of a curve of best fit for a set of data.

A.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear and quadratic functions.

Additional Content Background and Instructional Guidance:

- Data that fit linear $y = mx + b$ and quadratic $y = ax^2 + bx + c$ functions arise from practical situations.
- The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate.
- Rounding that occurs during intermediate steps of problem solving may reduce the accuracy of the final answer.
- Interpolation is a method of estimating values within a set of data points based on the known values of the surrounding points. In other words, it involves using the data points that are available to make an educated guess about the value of a data point that is not explicitly given.
- Extrapolation is the process of estimating values outside the range of known data by extending a curve or trend line beyond the observed data points. In other words, it involves making predictions about values that are outside the range of the available data based on the assumption that the same trend will continue beyond the observed data.
- The validity of predictions decreases as the degree of extrapolation increases.
- Evaluation of the reasonableness of a mathematical model of a practical situation involves asking questions including:
- "Is there another linear or quadratic curve that better fits the data?"
- "Does the curve of best fit make sense?"
- "Could the curve of best fit be used to make reasonable predictions?"