2023 Mathematics *Standards of Learning*
Understanding the Standards – Algebra 2

The Understanding the Standards document includes the mathematics understandings and key concepts that assist teachers in planning standards-focused instruction of the 2023 Mathematics *Standards of Learning* for Algebra 2. The Understanding the Standards document includes definitions, explanations, and examples regarding each mathematics standard and describes what students should know (core knowledge) as a result of the instruction specifically aligned to the course/grade level.

Expressions and Operations

A2.EO.1 The student will perform operations on and simplify rational expressions.

Students will demonstrate the following Knowledge and Skills:

a) Add, subtract, multiply, or divide rational algebraic expressions, simplifying the result.

b) Justify and determine equivalent rational algebraic expressions with monomial and binomial factors. Algebraic expressions should be limited to linear and quadratic expressions.

c) Recognize a complex algebraic fraction and simplify it as a product or quotient of simple algebraic fractions.

d) Represent and demonstrate equivalence of rational expressions written in different forms.

A2.EO.1 The student will perform operations on and simplify rational expressions.

Additional Content Background and Instructional Guidance:

- A rational algebraic expression is the ratio of two polynomial expressions.
- Computational skills applicable to numerical fractions also apply to rational algebraic expressions.
- In this standard, denominators are assumed to be non-zero; however, students would benefit from learning experiences that make connections between values of the variable that would lead to having a zero in the denominator of a rational expression and domain restrictions of the corresponding rational function.
- A complex algebraic fraction is a rational algebraic expression where one or both of the numerator or denominator is also a rational algebraic expression.
- A complex algebraic fraction can be rewritten in an equivalent form as the quotient or product of simple algebraic fractions and then simplified to create a third equivalent form.
- Rewriting a rational expression in different but equivalent forms allows some aspects of the expression to be more apparent. For example, it may be easier to visualize the simplified form of a rational expression, but this may not reveal possible domain restrictions.

A2.EO.2 The student will perform operations on and simplify radical expressions.

Students will demonstrate the following Knowledge and Skills:

a) Simplify and determine equivalent radical expressions that include numeric and algebraic radicands.

b) Add, subtract, multiply, and divide radical expressions that include numeric and algebraic radicands, simplifying the result. Simplification may include rationalizing the denominator.

c) Convert between radical expressions and expressions containing rational exponents.

A2.EO.2 The student will perform operations on and simplify radical expressions. *Additional Content Background and Instructional Guidance:*
• When simplifying radicals that have even indices (such as a square root), examine the radicand. If the radicand contains an algebraic expression with a negative coefficient, then use the imaginary unit, i to simplify the radical completely. • In Algebra 2, students may benefit from conversations about how techniques used to simplify square root and cube root expressions can be applied to simplify radical expressions with higher indices. • Only radicals with a common radicand and index can be added or subtracted, which may require writing the radical in an equivalent form using a lower base and different index. • Multiplying and dividing radical expressions containing different indices may require writing the expression in an equivalent form using rational exponents. • Radical expressions can be written in an equivalent form using rational exponents. • In Algebra 2, any variables in a radical expression will be assumed to be non-negative, but students may benefit from conversations about when absolute value notation would be necessary in simplifying a radical expression.

A2.EO.3 The student will perform operations on polynomial expressions and factor polynomial expressions in one and two variables.

Students will demonstrate the following Knowledge and Skills:

a) Determine sums, differences, and products of polynomials in one and two variables.

b) Factor polynomials completely in one and two variables with no more than four terms over the set of integers.

c) Determine the quotient of polynomials in one and two variables, using monomial, binomial, and factorable trinomial divisors.

d) Represent and demonstrate equality of polynomial expressions written in different forms and verify polynomial identities including the difference of squares, sum and difference of cubes, and perfect square trinomials.

A2.EO.3 The student will perform operations on polynomial expressions in two or more variables and factor polynomial expressions in one and two variables. *Additional Content Background and Instructional Guidance:*
• Combining like terms is a method which should be employed when adding or subtracting polynomial expressions. • Applying laws of exponents is required when multiplying or dividing polynomial expressions. • Factoring polynomials completely assists with dividing and simplifying polynomial expressions. • The complete factorization of polynomials occurs when each factor cannot be written as the product of polynomials of lower degree. • Polynomials may be factored in various ways, including but not limited to, grouping or recognizing general patterns such as difference of squares, sum and difference of cubes, and perfect square trinomials. • Techniques for factoring quadratic expressions can be extended to factoring some higher degree binomials and trinomials. For example, $x^4 + 2x^2 - 8$ can be expressed in an equivalent form as $(x^2 + 4)(x^2 - 2)$. • For division of polynomials in this standard, students may benefit from experiences with multiple methods, to include, but are not limited to long or synthetic division. • Polynomial expressions can be used to define functions and these functions can be represented graphically. • Rewriting a polynomial expression in different but equivalent forms allows some aspects of the expression to be more apparent. For example, a quadratic expression written in vertex form allows the vertex to be found by visual inspection of the expression. The quadratic written in standard form allows for the y-intercept to be found by visual inspection.

A2.EO.4 The student will perform operations on complex numbers.

Students will demonstrate the following Knowledge and Skills:

a) Explain the meaning of i.

b) Identify equivalent radical expressions containing negative rational numbers and expressions in $a + bi$ form.

c) Apply properties to add, subtract, and multiply complex numbers.

A2.EO.4 The student will perform operations on complex numbers. *Additional Content Background and Instructional Guidance:*

• i is the imaginary unit that satisfies the equation $i^2 = -1$, where $i = \sqrt{-1}$.
• i can be described using a cyclical approach:
  ○ $i = \sqrt{-1}$
  ○ $i^2 = -1$
  ○ $i^3 = -i$
  ○ $i^4 = 1$
  ○ $i^5 = \sqrt{-1}$
• All complex numbers can be written in the form $a + bi$, where a and b are real numbers.
  ○ a is considered to be the real part of the complex number
  ○ bi is considered to be the imaginary part of the complex number
• Real numbers and pure imaginary numbers are subsets of the complex number system. For example:
  ○ $5 = 5 + 0i$
  ○ $\pm\sqrt{-9} = 0 \pm 3i$
• The conjugate of the complex number $a + bi$ is $a - bi$.
• A complex number multiplied by its conjugate is a non-negative real number.
• Algebraic properties apply to complex numbers as well as real numbers.

Equations and Inequalities

A2.EI.1 The student will represent, solve, and interpret the solution to absolute value equations and inequalities in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Create an absolute value equation in one variable to model a contextual situation.
b) Solve an absolute value equation in one variable algebraically and verify the solution graphically.
c) Create an absolute value inequality in one variable to model a contextual situation.
d) Solve an absolute value inequality in one variable and represent the solution set using set notation, interval notation, and using a number line.
e) Verify possible solution(s) to absolute value equations and inequalities in one variable algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

A2.EI.1 The student will represent, solve, and interpret the solution to absolute value equations and inequalities in one variable.

Additional Content Background and Instructional Guidance:

- The definition of absolute value (for any real numbers a and b, where $b \geq 0$, if $|a| = b$, then $a = b$ or $a = -b$) is used in solving absolute value equations and inequalities.
- The absolute value of any number is the distance from that number to zero on a number line.
- Absolute value inequalities in one variable can be solved algebraically using a compound statement.
- Compound statements representing solutions of an inequality in one variable can be represented graphically on a number line.
- Practical problems can be interpreted, represented, and solved using equations and inequalities.
- Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation.
- Interval notation is used when describing a range of values. Thus, interval notation could be used to define solutions to inequalities.
- Examples may include:

Equation/Inequality	Set Notation	Interval Notation
$x = 3$	$\{3\}$	
$x = 3$ or $x = 5$	$\{3, 5\}$	
$0 \leq x < 3$	$\{x \mid 0 \leq x < 3\}$	$[0, 3)$
$y \geq 3$	$\{y : y \geq 3\}$	$[3, \infty)$
Empty (null) set $\emptyset$	$\{\ \}$	

- The process of solving equations or inequalities can lead to extraneous solutions.
- An extraneous solution is a solution of the simplified form of an equation/inequality that does not satisfy the original equation/inequality. Use substitution to verify solutions.

A2.EI.2 The student will represent, solve, and interpret the solution to quadratic equations in one variable over the set of complex numbers and solve quadratic inequalities in one variable.

Students will demonstrate the following Knowledge and Skills:

a) Create a quadratic equation or inequality in one variable to model a contextual situation.

b) Solve a quadratic equation in one variable over the set of complex numbers algebraically.

c) Determine the solution to a quadratic inequality in one variable over the set of real numbers algebraically.

d) Verify possible solution(s) to quadratic equations or inequalities in one variable algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

A2.EI.2 The student will represent, solve, and interpret the solution to quadratic equations in one variable over the set of complex numbers and solve quadratic inequalities in one variable.

Additional Content Background and Instructional Guidance:

- Quadratic equations and inequalities can be used to represent, interpret, and solve contextual problems.
- Quadratic equations can be solved in a variety of ways, including graphing, factoring, the quadratic formula, and completing the square.
- The quadratic formula and completing the square can be used to solve any quadratic equation over the set of complex numbers.
- The quadratic formula shows that the solutions to the equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- The discriminant of the equation $ax^2 + bx + c = 0$ is defined as $b^2 - 4ac$.
- The value of the discriminant of a quadratic equation can be used to describe the number and type of solutions to the equation:
  - If $b^2 - 4ac > 0$, then the equation has two real solutions. Further, if the discriminant is a rational number, then the original equation could be solved by factoring.
  - If $b^2 - 4ac = 0$, then the equation has one real solution with a multiplicity of 2. Further, the original equation is a perfect square trinomial.
  - If $b^2 - 4ac < 0$, then the equation has two complex, non-real, solutions.
- The quadratic formula can be derived by applying the completion of squares to any quadratic equation in standard form.
- Solutions of quadratic equations are real or a sum or difference of a real and imaginary component.
- Complex solutions occur in conjugate pairs.
- Quadratic equations with exactly one real root can be referred to as having one distinct root with a multiplicity of two. This is called a double zero. For instance, the quadratic equation, $x^2 - 4x + 4$, has two identical factors, giving one real root with a multiplicity of two. In this case, the equation is a perfect square trinomial.
- Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation.

A2.EI.2 The student will represent, solve, and interpret the solution to quadratic equations in one variable over the set of complex numbers and solve quadratic inequalities in one variable.

Additional Content Background and Instructional Guidance:

- Interval notation is used when describing a range of values. Thus, interval notation could be used to define solutions to inequalities.
- Examples may include:

Equation/Inequality	Set Notation	Interval Notation
$x = 3$	$\{3\}$	
$x = 3$ or $x = 5$	$\{3, 5\}$	
$x = -2 - 5i$ or $x = -2 + 5i$	$\{-2 - 5i, -2 + 5i\}$ or $\{-2 \pm 5i\}$	
$0 \leq x < 3$	$\{x \mid 0 \leq x < 3\}$	$[0, 3)$
$y \geq 3$	$\{y: y \geq 3\}$	$[3, \infty)$
Empty (null) set ∅	$\{\ \}$	

- Sign charts can be used to solve quadratic inequalities.
  Example: $x^2 + 3x + 2 \geq 0$
  $(x + 2)(x + 1) \geq 0$
  Equality holds when $x = -2$ and $x = -1$
  Perform sign analysis:

Interval	$x < -2$	$-2 < x < -1$	$x > -1$
Test Value	-3	-1.5	0
Sign Analysis	$(-)(-) = +$	$(+)(-) = -$	$(+)(+) = +$

- Solution: $x \leq -2$ and $x \geq -1$

A2.EI.3 The student will solve a system of equations in two variables containing a quadratic expression.

Students will demonstrate the following Knowledge and Skills:

a) Create a linear-quadratic or quadratic-quadratic system of equations to model a contextual situation.

b) Determine the number of solutions to a linear-quadratic and quadratic-quadratic system of equations in two variables.

c) Solve a linear-quadratic and quadratic-quadratic system of equations algebraically and graphically, including situations in context.

d) Verify possible solution(s) to linear-quadratic or quadratic-quadratic system of equations algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

A2.EI.3 The student will solve a system of equations in two variables containing a quadratic expression.

Additional Content Background and Instructional Guidance:

- Systems of equations can be used to represent, interpret, and solve contextual problems.
- The coordinates of points of intersection in any system of equations are solutions to the system.
- Quadratic equations included in this standard will only include those that can be represented as parabolas of the form $y = ax^2 + bx + c$ where $a \neq 0$.
- Solutions of a system of equations are numerical values that satisfy every equation in the system.
- A linear-quadratic system of equations may have zero, one, or two solutions.
- A quadratic-quadratic system of equations may have zero, one, two, or an infinite number of solutions.
- Solving an equation or system of equations graphically with technology may lead to approximate solutions. The context of the problem should be considered when determining whether to report an exact or approximate solution.

A2.EI.4 The student will represent, solve, and interpret the solution to an equation containing rational algebraic expressions.

Students will demonstrate the following Knowledge and Skills:

a) Create an equation containing a rational expression to model a contextual situation.
b) Solve rational equations with real solutions containing factorable algebraic expressions algebraically and graphically. Algebraic expressions should be limited to linear and quadratic expressions.
c) Verify possible solution(s) to rational equations algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.
d) Justify why a possible solution to an equation containing a rational expression might be extraneous.

A2.EI.4 The student will represent, solve, and interpret the solution to an equation containing rational algebraic expressions. *Additional Content Background and Instructional Guidance:*
• Equations that contain rational expressions can be used to represent, interpret, and solve contextual problems. • Equations that contain rational expressions can be solved in a variety of ways. • The process of solving equations that contain rational expressions can lead to extraneous solutions. • An extraneous solution is a solution of the simplified form of an equation that does not satisfy the original equation. Use substitution to verify solutions. • The process used to solve an equation with a rational expression may lead to solving an equivalent equation containing a polynomial expression. In Algebra 2, these experiences should be limited to equivalent linear or quadratic equations. • Solving an equation graphically with technology may lead to approximate solutions. The context of the problem should be considered when determining whether to report an exact or approximate solution.

A2.EI.5 The student will represent, solve, and interpret the solution to an equation containing a radical expression.

Students will demonstrate the following Knowledge and Skills:

a) Solve an equation containing no more than one radical expression algebraically and graphically.

b) Verify possible solution(s) to radical equations algebraically, graphically, and with technology, to justify the reasonableness of answer(s). Explain the solution method and interpret solutions for problems given in context.

c) Justify why a possible solution to an equation with a square root might be extraneous.

A2.EI.5 The student will represent, solve, and interpret the solution to an equation containing a radical expression.

Additional Content Background and Instructional Guidance:

- Equations that contain a radical expression can be used to represent, interpret, and solve contextual problems.
- Equations that contain a radical expression can be solved in a variety of ways.
- Radical expressions may be converted to expressions using rational exponents.
- The process of solving equations that contain a radical expression can lead to extraneous solutions.
- An extraneous solution is a solution of the simplified form of an equation that does not satisfy the original equation. Use substitution to verify solutions.
- In Algebra 2, solving equations involving radical expressions is limited to those with square root expressions, but students may benefit from conversations about how techniques used to solve equations containing square roots apply to solving equations containing radical expressions with higher indices.
- The process used to solve an equation containing the square root of an algebraic expression and a linear expression may involve solving an equivalent quadratic equation.
- Solving an equation graphically with technology may lead to approximate solutions. The context of the problem should be considered when determining whether to report an exact or approximate solution.

A2.EI.6 The student will represent, solve, and interpret the solution to a polynomial equation.

Students will demonstrate the following Knowledge and Skills:

a) Determine a factored form of a polynomial equation, of degree three or higher, given its zeros or the *x*-intercepts of the graph of its related function.

b) Determine the number and type of solutions (real or imaginary) of a polynomial equation of degree three or higher.

c) Solve a polynomial equation over the set of complex numbers.

d) Verify possible solution(s) to polynomial equations of degree three or higher algebraically, graphically, and with technology to justify the reasonableness of answer(s). Explain the solution method and interpret solutions in context.

A2.EI.6 The student will represent, solve, and interpret the solution to a polynomial equation. *Additional Content Background and Instructional Guidance:*
• Polynomial equations can be used to represent, interpret, and solve contextual problems. • Polynomial equations can be solved in a variety of ways. • The degree of a polynomial equation is the largest power or exponent of a variable in the equation. • The Fundamental Theorem of Algebra states that, including complex and repeated solutions, an n^{th} degree polynomial equation has exactly n roots (solutions). • Solutions of polynomial equations may be real or imaginary. • Imaginary solutions occur in conjugate pairs. • Polynomial equations may have fewer distinct roots than the degree of the polynomial. In these situations, a root may have "multiplicity." For instance, the polynomial equation $y = x^3 - 6x^2 + 9x$ has two identical factors, $(x - 3)$, and one other factor, x. This polynomial equation has two distinct, real roots, one with a multiplicity of 2. • Solving an equation graphically with technology may lead to approximate solutions. The context of the problem should be considered when determining whether to report an exact or approximate solution. • Polynomial equations can be solved using graphing, factoring, or the quadratic formula, or some combination of these. • In Algebra 2, students may benefit from experiences with multiple methods, to include, but are not limited to long or synthetic division to solve polynomial equations.

Functions

A2.F.1 The student will investigate, analyze, and compare square root, cube root, rational, exponential, and logarithmic function families, algebraically and graphically, using transformations.

Students will demonstrate the following Knowledge and Skills:

a) Distinguish between the graphs of parent functions for square root, cube root, rational, exponential, and logarithmic function families.

b) Write the equation of a square root, cube root, rational, exponential, and logarithmic function, given a graph, using transformations of the parent function, including $f(x) + k$; $f(kx)$; $f(x + k)$; and $kf(x)$, where k is limited to rational values. Transformations of exponential and logarithmic functions, given a graph, should be limited to a single transformation.

c) Graph a square root, cube root, rational, exponential, and logarithmic function, given the equation, using transformations of the parent function including $f(x) + k$; $f(kx)$; $f(x + k)$; and $kf(x)$, where k is limited to rational values. Use technology to verify transformations of the functions.

d) Determine when two variables are directly proportional, inversely proportional, or neither, given a table of values. Write an equation and create a graph to represent a direct or inverse variation, including situations in context.

e) Compare and contrast the graphs, tables, and equations of square root, cube root, rational, exponential, and logarithmic functions.

A2.F.1 The student will investigate, analyze, and compare square root, cube root, rational, exponential, and logarithmic function families, algebraically and graphically, using transformations.

Additional Content Background and Instructional Guidance:

- Connections between multiple representations (graphs, tables, and equations) of a function can be made.
- The transformation of a function, called a pre-image, changes the size, shape, and/or position of the function to a new function, called the image.
- The graphs/equations for a family of functions can be determined using a transformational approach.
- The graph of a parent function is an anchor graph from which other graphs are derived using transformations.
- Function families consist of a parent function and all transformations of the parent function.
- Transformations of functions act in a similar way as transformations on plane figures which students studied in Geometry and middle school mathematics.
- Transformations of graphs include:
  - Translations (horizontal and/or vertical shifting of a graph) which is represented by the function notation $f(x) + k$ and $f(x + k)$;
  - Reflections over the y-axis which is represented by the function notation $f(-x)$;
  - Reflections over the x-axis which is represented by the function notation $-f(x)$; and
  - Dilations (vertical stretching and compressing of graphs) which is represented by the function notation $f(kx)$ and $kf(x)$.

A2.F.1 The student will investigate, analyze, and compare square root, cube root, rational, exponential, and logarithmic function families, algebraically and graphically, using transformations. *Additional Content Background and Instructional Guidance:*
• A direct variation is a linear relationship that represents a proportional relationship between two quantities. If y is directly proportional to x, then $y = kx$, where k is the constant of proportionality. • The constant of proportionality (k) in a direct variation is represented by the ratio of the dependent variable to the independent variable. This is also referred to as the constant of variation. • A direct variation represents a linear relationship, where the constant of proportionality (k) is the slope of the line and the y-intercept is zero. • A direct variation can be represented graphically by a line passing through the origin. • An inverse variation represents an inversely proportional relationship between two quantities and is represented by a rational function. If y is inversely proportional to x, then $y = \frac{k}{x}$. • The constant of proportionality (k) in an inverse variation is represented by the product of the dependent variable and the independent variable. This is also referred to as the constant of variation. • The value of the constant of proportionality is typically positive when applied in contextual situations. • Contextual situations involving direct and inverse variation occur in Chemistry.

A2.F.2 The student will investigate and analyze characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions algebraically and graphically.

Students will demonstrate the following Knowledge and Skills:

a) Determine and identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically, including graphs with discontinuities.

b) Compare and contrast the characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions.

c) Determine the intervals on which the graph of a function is increasing, decreasing, or constant.

d) Determine the location and value of absolute (global) maxima and absolute (global) minima of a function.

e) Determine the location and value of relative (local) maxima or relative (local) minima of a function.

f) For any value, x, in the domain of f, determine $f(x)$ using a graph or equation. Explain the meaning of x and $f(x)$ in context, where applicable.

g) Describe the end behavior of a function.

h) Determine the equations of any vertical and horizontal asymptotes of a function using a graph or equation (rational, exponential, and logarithmic).

i) Determine the inverse of a function algebraically and graphically, given the equation of a linear or quadratic function (linear, quadratic, and square root). Justify and explain why two functions are inverses of each other.

j) Graph the inverse of a function as a reflection over the line $y = x$.

k) Determine the composition of two functions algebraically and graphically.

A2.F.2 The student will investigate and analyze characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions algebraically and graphically.

Additional Content Background and Instructional Guidance:

- Functions may be used to represent contextual situations.
- Functions describe the relationship between two variables where each input is paired to a unique output.
- The domain of a function is the set of all possible values of the independent variable.
- The range of a function is the set of all possible values of the dependent variable.
- Given f is a function: for each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair $(x, f(x))$ is a member of f. In other words, $f(x)$ represents values of the range and x represents values of the domain.
- The domain of a function may be restricted algebraically, graphically, or by the contextual situation represented by a function.
- Given a polynomial function $f(x)$, the following statements are equivalent for any real number, k, such that $f(k) = 0$:
    - k is a zero of the polynomial function $f(x)$ located at $(k, 0)$;
    - k is a solution or root of the polynomial equation $f(x) = 0$;
    - the point $(k, 0)$ is an x-intercept for the graph of $f(x) = 0$; and

A2.F.2 The student will investigate and analyze characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions algebraically and graphically.

Additional Content Background and Instructional Guidance:

- ($x - k$) is a factor of $f(x)$.

- A function is said to be continuous on an interval if its graph has no jumps or holes in that interval.
- Discontinuous domains and ranges include those with removable (holes) and nonremovable (asymptotes) discontinuities.
- A function can be described on an interval as increasing, decreasing, or constant over a specified interval or over the entire domain of the function.
- A function, $f(x)$, is increasing over an interval if the values of $f(x)$ consistently increase over the interval as the x values increase.
- A function, $f(x)$, is decreasing over an interval if the values of $f(x)$ consistently decrease over the interval as the x values increase.
- A function, $f(x)$, is constant over an interval if the values of $f(x)$ remain constant over the interval as the x values increase.
- Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation.
- Interval notation is used when describing a range of values.
- Examples may include:

Equation/Inequality	Set Notation	Interval Notation
$x = 3$	$\{3\}$	
$x = 3$ or $x = 5$	$\{3, 5\}$	
$0 \leq x < 3$	$\{x \mid 0 \leq x < 3\}$	$[0, 3)$
$y \geq 3$	$\{y: y \geq 3\}$	$[3, \infty)$
Empty (null) set $\emptyset$	$\{\ \}$	

- A function, f, has an absolute maximum located at $x = a$ if $f(a)$ is the largest value of f over its domain.
- A function, f, has an absolute minimum located at $x = a$ if $f(a)$ is the smallest value of f over its domain.
- Relative maximum points occur where the function changes from increasing to decreasing.
- A function, f, has a relative maximum located at $x = a$ over some open interval of the domain if $f(a)$ is the largest value of f on the interval.
- Relative minimum points occur where the function changes from decreasing to increasing.
- A function, f, has a relative minimum located at $x = a$ over some open interval of the domain if $f(a)$ is the smallest value of f on the interval.
- A value x in the domain of f is an x-intercept or a zero of a function f if and only if $f(x) = 0$.
- End behavior describes the values of a function as x approaches positive or negative infinity.
- If (a, b) is an element of a function, then (b, a) is an element of the inverse of the function.
- The reflection of a function over the line $y = x$ represents the inverse of the reflected function.

A2.F.2 The student will investigate and analyze characteristics of square root, cube root, rational, polynomial, exponential, logarithmic, and piecewise-defined functions algebraically and graphically. *Additional Content Background and Instructional Guidance:*
• A function is invertible if its inverse is also a function. For an inverse of a function to be a function, the domain of the function may need to be restricted. • Functions can be combined using composition of functions. • Two functions, $f(x)$ and $g(x)$, are inverses of each other if $f(g(x)) = g(f(x)) = x$. • For contextual situations, exponential functions can be used to represent compound interest, population growth, and radioactive decay. • Restrictions on the domain may need to be added to the function to appropriately represent the contextual situation.

Statistics

A2.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on univariate quantitative data represented by a smooth curve, including a normal curve.

Students will demonstrate the following Knowledge and Skills:

a) Formulate investigative questions that require the collection or acquisition of a large set of univariate quantitative data or summary statistics of a large set of univariate quantitative data and investigate questions using a data cycle.
b) Collect or acquire univariate data through research, or using surveys, observations, scientific experiments, polls, or questionnaires.
c) Examine the shape of a data set (skewed versus symmetric) that can be represented by a histogram, and sketch a smooth curve to model the distribution.
d) Identify the properties of a normal distribution.
e) Describe and interpret a data distribution represented by a smooth curve by analyzing measures of center, measures of spread, and shape of the curve.
f) Calculate and interpret the *z*-score for a value in a data set.
g) Compare two data points from two different distributions using *z*-scores.
h) Determine the solution to problems involving the relationship of the mean, standard deviation, and *z*-score of a data set represented by a smooth or normal curve.
i) Apply the Empirical Rule to answer investigative questions.
j) Compare multiple data distributions using measures of center, measures of spread, and shape of the distributions.

A2.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on univariate quantitative data represented by a smooth curve, including a normal curve.

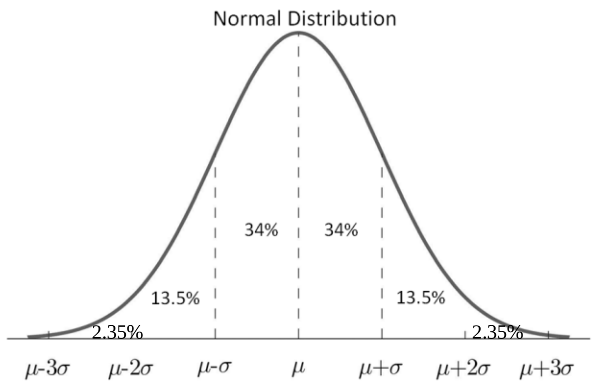
Additional Content Background and Instructional Guidance:

- There are data sets that cannot be represented by a smooth or normal curve.
- A very large data set provides a representation that can closely approximate the population.
- Summary statistics include measures of center (mean, median, mode) and dispersion or spread (variance and standard deviation). These statistics can be used to approximate the shape of the distribution.
- Descriptive statistics include measures of center (mean, median, mode) and dispersion or spread (variance and standard deviation).
- Variance (σ^2) and standard deviation (σ) measure the spread of data about the mean in a data set.
- Standard deviation is expressed in the original units of measurement of the data.
- The greater the value of the standard deviation, the further the data tends to be dispersed from the mean.
- In order to develop an understanding of standard deviation as a measure of dispersion (spread), students should have experience analyzing the formulas for and the relationship between variance and standard deviation.

A2.ST.1 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on univariate quantitative data represented by a smooth curve, including a normal curve.

Additional Content Background and Instructional Guidance:

- A normal distribution curve is the family of symmetrical, bell-shaped curves defined by the mean and the standard deviation of a data set. The arithmetic mean (μ) is located on the line of symmetry of the curve and is approximately equivalent to the median and mode of the data set.
- The normal curve is a probability distribution and the total area under the curve is 1.
- For a normal distribution, approximately 68 percent of the data fall within one standard deviation of the mean, approximately 95 percent of the data fall within two standard deviations of the mean, and approximately 99.7 percent of the data fall within three standard deviations of the mean. This is often referred to as the Empirical Rule or the 68 – 95 – 99.7 rule.



NOTE: This chart illustrates percentages that correspond to subdivisions in one standard deviation increments. Percentages for other subdivisions require the table of Standard Normal Probabilities or a graphing utility.

- The mean and standard deviation of a normal distribution affect the location and shape of the curve. The vertical line of symmetry of the normal distribution falls at the mean. The greater the standard deviation, the wider (“flatter” or “less peaked”) the distribution of the data.
- A z-score derived from a particular data value tells how many standard deviations that data value falls above or below the mean of the data set. It is positive if the data value lies above the mean and negative if the data value lies below the mean.
- A standard normal distribution is the set of all z-scores. The mean of the data in a standard normal distribution is 0 and the standard deviation is 1. This allows for the comparison of unlike normal data.
- Graphing utilities can be used to represent a normally distributed data set and explore relationships between the data set and its descriptive statistics.

A2.ST.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear, quadratic, exponential, or a combination of these functions.

Students will demonstrate the following Knowledge and Skills:

a) Formulate investigative questions that require the collection or acquisition of bivariate data and investigate questions using a data cycle.
b) Collect or acquire bivariate data through research, or using surveys, observations, scientific experiments, polls, or questionnaires.
c) Represent bivariate data with a scatterplot using technology.
d) Determine whether the relationship between two quantitative variables is best approximated by a linear, quadratic, exponential, or a combination of these functions.
e) Determine the equation(s) of the function(s) that best models the relationship between two variables using technology. Curves of best fit may include a combination of linear, quadratic, or exponential (piecewise-defined) functions.
f) Use the correlation coefficient to designate the goodness of fit of a linear function using technology.
g) Make predictions, decisions, and critical judgments using data, scatterplots, or the equation(s) of the mathematical model.
h) Evaluate the reasonableness of a mathematical model of a contextual situation.

A2.ST.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear, quadratic, exponential, or a combination of these functions.

Additional Content Background and Instructional Guidance:

- Data and scatterplots may indicate patterns that can be represented with an algebraic equation.
- Categorical variables can be added to a scatterplot using color or different symbols.
- Technology such as spreadsheets and graphing utilities can be used to collect, organize, represent, and generate a mathematical model for a set of data.
- Knowledge of transformational graphing using parent functions can be used to verify a mathematical model from a scatterplot that approximates the data.
- Data that fit linear ($y = ax + b$), quadratic ($y = ax^2 + bx + c$), and exponential ($y = ab^x$) represent arise from contextual situations.
- Correlation coefficient measures the strength of a linear correlation of variables. Correlation coefficients can range from -1 to 1, where -1 is a perfectly linear negative correlation, 0 suggests little to no correlation, and 1 is a perfectly linear positive correlation.
- The mathematical model of the relationship among a set of data points can be used to make predictions, decisions, and critical judgements where appropriate.
- The curve of best fit for the relationship among a set of data points can be used to make predictions where appropriate.
- Rounding that occurs during intermediate steps of problem solving may reduce the accuracy of the final answer.

A2.ST.2 The student will apply the data cycle (formulate questions; collect or acquire data; organize and represent data; and analyze data and communicate results) with a focus on representing bivariate data in scatterplots and determining the curve of best fit using linear, quadratic, exponential, or a combination of these functions. *Additional Content Background and Instructional Guidance:*
• Evaluation of the reasonableness of a mathematical model of a contextual situation involves asking questions including: o “Is there another curve (quadratic or exponential) that better fits the data?” o “Does the curve of best fit make sense?” o “Could the curve of best fit be used to make reasonable predictions?” o “Is some subset of the data better represented by a different function?” o “For what values of the domain is the model appropriate?”

A2.ST.3 The student will compute and distinguish between permutations and combinations.

Students will demonstrate the following Knowledge and Skills:

a) Compare and contrast permutations and combinations to count the number of ways that events can occur.

b) Calculate the number of permutations of n objects taken r at a time.

c) Calculate the number of combinations of n objects taken r at a time.

d) Use permutations and combinations as counting techniques to solve contextual problems.

e) Calculate and verify permutations and combinations using technology.

A2.ST.3 The student will compute and distinguish between permutations and combinations. *Additional Content Background and Instructional Guidance:*
• The *Fundamental Counting Principle* states that if one decision can be made n ways and another can be made m ways, then the two decisions can be made nm ways. • A permutation is the number of possible ways to arrange a group of objects without repetition and when order matters (e.g., the outcome 1, 2, 3 is different from the outcome 3, 2, 1 when order matters; therefore, both arrangements would be included in the possible outcomes). • A combination is the number of possible ways to select or arrange objects when there is no repetition and order does not matter (e.g., the outcome 1, 2, 3 is the same as the outcome 3, 2, 1 when order does not matter; therefore, both arrangements would not be included in the possible outcomes).