

Types of Variation

Strand: Functions

Topic: Exploring variations

Primary SOL: A2.F.1 The student will investigate, analyze, and compare square root, cube root, rational, exponential, and logarithmic function families, algebraically and graphically, using transformations.

d) Determine when two variables are directly proportional, inversely proportional, or neither, given a table of values. Write an equation and create a graph to represent a direct or inverse variation, including situations in context.

Materials

- Directly Proportional Relationships activity sheet (attached)
- Directly and Inversely Relationships activity sheet (attached)
- Direct and Inverse Variations activity sheet (attached)
- Graphing utility

Vocabulary

constant of proportionality, direct proportion, direct variation, inverse variation, proportion, proportional, ratio, rate

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Suggested Time: 90 minutes

1. Distribute the Directly Proportional Relationships activity sheet to students. Ask what prior knowledge students can recall about direct variation. After students have had time to work on this, in groups or individually, summarize the big ideas they need to know about direct variation.
2. Distribute the Directly and Inversely Proportional Relationships activity sheet. Allow students to start this individually and then get into groups or share out whole class, with what they noticed about each column. Have students also share what claim they made and encourage students to give evidence and reasoning to support their claim.
3. Provide any needed instruction related to inverse variation. Include examples of finding the equation from a variation statement and of creating a graph to represent variations. Some examples of data include the force of springs and the length they are stretched, the speed of an object in free fall and time, and the average speed of a trip and the time it takes to travel.
4. Distribute the Direct and Inverse Variations activity sheet. Have students complete the table at the top of this activity, then discuss as a class before moving on. (*Note: This is a good place for students to consolidate their understanding.*)
5. Have students continue to part 1 on the Direct and Inverse Variations activity sheet. Allow students to work collaboratively, using knowledge of other disciplines to fill in the blanks. Have students work individually to complete the activity in whole or in part.

(Note: You may choose to use parts of this handout for assessment or for another activity.)

Assessment

- **Questions**
 - Direct variation and inverse variation are names of specific types of mathematical models. What patterns in given data would lead you to choose one of these to model the data?
 - What are some formulas used in practical, everyday activities that are examples of direct and inverse variation?
- **Journal/writing prompts**
 - Explain the information you use to determine whether a variation exists.
 - In your own words, explain what is meant by a “constant of variation.”
- **Other Assessments**
 - Use part or all of the Direct and Inverse Variations activity sheet for assessment.
 - Have students find examples of variation that could be used in a career they are interested in pursuing.

Extensions and Connections (for all students)

- Have students look at half-lives and decay to correspond with the Earth Science curriculum.
- Ask students to research average vehicle stopping distance and the effect of speed.

Strategies for Differentiation

- Have students create and use flash cards, each with a variation type listed on one side and several examples of that type on the other.
- Provide students with sentence frames of the exact language they should use as they describe the variation relationships listed in the A2.F.1d standard.
- Create an “I Have, Who Has?” game to provide students with additional practice verbalizing variation relationships.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Directly Proportional Relationships

1. Identify the slope of following proportional relationships.

x	y
0	0
6	15
10	25
12	30

2. The distance a car travels at 60 mph is proportional to the time driven.

3. The interest earned at a rate of 5 percent with a savings account is proportional to the principal amount in the account.

- 4.

KWH	Electric Bill (\$)
200	24.00
500	60.00
750	90.00
1104	132.48

- 5.

Hours Worked	Pay (\$)
2	15.00
5	37.50
10	75.00
40	300.00

Directly and Inversely Proportional Relationships

Look at the tables below. Try to notice patterns within each column and what makes a column different from the others. You can try graphing the tables to help you see patterns.

Directly Proportional	Inversely Proportional	Neither																																				
<table border="1" style="margin: auto;"> <tr><th>x</th><th>y</th></tr> <tr><td>0</td><td>0</td></tr> <tr><td>2</td><td>-6</td></tr> <tr><td>3</td><td>-9</td></tr> <tr><td>4</td><td>-12</td></tr> <tr><td>5</td><td>-15</td></tr> </table>	x	y	0	0	2	-6	3	-9	4	-12	5	-15	<table border="1" style="margin: auto;"> <tr><th>x</th><th>y</th></tr> <tr><td>1</td><td>20</td></tr> <tr><td>2</td><td>10</td></tr> <tr><td>4</td><td>5</td></tr> <tr><td>5</td><td>4</td></tr> <tr><td>8</td><td>2.5</td></tr> </table>	x	y	1	20	2	10	4	5	5	4	8	2.5	<table border="1" style="margin: auto;"> <tr><th>x</th><th>y</th></tr> <tr><td>0</td><td>5</td></tr> <tr><td>2</td><td>11</td></tr> <tr><td>4</td><td>17</td></tr> <tr><td>5</td><td>20</td></tr> <tr><td>8</td><td>29</td></tr> </table>	x	y	0	5	2	11	4	17	5	20	8	29
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What claim do you make about two variables that are directly proportional or inversely proportional? Use evidence and reasoning to support your claim.

- Directly Proportional:

- Inversely Proportional:

Direct and Inverse Variations

Type of Variation	Write three things you think are important to remember about these variation types.	Sketch what a sample graph might look like for these variations.
Direct	1. 2. 3.	
Inverse	1. 2. 3.	

Part I

Identify each of the following statements as a direct or inverse variation by filling in the blank with the words *directly* or *inversely*.

1. The amount of sales tax paid varies _____ to the total of the goods purchased.
2. The time to complete a job varies _____ to the number of workers working.
3. The time it takes to complete a specific trip varies _____ to the speed of travel.
4. The cost of gas on a trip varies _____ to the length of the trip.
5. The number of people invited to dinner varies _____ to the amount of space each guest has at the table.

Part II

Write the equation being described by each of the following statements.

6. The number, n , of grapefruit that can fit into a box is inversely proportional to the cube of the diameter, d , of each grapefruit.

7. The time, t , that a plane spends on the runway varies inversely to the take-off speed.

8. The lift of an airplane wing, L , is directly proportional to the square of the velocity, v .

9. The velocity, v , of a satellite in orbit around Earth is inversely proportional to the orbit radius, r .

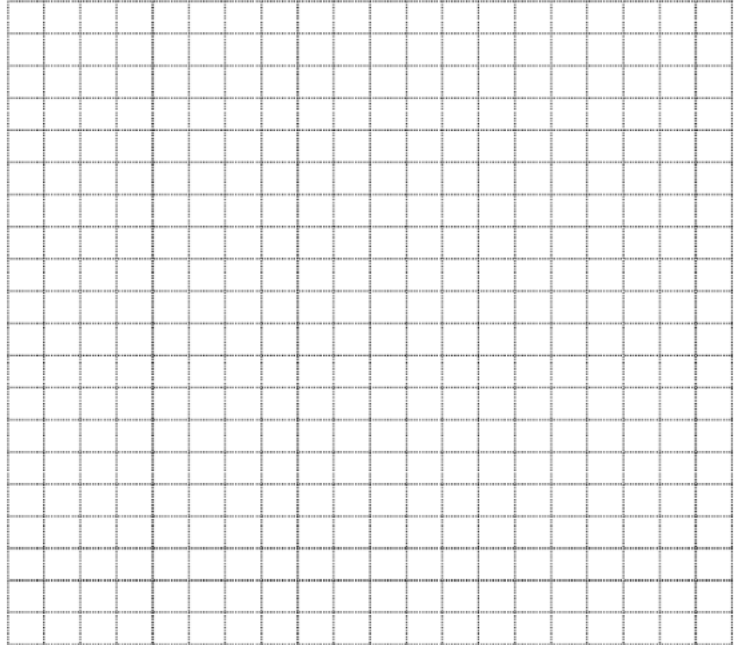
10. The intensity of a sound, I , varies inversely with the square of the distance, d , from the source.

11. The speed of an object, s , varies directly with the square of the time, t , since it has been dropped.

Part III

Create the graph for the following problems.

12. The total dollars you earn, d , varies directly to the time, t , in hours, you work. The total earned for 10 hours is \$125.



13. The time, t , in hours varies inversely to the speed, s , in mph. Traveling at 50 mph, it will take 2.4 hours to complete the trip.

