Radical Equations

Strand: Equations and Inequalities

Topic: Solving equations containing radical expressions

Primary SOL: A2.EI.5 The student will represent, solve, and interpret the solution to an equation containing a radical expression.

Related SOL: A2.EO.4

Materials

- Solving Radical Equations (Introductory Exercise) activity sheet (attached)
- Steps for Solving Radical Equations Algebraically activity sheet (attached)
- Solving Radical Equations: Practice Problems with Hints activity sheet (attached)
- Graphing utility

Vocabulary

cube, cube root, exponent, domain, extraneous solution, index, power, radical, radical algebraic equations, radicand, square, square root

Student/Teacher Actions: What should students be doing? What should teachers be doing?

Suggested Time: 90 minutes

1. On the board, write the equations $\sqrt{x} = 4$ and $x^2 = 4$. Ask students to name things that are alike and different between the equations. Expect answers such as "both equal 4" and "one is square and the other is square root."
2. Distribute the Solving Radical Equations (Introductory Exercise) activity sheet. Have students complete the activity, working individually and then in pairs to share and confirm or revise their responses. Have them place emphasis on the justification of their answers.
3. Create three columns on the board, writing number 1 from the Solving Radical Equations activity sheet, $\left(\frac{x}{2}\right)^2 - 3 = 1$ on the left, number 4 ($5 + \sqrt{9x} = 11$) on the right, and steps to solve in the middle. Lead the class in a discussion of how you would solve these equations algebraically. The finished work may look like this:

$\left(\frac{x}{2}\right)^2 - 3 = 1$	Steps to solve	$5 + \sqrt{9x} = 11$
$\left(\frac{x}{2}\right)^2 = 4$	Isolate the term containing the variable.	$\sqrt{9x} = 6$
$\frac{x}{2} = \pm 2$	Perform the inverse operation.	$9x = 36$
$x = \pm 4$	Solve for *x*.	$x = 4$

4. Divide the class into groups of 3. Provide each group with a vertical white board, chart paper, or something similar. Provide groups with the first of the following equations. Once they finish the equation and you check their work, provide them with the next equation.

$$\sqrt{2x} = 4$$

$$3 - \sqrt{x} = 5$$

$$\sqrt{2x - 7} + 2 = 9$$

$$\sqrt{\frac{x}{3}} + 5 = 2$$

5. Lead the class in a group discussion about the last equation. Say that you have some groups who found the solution of $x = 27$. Substitute this value for and determine that it does not lead to a true statement. Then walk through solving the equation algebraically to determine where this possible solution came up. Define this as an extraneous solution.
6. Provide students with the Steps for Solving Radical Equations Algebraically activity to help synthesize their learning. Students may work on this individually, with a partner, or as part of a teacher-led small group.
7. Use some of the problems from the Steps for Solving Radical Equations Algebraically activity sheet to introduce students to solving radical equations by graphing. Have students graph $f(x) = \sqrt{3x - 5}$. Ask students about the domain of the function. Have the students evaluate $f(3)$. Ask students to find the value of x when $f(x) = 2$. Discuss how these two are related on the graph. Ask the students to find the solution to the equation $\sqrt{3x - 5} = 2$ using the graph and to describe how they would use the graph to solve the equation. Have students use a graph to find the solution to $\sqrt{x + 2} = x$. Discuss how this problem is similar to and different from the first example. Remind students of the importance of identifying the domain for each function and how that relates to the possible solutions to the equation and any extraneous solutions.
8. Distribute the Solving Radical Equations Practice Problems activity sheet, and have students complete it.

Assessment

- **Questions**
  - How can you determine the solution set to a radical equation algebraically?
  - How can you determine the solution set to a radical equation graphically?
- **Journal/writing prompts**
  - Explain how you can use a graph's points of intersection to solve a radical equation.
  - In your own words, explain what is meant by the term *extraneous solution.* Is it a solution or not? Explain why.
- **Other Assessments**
  - Give students solution sets and ask them to create matching radical equations. (Note: Such open-ended problems allow students to be creative and differentiate the task based upon their own level of understanding.)

Extensions and Connections (for all students)

- Give advanced students equations with multiple radicals.
- Guide students to make connections to graphing functions containing radicals, paying particular attention to restrictions on the domain.

Strategies for Differentiation

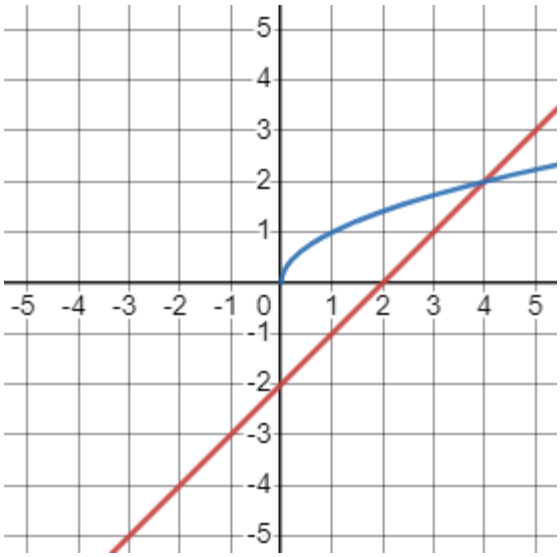
- Construct additional introductory problems to reinforce similar concepts.
- Create an additional handout similar to the Steps for Solving Radical Equations Algebraically activity sheet with examples in the left-hand column and similar radical equations in the right-hand column for students to solve.
- Have students create flash cards, each with a radical equation on one side and the first step on the other, to help them take the initial steps.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Solving Radical Equations
(Introductory Exercise)

For each equation, determine whether $x = 4$ and/or $x = -4$ is a solution. Justify your answers to your partner.

#	Equation	$x = 4$ is a solution	$x = -4$ is a solution
1.	$\left(\frac{x}{2}\right)^2 - 3 = 1$		
2.	$\sqrt{x} = 2$		
3.	$\sqrt{x} = -2$		
4.	$5 + \sqrt{9x} = 11$		

#	Equation	$x = 4$ is a solution	$x = -4$ is a solution
5.			
6.	$\sqrt[3]{2x} = 8$		
7.	$\sqrt[3]{2x} = -8$		

Steps for Solving Radical Equations Algebraically

Example 1: $\sqrt{3x-5}=5$

Step 1: Square both sides to eliminate the radical.

$\left(\sqrt{3x-5}\right)^2=(5)^2$

Step 2: Simplify and solve the familiar equation.

$3x-5=25$

$3x=30$

$x=10$

Step 3: Verify the solution.

Does $\sqrt{3(10)-5}=5$?

Problem 1: Now, you follow the steps to solve $\sqrt{5x+1}=4$.

Step 1:

Step 2:

Step 3:

Example 2: $\sqrt[3]{5x+2}=3$

Step 1: Cube both sides to eliminate the radical.

$\left(\sqrt[3]{5x+2}\right)^3=3^3$

Step 2: Simplify and solve the familiar equation.

$5x+2=27$

$5x=25$

$x=5$

Step 3: Verify the solution.

Does $\sqrt[3]{5(5)+2}=3$?

Problem 2: Now, you follow the steps to solve $\sqrt[3]{2x-1}=5$.

Step 1:

Step 2:

Step 3:

Example 3: $\sqrt{x+2}=x$

Step 1: Square both sides to eliminate the radical.

$\left(\sqrt{x+2}\right)^2=x^2$

Step 2: Simplify and solve the familiar equation.

$x+2=x^2$

$0=x^2-x-2=(x-2)(x+1)$

$x=2$ or $x=-1$

Step 3: Verify the solution.

Check with $x=2$; $\sqrt{2+2}=2$: True

Check with $x=-1$; $\sqrt{-1+2}=-1$: False

So, the only solution is $x=2$.

(−1 is extraneous.)

Problem 3: Now, you follow the steps to solve $\sqrt{2x+8}=x$

Step 1:

Step 2:

Step 3:

Solving Radical Equations Practice Problems

Solve each of the following and check your solutions.

1. $2\sqrt{x} - 5 = 3$

2. $x - 2\sqrt{x} - 15 = 0$

3. $\sqrt{x^2 + 2} = x + 1$

4. $\sqrt{2x + 1} = 2x - 1$

5. $x^{\frac{1}{2}} = x - 2$