Mathematics *Standards of Learning* for Virginia Public Schools 2023 Mathematical Analysis

Students enrolled in Mathematical Analysis are assumed to have mastered Geometry and Algebra 2 concepts. Mathematical Analysis develops students' understanding of algebraic and transcendental functions, parametric and polar equations, sequences and series, and vectors. The content of this course serves as appropriate preparation for a calculus course.

Technology tools will be used to assist in teaching and learning. Graphing technologies facilitate visualizing, analyzing, and understanding algebraic and statistical behaviors and provide a powerful tool for solving and verifying solutions.

Characteristics of Functions

MA.CF.1 The student will identify and analyze the properties of polynomial, rational, piecewise-defined, absolute value, radical, and step functions and sketch the graphs of the functions.

Students will demonstrate the following Knowledge and Skills:

a) Use mathematical reasoning to identify polynomial, rational, piecewise-defined, absolute value, radical, and step functions, given an equation or graph.

b) Given multiple representations of a polynomial, rational, piecewise-defined, absolute value, radical, and step function, analyze:
   i) domain and range;
   ii) roots (including complex roots);
   iii) intercepts;
   iv) symmetry (including even and odd functions);
   v) asymptotes (horizontal, vertical, and oblique/slant;
   vi) points of discontinuity;
   vii) intervals for which the function is increasing, decreasing or constant;
   viii) end behavior; and
   ix) relative and/or absolute maximum and minimum points.

c) Sketch the graph of a polynomial, rational, piecewise-defined, absolute value, radical, and step function.

MA.CF.2 The student will determine the limit of a function if it exists.

Students will demonstrate the following Knowledge and Skills:

a) Verify estimates about the limit of a function using graphing technology.

b) Determine the limit of a function algebraically and verify with graphing technology.

c) Determine the limit of a function numerically and verify with graphing technology.

d) Use proper limit notation, including when describing the end behavior of a function.

e) As the variable approaches a finite number,
   i) determine the limit of a function numerically by direct substitution;
   ii) determine the limit of a function using algebraic manipulation;
   iii) estimate the limit of a function using a table; and

iv) determine the limit of a function from a given graph.

f) As the variable approaches positive or negative infinity, analyze the limit of a function to describe the end behavior.

MA.CF.3 The student will analyze and describe the continuity of functions.

Students will demonstrate the following Knowledge and Skills:

a) Describe continuity of a function.

b) Use mathematical notation to communicate and describe the continuity of functions including polynomial, rational, piecewise, absolute value, radical, and step function, using graphical and algebraic methods.

c) Prove continuity at a point, using the definition.

d) Classify types of discontinuity based on which condition of continuity is violated.

Functional Relationships

MA.FR.1 The student will analyze compositions of functions to determine and verify inverses of functions.

Students will demonstrate the following Knowledge and Skills:

a) Construct the composition of functions algebraically and graphically.

b) Determine the domain and range of composite functions algebraically and graphically.

c) Develop the inverse of a function algebraically and graphically.

d) Compare the domain and range of the inverse of a function with the original function, both algebraically and graphically.

e) Use mathematical reasoning to generalize and communicate the criteria for an inverse function to exist.

MA.FR.2 The student will analyze the characteristics of exponential and logarithmic functions, and sketch the graphs of the functions.

Students will demonstrate the following Knowledge and Skills:

a) Generalize characteristics of exponential and logarithmic functions from an equation or a graph.

b) Define *e* and estimate its value.

c) Convert between equations written in logarithmic and exponential form.

d) Use laws of exponents and properties of logarithms to solve equations and simplify expressions.

e) Represent contextual problems, using exponential and logarithmic functions, to include common and natural logarithms.

f) Sketch the graph of exponential and logarithmic functions and identify asymptotes, end behavior, intercepts, domain, and range.

MA.FR.3 The student will analyze sequences and finite series, and model and solve problems in context using sequences and series.

Students will demonstrate the following Knowledge and Skills:

a) Use and interpret the notation: $\sum$, n, n^{th}, and a_n.

b) Derive the formulas associated with arithmetic and geometric sequences and series.

c) Determine the nth term, a_n, for an arithmetic or geometric sequence.

d) Determine the sum, S_n, if it exists, of an arithmetic or geometric series.

e) Model and solve problems in context, using sequences and series.

f) Distinguish between a convergent and divergent series.

g) Describe convergent series in relation to the concept of a limit.

Analytic Geometry

MA.AG.1 The student will identify and analyze the properties of conic sections and sketch a graph given an equation.

Students will demonstrate the following Knowledge and Skills:

a) Given a translation or rotation matrix, determine an equation for the transformed function or conic section.

b) Convert between standard and general forms of conic equations by completing the square.

c) Graph conic sections from equations written in general or standard form using transformations.

d) Identify characteristics of conic sections including center, vertices, axes, symmetry, foci, directrix, eccentricity, and asymptotes.

e) Represent applications of conic sections.

MA.AG.2 The student will use parametric equations to model and solve problems in context.

Students will demonstrate the following Knowledge and Skills:

a) Graph and analyze parametric equations and use the graph to determine solutions.

b) Use parametric equations to model contextual problems, including motion over time.

MA.AG.3 The student will perform operations with vectors in the coordinate plane.

Students will demonstrate the following Knowledge and Skills:

a) Use vector notation.

b) Perform the operations of addition, subtraction, and scalar multiplication, graphically and algebraically on vectors.

c) Find the dot (inner) product of two vectors and use it to determine the angle between two vectors.

d) Determine if two vectors are orthogonal.

e) Express complex numbers in vector notation.

f) Verify properties of the dot product.

g) Determine the components of a vector.

h) Determine the norm (magnitude) of a vector.

i) Find a unit vector in the same direction of a given vector.

j) Apply vectors to problems in context.

MA.AG.4 **The student will investigate and identify the characteristics of the graphs of polar equations.**

Students will demonstrate the following Knowledge and Skills:

a) Classify polar equations (rose, cardioid, limaçon, lemniscate, spiral, and circle), given the graph or the equation.

b) Determine the effects of changes in the parameters of polar equations on the graph, using graphing technology.

c) Convert between polar and rectangular forms of coordinates.

d) Convert between complex numbers written in rectangular form and polar form.

e) Convert equations between polar and rectangular forms.

f) Determine and verify the intersection of the graphs of two polar equations, using graphing technology.

MA.AG.5 **The student will use matrices to organize data and will add and subtract matrices, multiply matrices, multiply matrices by a scalar, and use matrices to solve systems of equations.**

Students will demonstrate the following Knowledge and Skills:

a) Multiply matrices by a scalar.

b) Add, subtract, and multiply matrices.

c) Represent problems with a system of no more than three linear equations.

d) Express a system of linear equations as a matrix equation.

e) Solve a system of equations using matrices.

f) Determine the inverse of a two-by-two or three-by-three matrix using paper and pencil.

g) Verify two matrices are inverses using matrix multiplication.

h) Verify the commutative and associative properties for matrix addition and multiplication.