



The *PRAXIS*<sup>®</sup> Study Companion

# Middle School Mathematics (5164)



**ETS.**  
Professional  
Educator  
PROGRAMS

[www.ets.org/praxis](http://www.ets.org/praxis)

## Table of Contents

Middle School Mathematics (5164).....	3
Test at a Glance .....	3
About This Test.....	4
On-Screen Graphing Calculator .....	5
Using Your Calculator .....	5
Content Topics .....	6
Discussion Questions.....	6
Tasks of Teaching Mathematics .....	16
Middle School Mathematics (5164) Sample Test Questions .....	18
Information about Questions That Is Specific to the Middle School Mathematics Test.....	18
Unit Conversions .....	20
Formulas.....	20
Mathematics (5164) Sample Test Questions .....	22
Middle School Mathematics (5164) Answers.....	30
Understanding Question Types.....	38
Understanding Selected-Response and Numeric-Entry Questions.....	38
Understanding Constructed-Response Questions .....	39

## Middle School Mathematics (5164)

### Test at a Glance

The *Praxis*® Middle School Mathematics test is designed to measure knowledge and competencies that are important for safe and effective beginning practice as a middle school mathematics teacher. Test takers have typically completed a bachelor's degree program with appropriate coursework in mathematics and education.

<b>Test Name</b>	Middle School Mathematics		
<b>Test Code</b>	5164		
<b>Time</b>	180 minutes		
<b>Number of Questions</b>	66 selected-response and numeric-entry questions		
<b>Format</b>	The test consists of a variety of selected-response questions, where you select one or more answer choices; questions where you enter a numeric answer in a box; and other types of questions. You can review the possible question types in Understanding Question Types.		
<b>Calculator</b>	An on-screen graphing calculator is provided.		
<b>Test Delivery</b>	Computer Delivered		
	<b>Content Categories</b>	<b>Approximate Number of Questions</b>	<b>Approximate Percentage of Examination</b>
	I. Numbers and Operations	16	23%
	II. Algebra	15	23%
	III. Functions	11	17%
	IV. Geometry and Measurement	13	20%
	V. Statistics and Probability	11	17%
All questions assess content from the above Middle School Mathematics domains. Approximately 30% of questions assess content applied to a Task of Teaching Mathematics.			

## About This Test

The Middle School Mathematics content topics span the middle school mathematics curriculum, including content related to (I) Numbers and Operations, (II) Algebra, (III) Functions, (IV) Geometry and Measurement, and (V) Statistics and Probability. A full list of the mathematics topics covered is provided in [Content Topics](#).

Test takers will find that approximately 30% of the questions call for application of mathematics within a teaching scenario or an instructional task. Such questions—designed to measure applications of mathematics knowledge and skills to the kinds of decisions and evaluations a teacher must make during work with students, curriculum, and instruction—situate mathematics content questions in tasks that are critical for teaching. A full list of the teaching tasks covered, which have been identified based on research on mathematics instruction and are a routine part of mathematics instruction, is provided in [Tasks of Teaching Mathematics](#).

Test takers have access to an on-screen graphing calculator and a list of selected unit conversions and formulas. This list is also provided in the [Middle School Mathematics \(5164\) Sample Test Questions](#) section.

The assessment is designed and developed through work with practicing teachers and teacher educators to reflect the mathematics curriculum as well as state and national standards for mathematics, including the Standards for the Preparation of Middle Level Mathematics Teachers (2020), by the National Council of Teachers of Mathematics (NCTM) and the Council for the Accreditation of Educator Preparation (CAEP).

This test may contain some questions that will not count toward your score.

## On-Screen Graphing Calculator

An on-screen graphing calculator is provided for the computer-delivered test. Please consult the *Praxis*® Calculator Use web page ([http://www.ets.org/praxis/test\\_day/policies/calculators/](http://www.ets.org/praxis/test_day/policies/calculators/)) for further information and for a link to download the calculator and view tutorials on using the calculator.

You are expected to know how and when to use the calculator since it will be helpful for some questions. The calculator is available as a free download for a 30-day trial period. You are expected to become familiar with its functionality before taking the test. The calculator may be used to perform calculations (e.g., division, exponents, roots, finding the mean of a data set), to graph and analyze functions, to find numerical solutions to equations, and to generate a table of values for a function.

### Using Your Calculator

Take time to download the trial version of the calculator. View the tutorials on the website. Practice with the calculator so that you are comfortable using it on the test.

There are only some questions on the test for which a calculator is helpful or necessary. First, decide how you will solve a problem, then determine if you need a calculator. For many questions, there is more than one way to solve the problem. Don't use the calculator if you don't need to; you may waste time.

Sometimes answer choices are rounded, so the answer that you get might not match the answer choices in the question. Since the answer choices are rounded,

substituting the choices into the question might not produce an exact answer.

Don't round any intermediate calculations. For example, if the calculator produces a result for the first step of a solution, keep the result in the calculator and use it for the second step. If you round the result from the first step and the answer choices are close to each other, you might choose the incorrect answer.

Read the question carefully so that you know what you are being asked to do. Sometimes a result from the calculator is NOT the final answer. If an answer you get is not one of the choices in the question, it may be that you didn't answer the question being asked. Read the question again. It might also be that you rounded at an intermediate step in solving the problem.

Think about how you are going to solve the question before using the calculator. You may only need the calculator in the final step or two. Don't use it more than necessary.

Check the calculator modes (degree versus radian, floating decimal versus scientific notation) to see that these are correct for the question being asked.

Make sure that you know how to perform the basic arithmetic operations and calculations (e.g., division, exponents, roots). Your test may involve questions that require you to do some of the following: graph functions and analyze the graphs, find zeros of functions, find points of intersection of graphs of functions, find minima/maxima of functions, find numerical solutions to equations, and generate a table of values for a function.

## Content Topics

This list details the topics that may be included on the test. All test questions will cover one or more of these topics.

## Discussion Questions

In this section, discussion questions are open-ended questions or statements intended to help test your knowledge of fundamental concepts and your ability to apply those concepts to classroom or real-world situations. **We do not provide answers for the discussion questions** but thinking about the answers will help improve your understanding of fundamental concepts and may help you answer a broad range of questions on the test. Most of the questions require you to combine several pieces of knowledge to formulate an integrated understanding and response. They are written to help you gain increased understanding and facility with the test's subject matter. You may want to discuss these questions and possible areas with a teacher or mentor.

### I. Numbers and Operations

#### A. Understands operations and properties of the real number system

1. Represents and solves word problems involving addition, subtraction, multiplication, and division of real numbers
2. Represents and identifies the effect that an operation has on a given number (e.g., adding a negative, adding the inverse, dividing by a nonzero fraction)
3. Uses the order of operations to simplify computations and solve problems

4. Identifies and applies properties of operations on a number system (e.g., commutative, associative, distributive, identity)
5. Compares and orders real numbers, including absolute values of real numbers
6. Classifies real numbers (e.g., natural, whole, integer, rational, irrational)
7. Identifies whether the sum or product of rational and/or irrational numbers must be rational, must be irrational, or can be rational or irrational (e.g., the sum of two rational numbers must be rational, the product of two irrational numbers can be rational or irrational)
8. Performs operations involving integer exponents
9. Approximates the value of a radical expression
10. Uses scientific notation to represent and compare numbers and to perform calculations

#### B. Understands the relationships among fractions, decimals, and percents

1. Converts among fractions, decimals, and percents
2. Represents repeating decimals as fractions
3. Represents fractions, decimals, and percents with models (e.g., area models, base-10 blocks, set models, colored rods)

**C. Understands how to use ratios and proportional relationships to solve problems**

1. Uses the language of ratio and rate to describe relationships between two quantities
2. Identifies and represents proportional relationships and uses them to solve problems (e.g., unit rates, scale factors, constant of proportionality)
3. Solves percent problems (e.g., expressing a percent as a ratio per 100, discounts, markups, taxes, tips, simple interest, percent error)

**D. Understands how to reason quantitatively and use units to solve problems**

1. Chooses and interprets units consistently in formulas
2. Chooses and interprets the scale in graphs and data displays
3. Solves problems involving dimensional analysis (e.g., feet per second to miles per hour, feet per second to kilometers per hour)

**E. Understands how to use basic concepts of number theory (e.g., divisibility, prime factorization, multiples) to solve problems**

1. Uses the definitions of prime and composite numbers to solve problems
2. Solves problems involving factors, multiples, and divisibility

**Discussion Questions: Numbers and Operations**

Note that the use of “e.g.” to start a list of examples implies that only a few examples are offered and not an exhaustive list.

- Be able to convert repeating decimals into fractions (e.g.,  $0.5\overline{83} = \frac{7}{12}$ ).
- Be able to distinguish between a ratio and a rate.
- Be able to calculate percent change, percent (relative) error, and percents of percents.
- Be able to determine the correct units in an answer based on the units of the initial measurements given in a problem.
- Be able to identify a scale for a graph that allows an entire set of data to be represented on the graph.
- Be familiar with what unit conversions are given on the math reference sheet. Note that some other common unit conversions (e.g., 1 yard = 3 feet, 1 minute = 60 seconds) are expected to be known, and other unit conversions (e.g., 1 mile = 1,760 yards, 1 gallon = 128 fluid ounces, 1 hour = 3,600 seconds) are expected to be determined based on what is known or what is given on the math reference sheet.

**II. Algebra****A. Understands how to create, evaluate, and manipulate algebraic expressions, equations, and formulas**

1. Adds, subtracts, and multiplies linear and quadratic polynomials, including polynomials with rational coefficients

2. Evaluates, manipulates, and compares algebraic expressions involving rational exponents (e.g., radicals, negative exponents)
3. Uses variables to construct and solve equations and inequalities in real-world contexts
4. Translates verbal relationships into algebraic equations or expressions
5. Interprets parts of expressions and equations in terms of a real-world setting
6. Rewrites linear, quadratic, and exponential expressions in equivalent forms to reveal properties of the quantity represented by the expression
7. Determines the nature of the solutions of a quadratic equation (e.g., interprets the graph, finds the discriminant, writes the equation in factored form)
8. Rearranges formulas to solve for a specified variable (e.g., solve  $d = rt$  for  $t$ )

**B. Understands how to recognize and represent linear relationships algebraically**

1. Determines the equation of a line from information presented in various forms (e.g., table, graph, description)
2. Recognizes and is able to extract information about a linear equation when it is presented in various forms (e.g., slope-intercept, point-slope, standard)
3. Converts among various forms of linear equations (e.g., slope-intercept, point-slope, standard)

**C. Understands how to solve equations and inequalities**

1. Solves one-variable linear equations and inequalities
2. Solves one-variable nonlinear equations and inequalities (e.g., absolute value, quadratic)
3. Represents solutions to equations and inequalities (e.g., on a number line, in the  $xy$ -plane)
4. Justifies each step in solving equations and inequalities

**D. Understands how to solve systems of equations and inequalities**

1. Solves a system of two linear equations or inequalities in two variables algebraically and graphically
2. Solves a system consisting of a linear equation and a quadratic equation in two variables graphically
3. Finds the solutions of  $f(x) = g(x)$  approximately (e.g., uses technology to graph the functions); includes cases where  $f(x)$  and/or  $g(x)$  are linear, quadratic, or exponential functions
4. Graphs the solution set to a system of linear inequalities in two variables in the  $xy$ -plane
5. In a modeling context, represents constraints by systems of equations and/or inequalities and interprets solutions as viable or nonviable options



### Discussion Questions: Algebra

Note that the use of “e.g.” to start a list of examples implies that only a few examples are offered and not an exhaustive list.

- Be able to identify expressions that are equivalent to expressions such  $x^{\frac{2}{3}}$ ,  $x^{\frac{5}{2}}$ ,  $x^{-4}$ ,  $(x^3)^{-1}$ , and  $\sqrt[5]{x^2}$ .
- Be able to write and solve equations, inequalities, and systems of equations or inequalities that represent real-world problems.
- Be able to identify what parts of expressions and equations (e.g., coefficients, terms, factors) represent in the context of a real-world situation.
- Be able to use the quadratic formula, which is given on the math reference sheet.
- Be able to determine the equation of a line given two points on the line or one point on the line and the slope of the line.
- Be able to determine the slope of a line or points on a line when an equation of the line is given in standard form, slope-intercept form, or point-slope form.
- Be able to solve one-variable linear equations and inequalities that have variables on both sides, involve combining like terms, and involve using the distributive property.
- Be able to graph the solutions to linear equations, linear inequalities, systems of linear equations, and systems of linear inequalities in two variables in the  $xy$ -plane.
- Be able to graph the solutions to one-variable inequalities on the number line.

- Be able to identify the properties (e.g., commutative property, distributive property) that justify each step in a given method for solving an equation or inequality.
- Be able to solve systems of linear equations graphically, by substitution, or by elimination.
- Remember that the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ .

### III. Functions

#### A. Understands how to identify, define, and evaluate functions

1. Determines whether a relation is a function
2. Given a function (presented as a table of values, algebraically, or graphically), determines if the function is linear, quadratic, or exponential
3. Determines the value of a function for a specified value in its domain

#### B. Knows how to determine and interpret the domain and the range of a function presented as a table of values, algebraically, or graphically

1. Determines the domain and range of a function
2. Interprets domain and range in real-world settings

**C. Understands basic characteristics of linear functions (e.g., intercepts, slope)**

1. Calculates the intercepts of a line and interprets them in a modeling context
2. Calculates the slope of a line presented as a table of values, algebraically, or graphically and interprets it in a modeling context
3. Interprets what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$ , where  $r$  is the unit rate

**D. Understands the relationships among functions, tables, and graphs**

1. Determines an equation to represent a linear or quadratic function presented graphically
2. Determines the type of equation that best represents a given graph
3. Sketches a graph, given an equation of a function (e.g., square root, absolute value)
4. Compares properties (e.g., intercepts, slope, maximum) of two functions presented as tables of values, algebraically, graphically, or by verbal descriptions
5. Identifies the symbolic representation of a linear function that is created when a graph is translated horizontally or vertically or reflected across the  $x$ -axis

**E. Knows how to analyze and represent functions (i.e., linear, quadratic, exponential) that model given information**

1. Interprets statements that use function notation in terms of a context
2. Interprets the parameters in a linear or exponential function in terms of a context
3. Calculates the rate of change of a function over a given interval and interprets it in a context
4. Determines and interprets the  $x$ - and  $y$ -intercepts of quadratic functions
5. Develops a function—represented by a graph, equation, or table—to model a given set of conditions
6. Evaluates whether a particular mathematical model (e.g., graph, equation, table) can be used to describe a given set of conditions
7. Interprets a particular mathematical model (e.g., graph, equation, table) in a given context

**F. Understands differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems**

1. Identifies situations in which one quantity changes at a constant rate per unit interval relative to another
2. Identifies situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another

3. Observes that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically

### G. Is familiar with how to represent arithmetic sequences as functions

1. Writes arithmetic sequences both recursively and with an explicit formula and uses them to model situations

#### Discussion Questions: Functions

Note that the use of “e.g.” to start a list of examples implies that only a few examples are offered and not an exhaustive list, whereas the use of “i.e.” to start a list of examples implies that the given list of examples is complete.

- Remember that a function assigns exactly one element of its range to each element of its domain.
- Be able to identify features of a function represented as a table, equation, or graph that indicate whether the function is linear, quadratic, or exponential (e.g., the second differences of a quadratic function are constant, a quadratic function has degree 2, a quadratic function has either a maximum or a minimum).
- Be able to determine the domain (x-values) and range (y-values) of a function.
- Be able to determine the domain and range of a function that is reasonable in the context of a given real-world situation (e.g., the domain in a certain situation consists of positive integers, the domain in a certain situation cannot include values that would result in a negative value for the height of an object).
- Be able to determine the intercepts and slope of a line represented as a table, equation, or graph.
- Be able to interpret the meaning of the intercepts and slope of a line in the context of a real-world situation.
- Be able to interpret the meaning of a point on the graph of a proportional relationship in the context of a real-world situation.
- Be able to determine whether a given graph is best represented using a linear equation, quadratic equation, exponential equation, absolute value equation, etc.
- Be able to write an equation of the function that results after an existing function is translated horizontally, translated vertically, or reflected across the x-axis.
- Be able to interpret the meaning of  $m$  and  $b$  in a function of the form  $f(x) = mx + b$  in the context of a real-world situation.
- Be able to interpret the meaning of  $a$  and  $b$  in a function of the form  $f(x) = a \cdot b^x$  in the context of a real-world situation.
- Be able to calculate the rate of change of a function  $f$  on the interval  $[a, b]$  by calculating  $\frac{f(b) - f(a)}{b - a}$ , and be able to interpret the meaning of the rate of change in the context of the problem.
- Be able to interpret the meaning of the intercepts of a quadratic function in the context of a real-world situation.
- Given a linear, quadratic, or exponential function represented as a table, equation, graph, or description, be able to determine a different representation of the function (e.g., determine the function that best matches a description of a real-world situation).

- Be able to interpret the meaning of a feature of a function (e.g., the maximum on a graph, an ordered pair in a table of values) in the context of a real-world situation.
- Be able to identify real-world situations that are best modeled by linear functions or that are best modeled by exponential functions.
- Be able to find the value of a term in an arithmetic sequence.
- Be able to write an expression, equation, or function that represents an arithmetic sequence.
- Be familiar with the differences between recursive and explicit rules for arithmetic sequences.
- Consider becoming familiar with the arithmetic sequence formula on the math reference sheet.

#### IV. Geometry and Measurement

##### A. Knows the properties of types of lines (e.g., parallel, perpendicular, intersecting) and angles

1. Solves problems involving parallel, perpendicular, and intersecting lines
2. Applies angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems

##### B. Understands the properties of triangles

1. Solves problems involving the Pythagorean theorem in two dimensions
2. Identifies characteristics of special triangles (e.g., equilateral, isosceles, right) and uses them to solve problems

3. Determines whether given side lengths or angle measures would produce a triangle (e.g., triangle inequality theorem) and classifies triangles by their sides or angles
4. Determines whether given conditions would produce a unique triangle, no triangle, or more than one triangle

##### C. Knows the properties of quadrilaterals and other polygons

1. Identifies the relationships among various quadrilaterals (e.g., parallelogram, rectangle, rhombus)
2. Solves problems involving sides and angles of polygons

##### D. Knows the concepts of transformations (i.e., translations, reflections, rotations, dilations)

1. Applies properties of translations, reflections, and rotations (e.g., line segments are taken to congruent line segments, angles are taken to congruent angles, parallel lines are taken to parallel lines)
2. Applies properties of dilations (e.g., angles are taken to congruent angles, parallel lines are taken to parallel lines)
3. Identifies a sequence of transformations that maps a preimage onto an image
4. Given a figure, describes the transformations that map the figure onto itself, including reflection over a line of symmetry
5. For a given transformation, determines the coordinates of a point on an image

**E. Understands the concepts of congruence and similarity**

1. Determines whether two figures are congruent or similar
2. Uses congruence and similarity to solve problems with two-dimensional and three-dimensional figures

**F. Understands the properties of circles**

1. Solves problems involving circles (e.g., circumference, area)

**G. Knows how to interpret relationships between geometric objects in the  $xy$ -plane (e.g., distance, midpoint)**

1. Uses coordinate geometry to represent and identify the properties of geometric shapes and to solve problems (e.g., Pythagorean theorem, perimeter, area)
2. Determines the distance between two points
3. Determines the midpoint of a segment

**H. Understands how to solve problems involving perimeter and area of polygons**

1. Calculates and interprets perimeter and area of polygons that can be composed of triangles and quadrilaterals, including in real-world situations
2. Calculates changes in perimeter and area as the dimensions of a polygon change

**I. Knows how to solve problems involving solids**

1. Calculates and interprets surface area and volume of solids (e.g., prisms, pyramids, cylinders, spheres) and composite solids, including in real-world situations
2. Calculates changes in surface area and volume as the dimensions of a solid change
3. Uses two-dimensional representations (e.g., nets) of three-dimensional objects to visualize and solve problems

**J. Understands systems of measurement (i.e., metric, United States customary)**

1. Solves measurement, estimation, and conversion problems involving time, length, temperature, volume, and mass in standard measurement systems
2. Uses appropriate units of measurement in a given context

**Discussion Questions: Geometry and Measurement**

Note that the use of “e.g.” to start a list of examples implies that only a few examples are offered and not an exhaustive list, whereas the use of “i.e.” to start a list of examples implies that the given list of examples is complete.

- Be familiar with the relationship between the slopes of parallel lines and the relationship between the slopes of perpendicular lines.
- Be able to identify congruent and supplementary angles given two parallel lines and a transversal.

- Be able to distinguish among acute, right, and obtuse triangles.
- Be able to identify and use special characteristics of triangles (e.g., equilateral, isosceles, right) to solve problems involving lengths of sides and measures of angles.
- Be able to distinguish among different types of quadrilaterals.
- Be able to identify and use special characteristics of squares, rectangles, parallelograms, rhombuses, and trapezoids to solve problems involving lengths of sides and measures of angles.
- Be able to find missing side lengths or angle measures in polygons with more than four sides.
- Be able to find the measures of interior and exterior angles of regular polygons.
- Be familiar with the effects of translations, reflections, rotations, and dilations on figures.
- Be able to translate, reflect, rotate, and dilate figures.
- Be able to distinguish between congruent and similar figures and use corresponding parts of congruent or similar figures to solve problems.
- Be familiar with what geometric formulas are given on the math reference sheet, and be able to apply these formulas to solve problems.
- Be able to solve problems that involve the circumference or area of a circle and the perimeter or area of a polygon (e.g., finding the difference between the area of a square and the area of a circle inscribed in the square).
- Be able to find the distance between any two points in the  $xy$ -plane by using a formula or the Pythagorean theorem.
- Be able to find the midpoint of a line segment in the  $xy$ -plane by using a formula or another approach.
- Be familiar with the effect on the perimeter, area, or volume of a figure as the dimensions of the figure change by different factors.
- Be able to use a net to find the surface area and volume of a solid.
- Be familiar with what unit conversions are given on the math reference sheet. Note that some other common unit conversions (e.g., 1 yard = 3 feet, 1 minute = 60 seconds) are expected to be known, and other unit conversions (e.g., 1 mile = 1,760 yards, 1 gallon = 128 fluid ounces, 1 hour = 3,600 seconds) are expected to be determined based on what is known or what is given on the math reference sheet.
- Be able to identify units that measure length, area, volume, weight, etc.

## V. Statistics and Probability

### A. Understands statistical processes and how to evaluate them

1. Recognizes a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers
2. Uses statistics to make inferences about population parameters based on a sample from that population
3. Distinguishes between random and biased sampling

**B. Understands how to interpret, analyze, and represent data presented in a variety of displays**

1. Represents and analyzes data in various displays (e.g., bar graphs, line graphs, circle graphs, boxplots, histograms, scatterplots, stem-and-leaf plots, two-way tables)
2. Calculates relative frequencies for rows or columns in two-way tables and uses the calculations to describe possible associations between the two variables
3. Uses the equation of a linear model to solve problems in the context of bivariate measurement data (e.g., interpreting the slope and intercept, interpolation)
4. Describes how two quantitative variables are related (e.g., fit a function to data, association, correlation)
5. Chooses appropriate graphs based on data type (e.g., numerical, categorical)

**C. Understands concepts associated with measures of central tendency and dispersion**

1. Solves for the mean and weighted average of given sets of data
2. Determines and interprets measures of center (e.g., mean, median, mode) and spread (e.g., range, interquartile range) in a variety of problems
3. Summarizes a given numerical data set in relation to its context
4. Describes the distribution of a set of data by its center and spread

5. Uses statistics appropriate to the shape of the data distribution to compare center and spread of two or more different data sets
6. Interprets differences in center and spread in the context of the data sets, accounting for possible effects of outliers

**D. Knows how to use and evaluate probability models**

1. Uses counting techniques (e.g., the counting principle, tree diagrams) to answer questions involving a finite sample space
2. Solves probability problems involving simple events
3. Solves probability problems involving compound events
4. Interprets a probability model and uses it to find probabilities of events
5. Compares probabilities from a model to observed frequencies and identifies possible sources of the discrepancy if the agreement is not good
6. Interprets a uniform probability model and uses it to determine probabilities of events

**Discussion areas: Statistics and Probability**

Note that the use of “e.g.” to start a list of examples implies that only a few examples are offered and not an exhaustive list.

- Be able to make inferences about a population based on a random sample from the population (e.g., estimate the number of people in a population for which a certain characteristic is true).
- Be able to interpret a line of best fit (trend line) and use it to solve problems.

- Be familiar with how to summarize numerical data sets in relation to their context (e.g., describe any overall pattern and any notable differences from the overall pattern, relate the chosen measures of center and variability to the shape of the data).
  - Be able to express the difference between the centers of two data sets as a multiple of a measure of variability.
  - Be able to solve counting problems by using counting techniques or by counting individual outcomes (e.g., construct or interpret a tree diagram that models a sample space).
  - Be able to solve probability problems involving independent events or dependent events.
  - Be able to solve problems involving a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.
  - Be able to solve problems involving a uniform probability model by assigning equal probability to all outcomes.
2. Evaluates or compares explanations and justifications for their validity, generalizability, coherence, or precision, including identifying flaws in explanations and justifications
  3. Determines the changes that would improve the validity, generalizability, coherence, and/or precision of an explanation or justification
  4. Evaluates whether counterarguments address a critique of a given justification
  5. Evaluates definitions or other mathematical language for validity, generalizability, precision, usefulness in a particular context, or support of key ideas

### **Mathematical problems, tasks, examples, and procedures**

## **Tasks of Teaching Mathematics**

This list includes instructional tasks that teachers engage in that are essential for effective teaching of middle school mathematics. Approximately 30% of test questions will measure content knowledge by assessing how that content knowledge is applied in the context of one or more of these tasks.

### **Mathematical explanations, justifications, and definitions**

1. Identifies valid explanations of mathematical concepts (e.g., explaining why a mathematical idea is considered to be true), procedures, representations, or models
6. Identifies problems or tasks that fit a particular structure, address the same concept, demonstrate desired characteristics, or elicit particular student thinking
7. Identifies two or more problems that systematically vary in difficulty or complexity
8. Evaluates the usefulness of examples for introducing a concept, illustrating an idea, or demonstrating a strategy, procedure, or practice
9. Identifies examples that support particular strategies or address particular student questions, misconceptions, or challenges with content
10. Identifies nonexamples or counterexamples that highlight a mathematical distinction or demonstrate why a student conjecture is incorrect or partially incorrect



11. Evaluates procedures for working with mathematics content to identify special cases in which the procedure might be problematic or for validity, appropriateness, or robustness

**Mathematical representations, models, manipulatives, and technology**

12. Evaluates representations and models (e.g., concrete, pictorial) in terms of validity, generalizability, usefulness for supporting students' understanding, or fit to the concept, calculation, etc. to be represented
13. Evaluates how representations and models (e.g., concrete, pictorial) have been used to show particular ideas, relationships between ideas, processes, or strategies
14. Evaluates the use of technology (e.g., graphing tools, software) for its appropriateness or its support of key ideas

**Students' mathematical reasoning**

15. Identifies likely misconceptions about or partial understanding of particular mathematics content and practices
16. Identifies how new mathematics content and practices can build on or connect to students' prior knowledge, including misconceptions and errors
17. Evaluates or compares student work (e.g., solutions, explanations, justifications, representations) in terms of validity, generalizability, coherence, and/or precision
18. Evaluates student work to identify the use of a particular concept, idea, or strategy

19. Identifies how a student's reasoning would replicate across similar problems
20. Identifies different pieces of student work that demonstrate the same reasoning
21. Identifies situations in which student work that seems valid might mask incorrect thinking

## Middle School Mathematics (5164) Sample Test Questions

### Information about Questions That Is Specific to the Middle School Mathematics Test

#### General

- All numbers used are real numbers.
- Unless otherwise stated, the domain of a given function  $f$  is the set of all real numbers  $x$  for which  $f(x)$  is a real number.
- Rectangular coordinate systems are used unless otherwise stated.
- Figures that accompany questions are intended to provide information that is useful in answering questions.
  - Figures are drawn to scale unless otherwise stated.
  - Lines shown as straight are straight, and angle measures are positive. Positions of points, angles, regions, etc., exist in the order shown.

#### Types of questions that may be on the test

- Selected-response questions—select one answer choice
  - These are questions that ask you to select only one answer choice from a list of four choices.
  - Note that in most selected-response questions that ask for numerical values, the exact answer should be found. If a selected-response question includes a word or phrase like “approximately,” “best approximates,” or “is closest to,” it generally indicates that the correct option will **not** be an exact value.
- Selected-response questions—select one or more answer choices
  - These are questions that ask you to select one or more answer choices from a list of choices. A question may or may not specify the number of choices to select. These questions are marked with square boxes beside the answer choices, not circles or ovals. See question 16 in the Sample Test Questions.
  - If a question of this type has exactly three answer choices, one, two, or three of the choices may be correct.
  - If a question of this type has more than three answer choices, the number of correct choices will be at least 2 but fewer than the number of choices. For example, if a question of this type has six answer choices, there will be two, three, four, or five correct choices.
- Selected-response questions—select an area
  - These are questions that ask you to select one or more locations on a picture or a figure (e.g., the  $xy$ -plane).

- Numeric-entry questions
  - Some of these questions ask you to enter your answer as an integer or a decimal in a single answer box. Equivalent forms of the correct answer, such as 2.5 and 2.50, are all correct. See questions 10 and 14 in the Sample Test Questions. Note that in these questions, the exact answer should be entered unless the question asks you to round your answer. Therefore, if one of these questions does **not** ask you to round your answer, you should be able to enter the exact answer in the numeric-entry box. If you are unable to do so, this may indicate that your answer is incorrect.
  - Some of these questions ask you to enter your answer as a fraction in two separate boxes—one for the numerator and one for the denominator. A negative sign can be entered in either box. Equivalent forms of the correct answer, such as  $\frac{1}{2}$  and  $\frac{6}{12}$ , are all correct, though there may be cases where you need to simplify your fraction so it fits in the boxes. See question 9 in the Sample Test Questions.
- Drag-and-drop questions
  - These questions ask you to pair up given phrases or expressions by dragging (with your computer mouse) phrases from one location and matching them with given phrases or expressions in another location. See question 8 in the Sample Test Questions.
- Table grid questions
  - These questions refer to a table in which statements appear in the first column. For each statement, select the correct properties by selecting the appropriate cell(s) in the table. See question 2 in the Sample Test Questions.
- Text completion questions
  - These questions ask you to select one or more answer choices to complete one or more sentences. The choices may be located in drop-down menus in the sentences or in columns at the end of the question. You will select one answer choice from each drop-down menu or column.

## Unit Conversions

1 mile = 5,280 feet	1 mile $\approx$ 1.61 kilometers	1 inch = 2.54 centimeters
1 pound = 16 ounces	1 ton = 2,000 pounds	1 kilogram $\approx$ 2.2 pounds
1 cup = 8 fluid ounces	1 quart = 2 pints	1 gallon $\approx$ 3.785 liters
1 pint = 2 cups	1 gallon = 4 quarts	1 liter = 1,000 cubic centimeters

## Formulas

### Area

Rectangle with length $\ell$ and width $w$ :	$A = \ell w$
Parallelogram with height $h$ and base of length $b$ :	$A = bh$
Triangle with height $h$ and base of length $b$ :	$A = \frac{1}{2}bh$
Trapezoid with height $h$ and bases of length $b_1$ and $b_2$ :	$A = \frac{1}{2}(b_1 + b_2)h$
Circle with radius $r$ :	$A = \pi r^2$

### Perimeter

Rectangle with length $\ell$ and width $w$ :	$P = 2\ell + 2w$
--	------------------

### Circumference

Circle with radius $r$ :	$C = 2\pi r$
--------------------------	--------------

### Volume

Right rectangular prism with length $\ell$ , width $w$ , and height $h$ :	$V = \ell wh$
Right prism with height $h$ and base of area $B$ :	$V = Bh$
Pyramid with height $h$ and base of area $B$ :	$V = \frac{1}{3}Bh$
Right circular cylinder with height $h$ and base of radius $r$ :	$V = \pi r^2 h$
Right circular cone with height $h$ and base of radius $r$ :	$V = \frac{1}{3}\pi r^2 h$
Sphere with radius $r$ :	$V = \frac{4}{3}\pi r^3$

**Surface Area**

Cube with side of length  $s$ : .....  $A = 6s^2$

Right rectangular prism with length  $\ell$ , width  $w$ , and height  $h$ : .....  $A = 2\ell w + 2\ell h + 2wh$

Right circular cylinder with height  $h$  and base of radius  $r$ : .....  $A = 2\pi rh + 2\pi r^2$

Sphere with radius  $r$ : .....  $A = 4\pi r^2$

**Other Formulas**

Quadratic formula: .....  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Arithmetic sequence: .....  $a_n = a_1 + (n-1)d$

Pythagorean theorem: .....  $a^2 + b^2 = c^2$

Sum of the measures of the interior angles of a polygon with  $n$  sides: .....  $S = 180^\circ(n-2)$

## Mathematics (5164) Sample Test Questions

1. A student used the same reasoning to evaluate four expressions. The four expressions and the student's answers are given as follows. The student incorrectly evaluated the first two expressions but correctly evaluated the next two expressions.

1.  $7 \times 2 - 6 + 3 = 5$
2.  $9 - 5 + 16 \div 8 = 2$
3.  $9 + 24 \div 3 - 1 = 16$
4.  $7 \times 2 - 18 \div 6 = 11$

If the student continues to use the same reasoning, which of the following expressions is the student most likely to evaluate **INCORRECTLY**?

- A.  $8 + 7 - 12 \div 3$
- B.  $13 - 3 \times 2 + 5$
- C.  $10 \times 6 \div 15 - 3$
- D.  $4 \times 5 + 10 - 12$

2. Ms. Kress asked her students to compare  $\frac{1}{3}$  and  $\frac{7}{8}$ . Four of her students correctly answered that  $\frac{7}{8}$  is greater than  $\frac{1}{3}$ , but they gave different explanations when asked to describe their strategies to the class.

Indicate whether each of the following student explanations provides evidence or does not provide evidence of a mathematically valid strategy for comparing  $\frac{1}{3}$  and  $\frac{7}{8}$ .

Student Explanation	Provides Evidence	Does Not Provide Evidence
When you look at the numbers, you see that 7 is bigger than 1, so $\frac{7}{8}$ is the bigger fraction.		
In the first fraction, 1 is less than half of 3, but in the second, 7 is more than half of 8, so $\frac{7}{8}$ is larger than $\frac{1}{3}$ .		
I multiplied 1 times 7 and 3 times 7, so $\frac{1}{3}$ is the same as $\frac{7}{21}$ . This means that $\frac{7}{8}$ is bigger than $\frac{1}{3}$ because $\frac{1}{8}$ is bigger than $\frac{1}{21}$ .		
I wanted to make a fraction equal to $\frac{1}{3}$ with the same bottom number as $\frac{7}{8}$ , so I added 5 to 3 and got 8. Then I added 5 to 1 and got 6, but 7 is greater than 6, so $\frac{7}{8}$ is greater.		

3. Ernesto bought 2 sport coats for \$88.95 each. One of the coats needed alterations that cost \$15.50, and a 6% sales tax is applied to the cost of the coats but not to the alterations.

Which of the following values is closest to the total cost for the sport coats and the alterations?

- A. \$190
- B. \$200
- C. \$205
- D. \$215

4. The maximum speed at which a horse can run is 36 miles per hour.

What is the maximum speed of the horse in feet per second?

- A. 2.4
- B. 24.5
- C. 37.5
- D. 52.8

5. A teacher wants to give an example in which the distributive property must be used to solve a literal equation for a given variable.

Which of the following examples best serves the teacher's purpose?

- A. Solve  $A = \frac{(a+b)h}{2}$  for  $b$ .
- B. Solve  $A = P(1+rt)$  for  $r$ .
- C. Solve  $P = 2\ell + 2w$  for  $w$ .
- D. Solve  $S = 2\ell w + 2\ell h + 2wh$  for  $h$ .

6. A line in the  $xy$ -plane passes through the point  $(4,5)$  and is parallel to the graph of  $3x + y = 4$ .

What is an equation of the line?

- A.  $y = -3x + 17$
- B.  $y = -3x + 7$
- C.  $y = 3x - 7$
- D.  $y = 3x - 17$



7. Mr. Keller’s class is learning about algebraic equations. In his teacher’s edition of the textbook, Mr. Keller finds a page that suggests he ask students to critique the following two solutions to determine whether they are valid.

$4x + 2 = 66$	$5 = 2x + 3$
$\frac{6x}{6} = \frac{66}{6}$	$\frac{5}{5} = \frac{5x}{5}$
$x = 11$	$x = 1$

Which of the following is best addressed by the preceding task?

- A. Misunderstanding of the properties of operations
  - B. Misunderstanding of the meaning of the equal sign
  - C. Misunderstanding of how to identify and combine like terms
  - D. Misunderstanding of how to use inverse operations to solve equations
8. The steps in a solution method for the equation  $\frac{1}{3}(11x + 20) = 2x$  follow.

Provide the justification for the result shown in each step in the solution method.

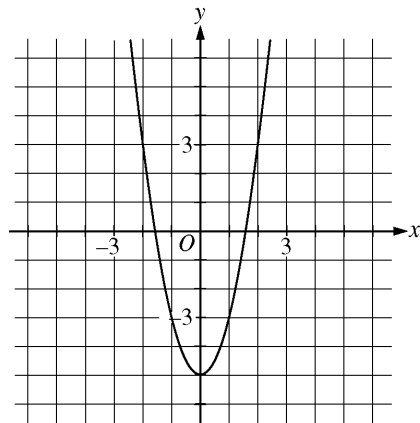
Addition/Subtraction Property of Equality  
 Multiplication/Division Property of Equality  
 Distributive Property

Step	Justification
$\frac{1}{3}(11x + 20) = 2x$	Given
$11x + 20 = 6x$	
$20 = -5x$	
$-4 = x$	

9. The graph of linear function  $f$  passes through the points  $(-3, 11)$  and  $(7, -4)$ .  
 What is the slope of the graph of  $f$ ?

Give your answer as a fraction.


10. The graph of the quadratic equation  $y = ax^2 + c$  is shown in the following  $xy$ -plane .



If  $a$  and  $c$  are integers, what are the values of  $a$  and  $c$  ?

$a =$

$c =$

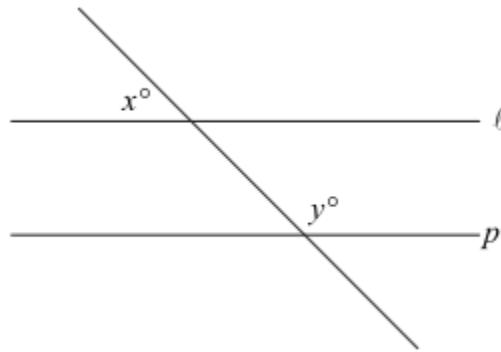
11. A teacher wants to show the students in an Algebra I class two examples of functions that are both linear and continuous. The teacher thinks of the following example.

The temperature in degrees Fahrenheit is a function of the temperature in degrees Celsius.

Which of the following is also an example of a function that is both linear and continuous?

- A. The height in inches of a person is a function of the person's age in years throughout the person's life.
- B. The number of calories consumed is a function of the volume, in ounces, of an energy drink that is consumed.
- C. The total cost in dollars for purchasing hot dogs is a function of the number of hot dogs purchased for \$2.00 each.
- D. The total number of games in a tournament is a function of the number of teams in the tournament when each team plays every other team once.

12. In the following figure, line  $\ell$  and line  $p$  are parallel, and  $y = 3x$ .



What is the value of  $x$ ?

- A. 75
- B. 60
- C. 45
- D. 30

13. When Ms. Slater gave her students a problem to assess their prior knowledge at the start of a lesson on finding the side length of a square given its area, several students incorrectly answered that the side length of a square with area 36 square units is 9 units. At the end of the lesson, Ms. Slater wanted to give a similar problem to assess whether her students were still making the same error.

Which of the following areas would be **LEAST** useful for assessing student learning in this situation?

- A. 4 square units
  - B. 16 square units
  - C. 64 square units
  - D. 100 square units
14. Reggie hiked 3,500 meters along a trail at a nearby park each day for the last 14 days.

How many **kilometers** did Reggie hike in the last 14 days?

kilometers

15. An automobile company sold 6 different models in the United States in a certain year. The following table shows the percent of total sales in the United States for each of the 6 models that year.

Model	Percent of Sales
B	49.5%
C	24.7%
D	16.1%
E	5.0%
F	3.5%
G	1.2%

If a circle graph is constructed using the data in the table, which of the following values best approximates the measure, in degrees, of the central angle for the sector representing the sales of Model D?

- A.  $4^\circ$
- B.  $16^\circ$
- C.  $29^\circ$
- D.  $58^\circ$

16. Each of the integers in list  $K$  (not shown) is greater than 75, and integers may appear more than once in the list. List  $M$  consists of the integers in list  $K$  and 4 additional integers that are each less than 75.

Which of the following statements about the centers or spreads of lists  $K$  and  $M$  must be true?

Select **ALL** that apply.

- A. The mean of the integers in list  $M$  is less than the mean of the integers in list  $K$ .
  - B. The median of the integers in list  $M$  is less than the median of the integers in list  $K$ .
  - C. The mode of the integers in list  $M$  is less than the mode of the integers in list  $K$ .
  - D. The range of the integers in list  $M$  is greater than the range of the integers in list  $K$ .
  - E. The interquartile range of the integers in list  $M$  is greater than the interquartile range of the integers in list  $K$ .
17. In a survey, 50 people were asked how many hours per day,  $h$ , they watched television. The survey results are shown in the following table.

**Hours of Television Watched per Day**

Number of Hours Watched per Day	Number of People
$h < 1$	5
$1 \leq h < 2$	12
$2 \leq h < 3$	16
$3 \leq h < 4$	14
$h \geq 4$	3

If a person is selected at random from those surveyed, what is the probability that the person selected will have watched at least 2 hours but less than 4 hours of television per day?

- A.  $\frac{3}{10}$
- B.  $\frac{8}{25}$
- C.  $\frac{1}{2}$
- D.  $\frac{3}{5}$

## Middle School Mathematics (5164) Answers

1. The correct answer is (B). A common misconception about the order of operations is that multiplication always comes before division and that addition always comes before subtraction. In the first expression, the student evaluated  $7 \times 2 - (6 + 3)$  instead of  $7 \times 2 - 6 + 3$ , and in the second expression, the student evaluated  $9 - (5 + 16 \div 8)$  instead of  $9 - 5 + 16 \div 8$ . In the next two expressions, the misconception described does not interfere with a student's ability to correctly evaluate the expressions, and the student obtained the correct answers, so it is likely that this misconception is the basis for the student's incorrect answers. In the expressions in (A), (C), and (D), this misconception does not interfere with a student's ability to correctly evaluate the expressions either, but in the expression in (B), it is incorrect to add before subtracting, so the expression in (B) is the one that the student is most likely to evaluate incorrectly.

<b>Task of Teaching Topic</b>	Students' mathematical reasoning
<b>Task of Teaching Subtopic</b>	19. Identifies how a student's reasoning would replicate across similar problems
<b>Category</b>	I. Numbers and Operations
<b>Topic</b>	A. Understands operations and properties of the real number system
<b>Subtopic</b>	3. Uses the order of operations to simplify computations and solve problems

2. The first and fourth explanations do not provide evidence of a mathematically valid strategy for comparing  $\frac{1}{3}$  and  $\frac{7}{8}$ , but the second and third explanations do. In the first explanation, the student compares only the numerators of the fractions, which is not a valid strategy because it does not take into account the effect of the denominator on the size of the pieces. In the second explanation, the student compares both fractions to the benchmark fraction  $\frac{1}{2}$ , which is a valid strategy since  $\frac{1}{3}$  is less than  $\frac{1}{2}$  and  $\frac{7}{8}$  is greater than  $\frac{1}{2}$ . In the third explanation, the student uses multiplicative reasoning to find a common numerator, and then the student compares the fractions by reasoning about the sizes of the unit

fractions  $\frac{1}{8}$  and  $\frac{1}{21}$ . This is a valid strategy. In the fourth explanation, the student uses additive reasoning to try to find a fraction equivalent to  $\frac{1}{3}$  that has a denominator of 8, but  $\frac{6}{8}$  is not equivalent to  $\frac{1}{3}$ , so this strategy is not valid.

<b>Task of Teaching Topic</b>	Students' mathematical reasoning
<b>Task of Teaching Subtopic</b>	17. Evaluates or compares student work (e.g., solutions, explanations, justifications, representations) in terms of validity, generalizability, coherence, and/or precision
<b>Category</b>	I. Numbers and Operations
<b>Topic</b>	A. Understands operations and properties of the real number system
<b>Subtopic</b>	5. Compares and orders real numbers, including absolute values of real numbers

3. The correct answer is (C). Based on the information in the question, the total cost can be calculated as  $(\$88.95)(2)(1.06) + \$15.50 = \$204.07$ . The choice that is closest to the total cost is \$205.

<b>Category</b>	I. Numbers and Operations
<b>Topic</b>	C. Understands how to use ratios and proportional relationships to solve problems
<b>Subtopic</b>	3. Solves percent problems (e.g., expressing a percent as a ratio per 100, discounts, markups, taxes, tips, simple interest, percent error)

4. The correct answer is (D). To find the maximum speed of the horse in feet per second, multiply the maximum speed of the horse in miles per hour by the appropriate conversion factors. Note that certain conversion factors are provided on the test, such as the conversion from miles to feet, but other conversion factors are not provided, such as the conversions from hours to minutes and from minutes to seconds.

Since 1 mile = 5,280 feet, 1 hour = 60 minutes, and 1 minute = 60 seconds, the maximum speed of the horse in feet per second is equal to

$$\frac{36 \text{ miles}}{1 \text{ hour}} \cdot \frac{5,280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}}$$

which is equal to 52.8 feet per second. Remember that when calculations like this are performed, the units in the numerators and denominators of the fractions need to be divided out so that only the required units remain.

<b>Category</b>	I. Numbers and Operations
<b>Topic</b>	D. Understands how to reason quantitatively and use units to solve problems
<b>Subtopic</b>	3. Solves problems involving dimensional analysis (e.g., feet per second to miles per hour, feet per second to kilometers per hour)

5. The correct answer is (D). Remember that both  $a(b + c) = ab + ac$  and  $ab + ac = a(b + c)$  demonstrate the distributive property. To solve  $S = 2\ell w + 2\ell h + 2wh$  for  $h$ , first subtract  $2\ell w$  from both sides of the equation to obtain  $S - 2\ell w = 2\ell h + 2wh$ . Then use the distributive property to factor the right-hand side of the equation to obtain  $S - 2\ell w = h(2\ell + 2w)$ . Finally, divide both sides of the equation by  $2\ell + 2w$  to obtain  $\frac{S - 2\ell w}{2\ell + 2w} = h$ . In this example, since  $h$  appears in two of the terms on the right-hand side of the equation but

does not appear in the third term, the distributive property must be used to isolate  $h$ , which means (D) is the example that best serves the teacher's purpose. Each of the other examples can be solved for the given variable without using the distributive property.

<b>Task of Teaching Topic</b>	Mathematical problems, tasks, examples, and procedures
<b>Task of Teaching Subtopic</b>	9. Identifies examples that support particular strategies or address particular student questions, misconceptions, or challenges with content
<b>Category</b>	II. Algebra
<b>Topic</b>	A. Understands how to create, evaluate, and manipulate algebraic expressions, equations, and formulas
<b>Subtopic</b>	8. Rearranges formulas to solve for a specified variable (e.g., solve $d = rt$ for $t$ )

6. The correct answer is (A). Since  $3x + y = 4$  is equivalent to  $y = -3x + 4$ , the slope of the graph of  $3x + y = 4$  is  $-3$ . This means that the slope of the line that passes through the point  $(4, 5)$  is also  $-3$  since the line is parallel to the graph of  $3x + y = 4$ . Substituting the slope of  $-3$  and the point  $(4, 5)$  into the point-slope form of the equation of a line yields  $y - 5 = -3(x - 4)$ . Applying the distributive property yields



$y - 5 = -3x + 12$ , and then adding 5 to both sides of the equation yields  $y = -3x + 17$ , which is the equation in (A).

<b>Category</b>	II. Algebra
<b>Topic</b>	B. Understands how to recognize and represent linear relationships algebraically
<b>Subtopic</b>	1. Determines the equation of a line from information presented in various forms (e.g., table, graph, description)

7. The correct answer is (C). In the first solution,  $4x$  and 2 are added to get  $6x$ , but the  $4x$  term contains a variable, whereas the 2 is a constant term; it is incorrect to add  $4x$  and 2 because they are not like terms. Similarly, in the second solution,  $2x$  and 3 are added to get  $5x$ , but  $2x$  and 3 are not like terms, so this strategy is not valid. Therefore, a misunderstanding of how to identify and combine like terms is the option that is best addressed by asking students to critique the two invalid strategies.

<b>Task of Teaching Topic</b>	Mathematical problems, tasks, examples, and procedures
<b>Task of Teaching Subtopic</b>	8. Evaluates the usefulness of examples for introducing a concept, illustrating an idea, or demonstrating a strategy, procedure, or practice
<b>Category</b>	II. Algebra
<b>Topic</b>	C. Understands how to solve equations and inequalities
<b>Subtopic</b>	1. Solves one-variable linear equations and inequalities

8. The correct answer, from top to bottom, is the Multiplication/Division Property of Equality, the Addition/Subtraction Property of Equality, and the Multiplication/Division Property of Equality. The equation  $\frac{1}{3}(11x + 20) = 2x$  is multiplied by 3 on both sides to obtain  $11x + 20 = 6x$ , so this step is justified by the Multiplication/Division Property of Equality. Then,  $11x$  is subtracted from both sides of the equation  $11x + 20 = 6x$  to obtain  $20 = -5x$ , so this step is justified by the Addition/Subtraction Property of Equality. Finally, both sides of the equation  $20 = -5x$  are divided by  $-5$  to obtain  $-4 = x$ , so this step is justified by the Multiplication/Division Property of Equality.

<b>Category</b>	II. Algebra
<b>Topic</b>	C. Understands how to solve equations and inequalities
<b>Subtopic</b>	4. Justifies each step in solving equations and inequalities

9. The correct answer is  $-\frac{3}{2}$ . The slope of a linear function can be found by substituting into the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } m \text{ is the slope and}$$

$(x_1, y_1)$  and  $(x_2, y_2)$  are two points on the linear function. Substituting the given points into the formula gives

$$m = \frac{-4 - 11}{7 - (-3)} = \frac{-15}{10} = -\frac{3}{2}.$$

<b>Category</b>	III. Functions
<b>Topic</b>	C. Understands basic characteristics of linear functions (e.g., intercepts, slope)
<b>Subtopic</b>	2. Calculates the slope of a line presented as a table of values, algebraically, or graphically and interprets it in a modeling context

10. The correct answer is  $a = 2$  and  $c = -5$ . Since the graph of the equation intersects the  $y$ -axis at the point  $(0, -5)$ , the value of  $c$  must be  $-5$ . Then, one method to find the value of  $a$  is to substitute the coordinates from another point on the graph into the equation and solve for  $a$ . Using the point  $(2, 3)$  and the fact that  $c = -5$ , it can be determined that  $3 = a(2^2) - 5$ , so  $4a - 5 = 3$ . To solve this equation for  $a$ , add 5 to both sides of the equation, and then divide both sides of the equation by 4, which leads to the answer  $a = 2$ .

<b>Category</b>	III. Functions
<b>Topic</b>	D. Understands the relationships among functions, tables, and graphs
<b>Subtopic</b>	1. Determines an equation to represent a linear or quadratic function presented graphically

11. The correct answer is (B). The function is linear because the number of calories consumed is proportional to the number of ounces of the energy drink that are consumed, and the function is continuous because the volume of the energy drink that is consumed can be measured with any level of accuracy (e.g., to the nearest hundredth of an ounce, not only the nearest ounce). The function in (A) is continuous but it is not linear because a person does not grow linearly over time. The function in (C) is linear but it is not continuous because it can be assumed that one can only buy a whole number of hot dogs. The function

in (D) is neither linear nor continuous because the function is quadratic and there can only be a whole number of teams in the tournament.

<b>Task of Teaching Topic</b>	Mathematical problems, tasks, examples, and procedures
<b>Task of Teaching Subtopic</b>	8. Evaluates the usefulness of examples for introducing a concept, illustrating an idea, or demonstrating a strategy, procedure, or practice
<b>Category</b>	III. Functions
<b>Topic</b>	F. Understands differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems
<b>Subtopic</b>	1. Identifies situations in which one quantity changes at a constant rate per unit interval relative to another

12. The correct answer is (C). The properties of angles associated with parallel and transversal lines can be used to show that the angle with measure  $x$  degrees and the angle with measure  $y$  degrees are supplementary angles. The sum of the measures of supplementary angles is  $180^\circ$ , so  $x + y = 180$ . It is given that  $y = 3x$ . Substituting  $3x$  for  $y$  in the equation  $x + y = 180$  yields  $4x = 180$ . Hence,  $x = 45$ .

<b>Category</b>	IV. Geometry and Measurement
<b>Topic</b>	A. Knows the properties of types of lines (e.g., parallel, perpendicular, intersecting) and angles
<b>Subtopic</b>	2. Applies angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems

13. The correct answer is (B). At the start of the lesson, several students answered that the side length of a square with area 36 square units is 9 units instead of giving the correct side length of 6 units. Since the side length of a square with **perimeter** 36 units is 9 units, the students are probably confusing area and perimeter. Therefore, the area that would be the least useful for assessing student learning in this situation is the one that would allow a student to find the correct answer by dividing the number of square units by 4, since one would divide the perimeter of a square by 4 to find the side length of the square. In (B),  $\sqrt{16} = 4$  and  $16 \div 4 = 4$ . Since the answers are the same, Ms. Slater would have no way of knowing whether students were thinking about area or thinking about perimeter when finding the answer, so this is not a useful problem for assessing student learning in this situation. In each of (A), (C), and (D), the square root of the number of square units is not equal to the result when the number of square units is divided by 4, so these areas would be useful for assessing student learning in this situation.

<b>Task of Teaching Topic</b>	Mathematical problems, tasks, examples, and procedures
<b>Task of Teaching Subtopic</b>	6. Identifies problems or tasks that fit a particular structure, address the same concept, demonstrate desired characteristics, or elicit particular student thinking
<b>Category</b>	IV. Geometry and Measurement
<b>Topic</b>	H. Understands how to solve problems involving perimeter and area of polygons
<b>Subtopic</b>	1. Calculates and interprets perimeter and area of polygons that can be composed of triangles and quadrilaterals, including in real-world situations

14. The correct answer is 49 kilometers. Reggie hiked 3,500 meters along a trail each day for 14 days, so Reggie hiked  $3,500 \times 14 = 49,000$  meters during that time. Since there are 1,000 meters in 1 kilometer, dividing 49,000 by 1,000 gives the final answer: that Reggie hiked 49 kilometers in the last 14 days.

<b>Category</b>	IV. Geometry and Measurement
<b>Topic</b>	J. Understands systems of measurement (i.e., metric, United States customary)
<b>Subtopic</b>	1. Solves measurement, estimation, and conversion problems involving time, length, temperature, volume, and mass in standard measurement systems

15. The correct answer is (D). There are  $360^\circ$  in a circle, and 16.1% of 360 is equal to  $0.161 \times 360 = 57.96$ . So of the values given,  $58^\circ$  is the value that best approximates the measure of the central angle of the sector representing the sales of Model D.

<b>Category</b>	V. Statistics and Probability
<b>Topic</b>	B. Understands how to interpret, analyze, and represent data presented in a variety of displays
<b>Subtopic</b>	1. Represents and analyzes data in various displays (e.g., bar graphs, line graphs, circle graphs, boxplots, histograms, scatterplots, stem-and-leaf plots, two-way tables)

16. The correct answers are (A) and (D). Since the 4 additional integers that list  $M$  contains are each less than each of the integers in list  $K$ , then each of the 4 additional integers that list  $M$  contains must be less than the mean of the integers in list  $K$ , so the statement in (A) must be true. Also, since the greatest integer in list  $M$  is equal to the greatest integer in list  $K$ , but the least integer in list  $M$  is less than the least integer in list  $K$ , the statement in (D) must be true. To demonstrate that the statements in (B), (C), and (E) may not be true, suppose that list  $K$  consists of the integer 76 listed 15 times and that list  $M$  consists of the integers 71, 72, 73, 74, and the integer 76 listed 15 times. In this case, the median of each list is 76 and the mode of each list is 76, so the statements in (B) and (C) are not true for this example. In addition, the first quartile of each list is 76 and the third quartile of each list is 76, which means the interquartile range of each list is 0, so the statement in (E) is not true for this example. Therefore, the statements in (B), (C), and (E) may not be true.

<b>Category</b>	V. Statistics and Probability
<b>Topic</b>	C. Understands concepts associated with measures of central tendency and dispersion
<b>Subtopic</b>	2. Determines and interprets measures of center (e.g., mean, median, mode) and spread (e.g., range, interquartile range) in a variety of problems

17. The correct answer is (D). Based on the data in the table, a total of  $16 + 14 = 30$  people surveyed watched at least 2 hours of television but less than 4 hours of television per day. If a person is selected at random from those surveyed, the probability that the person selected will have watched at least 2 hours but less than 4 hours per day is  $\frac{30}{50} = \frac{3}{5}$ .

<b>Category</b>	V. Statistics and Probability
<b>Topic</b>	D. Knows how to use and evaluate probability models
<b>Subtopic</b>	2. Solves probability problems involving simple events

## Understanding Question Types

The *Praxis*® assessments include a variety of question types: constructed response (for which you write a response of your own); selected response, for which you select one or more answers from a list of choices or make another kind of selection (e.g., by selecting a sentence in a text or by selecting part of a graphic); and numeric entry, for which you enter a numeric value in an answer field. You may be familiar with these question formats from taking other standardized tests. If not, familiarize yourself with them so you don't spend time during the test figuring out how to answer them.

### Understanding Selected-Response and Numeric-Entry Questions

For most questions, you respond by selecting an oval to select a single answer from a list of answer choices.

However, interactive question types may also ask you to respond by:

- Selecting more than one choice from a list of choices.
- Typing in a numeric-entry box. When the answer is a number, you may be asked to enter a numerical answer. Some questions may have more than one entry box to enter a response.
- Selecting parts of a graphic. In some questions, you will select your answers by selecting a location (or locations) on a graphic such as a map or chart, as opposed to choosing your answer from a list.
- Selecting sentences. In questions with reading passages, you may be asked to choose your answers by selecting a sentence (or sentences) within the reading passage.
- Dragging and dropping answer choices into targets on the screen. You may be asked to select answers from a list of choices and to drag your answers to the appropriate location in a table, paragraph of text or graphic.
- Selecting answer choices from a drop-down menu. You may be asked to choose answers by selecting choices from a drop-down menu (e.g., to complete a sentence).

Remember that with every question you will get clear instructions.

## Understanding Constructed-Response Questions

Constructed-response questions require you to demonstrate your knowledge in a subject area by writing your own response to topics. Essays and short-answer questions are types of constructed-response questions.

For example, an essay question might present you with a topic and ask you to discuss the extent to which you agree or disagree with the opinion stated. You must support your position with specific reasons and examples from your own experience, observations, or reading.

Review a few sample essay topics:

- *Brown v. Board of Education of Topeka*

“We come then to the question presented: Does segregation of children in public schools solely on the basis of race, even though the physical facilities and other ‘tangible’ factors may be equal, deprive the children of the minority group of equal educational opportunities? We believe that it does.”

- A. What legal doctrine or principle, established in *Plessy v. Ferguson* (1896), did the Supreme Court reverse when it issued the 1954 ruling quoted above?
- B. What was the rationale given by the justices for their 1954 ruling?

- *In his self-analysis, Mr. Payton says that the better-performing students say small-group work is boring and that they learn more working alone or only with students like themselves. Assume that Mr. Payton wants to continue using cooperative learning groups because he believes they have value for all students.*
  - Describe TWO strategies he could use to address the concerns of the students who have complained.
  - Explain how each strategy suggested could provide an opportunity to improve the functioning of cooperative learning groups. Base your response on principles of effective instructional strategies.
- *“Minimum-wage jobs are a ticket to nowhere. They are boring and repetitive and teach employees little or nothing of value. Minimum-wage employers take advantage of people because they need a job.”*
  - Discuss the extent to which you agree or disagree with this opinion. Support your views with specific reasons and examples from your own experience, observations, or reading.

Keep these things in mind when you respond to a constructed-response question:

1. **Answer the question accurately.** Analyze what each part of the question is asking you to do. If the question asks you to describe or discuss, you should provide more than just a list.
2. **Answer the question completely.** If a question asks you to do three distinct things in your response, you should cover all three things for the best score. Otherwise, no matter how well you write, you will not be awarded full credit.
3. **Answer the question that is asked.** Do not change the question or challenge the basis of the question. You will receive no credit or a low score if you answer another question or if you state, for example, that there is no possible answer.
4. **Give a thorough and detailed response.** You must demonstrate that you have a thorough understanding of the subject matter. However, your response should be straightforward and not filled with unnecessary information.
5. **Take notes on scratch paper** so that you don't miss any details. Then you'll be sure to have all the information you need to answer the question.
6. **Reread your response.** Check that you have written what you thought you wrote. Be sure not to leave sentences unfinished or omit clarifying information.



To search for the Praxis test prep resources  
that meet your specific needs, visit:

**[www.ets.org/praxis/testprep](http://www.ets.org/praxis/testprep)**

To purchase official test prep made by the creators  
of the Praxis tests, visit the ETS Store:

**[www.ets.org/praxis/store](http://www.ets.org/praxis/store)**



**[www.ets.org](http://www.ets.org)**