ETS PRAXIS

Multistate Standard-Setting Technical Report

PRAXIS® MATHEMATICS (5165)

Licensure and Credentialing Research

ETS

Princeton, New Jersey

February 2021

Executive Summary

To support the decision-making process of education agencies establishing a passing score (cut score) for the *Praxis*® Mathematics (5165) test, research staff from Educational Testing Service (ETS) designed and conducted a distance-based multistate standard-setting study.

Participating States

Panelists from 13 states and Washington, D.C., were recommended by their respective education agencies. The education agencies recommended panelists with (a) experience as either secondary mathematics teachers or college faculty who prepare secondary mathematics teachers and (b) familiarity with the knowledge and skills required of beginning secondary mathematics teachers.

Recommended Passing Score

ETS provides a recommended passing score from the multistate standard-setting study to help education agencies determine an appropriate operational passing score. For the *Praxis* Mathematics test, the recommended passing score[1] is 39 out of a possible 60 raw-score points. The scale score associated with a raw score of 39 is 159 on a 100–200 scale.

[1] Results from the two panels participating in the study were averaged to produce the recommended passing score.

INTRODUCTION

To support the decision-making process for education agencies establishing a passing score (cut score) for the *Praxis*® Mathematics (5165) test, research staff from ETS designed and conducted a distance-based multistate standard-setting study in January 2021. Education agencies[2] recommended panelists with (a) experience as either secondary mathematics teachers or college faculty who prepare secondary mathematics teachers and (b) familiarity with the knowledge and skills required of beginning secondary mathematics teachers. Thirteen states and Washington, D.C. (Table 1) were represented by 25 panelists. (See Appendix A for the names and affiliations of the panelists.)

Table 1
Participating States, Washington, D.C., and Number of Panelists

Alabama (2 panelists)	Mississippi (2 panelists)
Arkansas (2 panelists)	Nevada (1 panelist)
Delaware (2 panelists)	South Carolina (2 panelists)
Idaho (1 panelist)	South Dakota (2 panelists)
Indiana (1 panelist)	Tennessee (2 panelists)
Kansas (2 panelists)	Washington, D.C. (2 panelists)
Maryland (2 panelists)	West Virginia (2 panelists)

The following technical report contains three sections. The first section describes the content and format of the test. The second section describes the standard-setting processes and methods. The third section presents the results of the standard-setting study.

ETS provides a recommended passing score from the multistate standard-setting study to education agencies. In each state and D.C., the department of education, the board of education, or a designated educator licensure board is responsible for establishing the operational passing score in accordance with applicable regulations. This study provides a recommended passing score,[3] which represents the combined judgments of two panels of experienced educators. Each state and D.C., may want to consider the recommended passing score but also other sources of information when setting the final *Praxis* Mathematics passing score (see Geisinger & McCormick, 2010). A states and D.C., may accept the recommended passing score, adjust the score upward to reflect more stringent expectations, or

[2] States and jurisdictions that currently use *Praxis* tests were invited to participate in the multistate standard-setting study.
[3] In addition to the recommended passing score averaged across the two panels, the passing scores for each panel are presented.

adjust the score downward to reflect more lenient expectations. There is no *correct* decision; the appropriateness of any adjustment may only be evaluated in terms of its meeting the state and D.C.'s, needs.

Two sources of information to consider when setting the passing score are the standard error of measurement (SEM) and the standard error of judgment (SEJ). The former addresses the reliability of the *Praxis* Mathematics test score and the latter, the reliability of panelists' passing-score recommendation. The SEM allows a state and D.C., to recognize that any test score on any standardized test—including a *Praxis* Mathematics test score—is not perfectly reliable. A test score only *approximates* what a candidate truly knows or truly can do on the test. The SEM, therefore, addresses the question: How close of an approximation is the test score to the *true* score? The SEJ allows a state and D.C. to gauge the likelihood that the recommended passing score from a particular panel would be similar to the passing scores recommended by other panels of experts similar in composition and experience. The smaller the SEJ, the more likely that another panel would recommend a passing score consistent with the recommended passing score. The larger the SEJ, the less likely the recommended passing score would be reproduced by another panel.

In addition to measurement error metrics (e.g., SEM, SEJ), each state and D.C. should consider the likelihood of classification errors. That is, when adjusting a passing score, policymakers should consider whether it is more important to minimize a false-positive decision or to minimize a false-negative decision. A false-positive decision occurs when a candidate's test score suggests that he should receive a license/certificate, but his actual level of knowledge/skills indicates otherwise (i.e., the candidate does not possess the required knowledge/skills). A false-negative decision occurs when a candidate's test score suggests that she should not receive a license/certificate, but she actually does possess the required knowledge/skills. The state and D.C. need to consider which decision error is more important to minimize.

Overview of the *Praxis*® Mathematics Test

The Praxis® Mathematics Test at a Glance document (ETS, in press) describes the purpose and structure of the test. In brief, the test measures the knowledge and competencies important for safe and effective beginning practice as a secondary school mathematics teacher.

The three-hour assessment contains 66 selected-response items[4] covering four content areas: *Number & Quantity and Algebra* (approximately 20 items), *Functions and Calculus* (approximately 20 items), *Geometry* (approximately 13 items), and *Statistics & Probability* (approximately 13 items).[5] The reporting scale for the *Praxis* Mathematics test ranges from 100 to 200 scale-score points.

Processes and Methods

The design of the standard-setting study included two, independent expert panels of educators with experience with the test content and with new teachers or teacher candidates. Before the study, panelists received an email explaining the purpose of the standard-setting study and requesting that they review materials for the study, such as the test specifications and an overview presentation. This review helped familiarize the panelists with the general structure and content of the test. Additionally, panelists were asked to attend a brief, technology check meeting, to ensure that everyone could access the technology needed for the study.

For each panel, the first day of the standard-setting study began with a welcome by the meeting facilitator. After introductions of the panelists and ETS staff, the facilitator engaged the panel in a question and answer period about the overview presentation. Appendix B shows the agenda for the panel meeting.

Reviewing the Test

Test familiarization was the first activity for the panel. The purpose of test familiarization is for the panelists to review the test and become familiar with the manner in which a candidate would take the test. After the facilitator described the purpose of the review and how to access the test[6], the standard-

[4] Six of the 66 selected-response items are pretest items and do not contribute to a candidate's score.
[5] The number of items for each content area may vary slightly from form to form of the test.
[6] The computer-administered test items were available through the ETS IBIS Content Review Tool.

setting panelists took the test and had a discussion of the content measured. This discussion helped bring the panelists to a shared understanding of what the test measures.

The test discussion covered the major content areas being addressed by the test. Panelists were asked to remark on any content areas that would be particularly challenging for entry-level teachers or areas that address content particularly important for entry-level teachers. Overall, this discussion serves to reduce potential judgment errors later in the standard-setting process.

DEFINING THE JUST QUALIFIED CANDIDATE

Following the review of the test, panelists described the just qualified candidate. The *just qualified candidate description* plays a central role in standard setting (Perie, 2008); the goal of the standard-setting process is to identify the test score that aligns with this description.

Both panels worked together to create the final description of the just qualified candidate — the knowledge/skills that differentiate a *just* from a *not quite* qualified candidate. Each panel first worked separately by working in smaller and then a large group. Then both panels convened and, through whole-group discussion, combined the two descriptions in to the final version of the just qualified candidate to use for the remainder of the study.

The written description of the just qualified candidate summarized the panel discussion in a bulleted format. The description was not intended to describe all the knowledge and skills of the just qualified candidate but only highlight those that differentiate a *just* qualified candidate from a *not quite* qualified candidate. The written description was distributed to panelists to use during later phases of the study (see Appendix C for the just qualified candidate description).

PANELISTS' JUDGMENTS

The standard-setting process for the *Praxis* Mathematics test was a probability-based Modified Angoff method (Brandon, 2004; Hambleton & Pitoniak, 2006). In this study, each panelist judged each item on the likelihood (probability or chance) that the just qualified candidate would answer the item correctly. Panelists made their judgments using the following rating scale: 0, .05, .10, .20, .30, .40, .50, .60, .70, .80, .90, .95, 1. The lower the value, the less likely it is that the just qualified candidate would answer the item correctly because the item is difficult for the just qualified candidate. The higher the value, the more likely it is that the just qualified candidate would answer the item correctly.

Panelists were asked to approach the judgment process in two stages. First, they reviewed both the description of the just qualified candidate and the item and determined what was the probability that the just qualified candidate would answer the question correctly. The facilitator encouraged the panelists to consider the following rules of thumb to guide their decision:

- Items in the 0 to .30 range were those the just qualified candidate would have a low chance of answering correctly.
- Items in the .40 to .60 range were those the just qualified candidate would have a moderate chance of answering correctly.
- Items in the .70 to 1 range were those that the just qualified candidate would have a high chance of answering correctly.

Next, panelists decided how to refine their judgment within the range. For example, if a panelist thought that there was a high chance that the just qualified candidate would answer the question correctly, the initial decision would be in the .70 to 1 range. The second decision for the panelist was to judge if the likelihood of answering it correctly is .70, .80, .90, .95 or 1.

After the training, panelists made practice judgments and discussed those judgments and their rationales. All panelists completed a post-training evaluation to confirm that they had received adequate training and felt prepared to continue; the standard-setting process continued only if all panelists confirmed their readiness.

Following this first round of judgments (*Round 1*), item-level feedback was provided to the panel. The panelists' judgments were displayed for each item and summarized across panelists. Items were highlighted to show when panelists converged in their judgments (at least two-thirds of the panelists located an item in the same difficulty range) or diverged in their judgments.

The panelists discussed their item-level judgments. These discussions helped panelists maintain a shared understanding of the knowledge/skills of the just qualified candidate and helped to clarify aspects of items that might not have been clear to all panelists during the Round 1 judgments. The purpose of the discussion was not to encourage panelists to conform to another's judgment, but to understand the different relevant perspectives among the panelists.

In Round 2, panelists discussed their Round 1 judgments and were encouraged by the facilitator (a) to share the rationales for their judgments and (b) to consider their judgments in light of the rationales provided by the other panelists. Panelists recorded their Round 2 judgments only for items when they wished to change a Round 1 judgment. Panelists' final judgments for the study, therefore, consist of their Round 1 judgments and any adjusted judgments made during Round 2.

Other than the description of the just qualified candidate, results from Panel 1 were not shared with Panel 2. The item-level judgments and resulting discussions for Panel 2 were independent of judgments and discussions that occurred with Panel 1.

RESULTS

EXPERT PANELS

Table 2 presents a summary of the panelists' demographic information. The panel included 26 educators representing 12 states and D.C. (See Appendix A for a listing of panelists.) Twelve panelists were teachers, eight were college faculty, two were specialists, and three held another position. All of the faculty members' job responsibilities included the training of secondary mathematics teachers.

The number of experts by panel and their demographic information are presented in Appendix D (Table D1).

Table 2
Panel Member Demographics (Across Panels)

	N	%
Current position		
Teacher	12	48
College faculty	8	32
Mathematics Specialist	2	8
Other	3	12
Race		
White	23	92
Black or African American	2	8
Gender		
Female	18	72
Male	7	28
Are you currently certified to teach mathematics in your state?		
Yes	22	88
No	3	12

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Table 2 (continued)
Panel Member Demographics (Across Panels)

	N	%
Are you currently teaching mathematics in your state?		
Yes	19	76
No	6	24
Are you currently supervising or mentoring mathematics teachers?		
Yes	23	92
No	2	8
At what K–12 grade level are you currently teaching mathematics?		
Elementary (K–5 or K–6)	1	4
Middle school (6–8 or 7–9)	1	4
Middle and High school	1	4
High school (9–12 or 10–12)	10	40
All Grades	1	4
Not currently teaching at the K–12 level	11	44
Including this year, how many years of experience do you have teaching mathematics?		
3 years or less	0	0
4–7 years	0	0
8–11 years	7	28
12–15 years	3	12
16 years or more	15	60
Which best describes the location of your K–12 school?		
Urban	3	12
Suburban	7	28
Rural	4	16
Not currently working at the K–12 level	11	44
If you are college faculty, are you currently involved in the training/preparation of teacher candidates in mathematics?		
Yes	8	32
No	0	0
Not college faculty	17	68

Standard-Setting Judgments

Table 3 summarizes the standard-setting judgments (Round 2) of panelists. The table also includes estimates of the measurement error associated with the judgments: the standard deviation of the mean and the standard error of judgment (SEJ). The SEJ is one way of estimating the reliability or consistency of a panel's standard-setting judgments.[7] It indicates how likely it would be for several other panels of educators similar in makeup, experience, and standard-setting training to the current panel to recommend the same passing score on the same form of the test. The confidence intervals created by adding/subtracting two SEJs to each panel's recommended passing score overlap, indicating that they may be comparable.

Panelist-level results, for Rounds 1 and 2, are presented in Appendix D (Table D2).

Table 3
Summary of Round 2 Standard-setting Judgments

	Panel 1	Panel 2
Average	37.18	39.94
Lowest	30.40	33.95
Highest	43.70	46.10
SD	4.22	3.64
SEJ	1.17	1.05

Round 1 judgments are made without discussion among the panelists. The most variability in judgments, therefore, is typically present in the first round. Round 2 judgments, however, are informed by panel discussion; thus, it is common to see a decrease both in the standard deviation and SEJ. This decrease — indicating convergence among the panelists' judgments — was observed for each panel (see Table D2 in Appendix D). The Round 2 average score is the panel's recommended passing score.

The panels' passing score recommendations for the *Praxis* Mathematics test are 37.18 for Panel 1 and 39.94 for Panel 2 (out of a possible 60 raw-score points). The values were rounded to the next highest whole number, to determine the functional recommended passing score — 38 for Panel 1 and 40 for Panel 2. The scale scores associated with 38 and 40 raw points are 157 and 161, respectively.

In addition to the recommended passing score for each panel, the average passing score across the two panels is provided to help education agencies determine an appropriate passing score. The panels' average passing score recommendation for the *Praxis* Mathematics test is 38.56 (out of a possible 60 raw-score points). The value was rounded to 39 (next highest raw score) to determine the functional recommended passing score. The scale score associated with 39 raw points is 159.

[7] An SEJ assumes that panelists are randomly selected and that standard-setting judgments are independent. It is seldom the case that panelists are randomly sampled, and only the first round of judgments may be considered independent. The SEJ, therefore, likely underestimates the uncertainty of passing scores (Tannenbaum & Katz, 2013).

Table 4 presents the estimated conditional standard error of measurement (CSEM) around the recommended passing score. A standard error represents the uncertainty associated with a test score. The scale scores associated with one and two CSEM above and below the recommended passing score are provided. The conditional standard error of measurement provided is an estimate.

Table 4

***Passing Scores Within 1 and 2 CSEM of the Recommended Passing Score*[8]**

Recommended passing score (CSEM)		Scale score equivalent
	39 (3.73)	159
-2 CSEM	32	143
-1 CSEM	36	152
+ 1 CSEM	43	168
+ 2 CSEM	47	177

***Note.* CSEM = conditional standard error(s) of measurement.**

Final Evaluations

The panelists completed an evaluation at the conclusion of their standard-setting study. The evaluation asked the panelists to provide feedback about the quality of the standard-setting implementation and the factors that influenced their decisions. The responses to the evaluation provided evidence of the validity of the standard-setting process, and, as a result, evidence of the reasonableness of the recommended passing score.

Panelists were also shown the panel's recommended passing score and asked (a) how comfortable they are with the recommended passing score and (b) if they think the score was too high, too low, or about right. A summary of the final evaluation results is presented in Appendix D.

All panelists *strongly agreed* or *agreed* that they understood the purpose of the study and that the facilitator's instructions and explanations were clear. All panelists *strongly agreed* or *agreed* that they were prepared to make their standard-setting judgments. All panelists *strongly agreed* or *agreed* that the standard-setting process was easy to follow.

All panelists reported that the description of the just qualified candidate was at least *somewhat influential* in guiding their standard-setting judgments; 22 of the 25 panelists indicated the description was *very influential*. All of the panelists reported that between-round discussions were at least *somewhat influential* in guiding their judgments. More than half of the panelists (18 of the 25 panelists) indicated that their own professional experience was *very influential* in guiding their judgments.

[8] The unrounded CSEM value is added to or subtracted from the rounded passing-score recommendation. The resulting values are rounded up to the next-highest whole number and the rounded values are converted to scale scores.

All of the panelists indicated they were at least *somewhat comfortable* with the passing score they recommended; 22 of the 25 panelists were *very comfortable*. Twenty-four of the 25 panelists indicated the recommended passing score was *about right* with the remaining panelist indicating that the passing score was *too high*.

SUMMARY

To support the decision-making process for education agencies establishing a passing score (cut score) for the *Praxis* Mathematics test, research staff from ETS designed and conducted a multistate standard-setting study.

ETS provides a recommended passing score from the multistate standard-setting study to help education agencies determine an appropriate operational passing score. For the *Praxis* Mathematics test, the recommended passing score[9] is 39 out of a possible 60 raw-score points. The scale score associated with a raw score of 39 is 159 on a 100–200 scale.

[9] Results from the two panels participating in the study were averaged to produce the recommended passing score.

REFERENCES

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Appendix A

Panelists' Names & Affiliations

Participating Panelists With Affiliation

Panelist	**Affiliation**
Jodi Albers	Red Clay Consolidated School District (DE)
Holly Anthony	Tennesse Tech University (TN)
David Barnes II	Kansas State Department of Education (KS)
Stephen Bismarck	University of South Carolina Upstate (SC)
Sheila Blackmore	Bethany College (WV)
Tyesha Deas	A.C. Flora High School (SC)
Lacey Eckert	Sussex Technical High School (DE)
Ella Harris	Olathe Northwest High Sschool (KS)
Amanda Huffman	Pike High School (IN)
Paul Johanson	Brigham Young University - Idaho (ID)
Samantha Junkin	Auburn University at Montgomery (AL)
Melike Kara	Towson University (MD)
Ashley Kearney	Office of State Superintendent (DC)
Cindy Kroon	Montrose High School (SD)
Mary Martin	Middle Tennessee State University (TN)
Stephanie Marvel	Anne Arundel County Public Schools (MD)
Erin McCain	NW Arkansas Education Service Cooperative (AR)
Amanda Pendergrass	University of West Alabama (AL)
Adam Riazi	Cabell Midland High School (WV)
Amy Schander	Gayville-Volin High School (SD)
Thomas Schutt	DC Public Schools (DC)

(table continues)

Participating Panelists With Affiliation (continued)

Panelist	**Affiliation**
Sherra Shearer	Brandon High School (MS)
Douglas Speck	Southern Nevada Regional Professional Development Program (NV)
Rusty Young	Arkansas State University (AR)
Lauren Zarandona	Mississippi School for Math and Science (MS)

Appendix B

Study Agenda

AGENDA

Praxis® Mathematics (5165)
Standard-Setting Study

Day 1

Welcome and Introduction

Overview of Standard Setting and the *Praxis* Mathematics Test

Review the *Praxis* Mathematics Test

Discuss the *Praxis* Mathematics Test

Lunch

Define the Knowledge/Skills of a Just Qualified Candidate

Break

Define the Just Qualified Candidate *(continued)*

End of Day 1

AGENDA

Praxis® Mathematics (5165)
Standard-Setting Study

Day 2

Overview of Day 2

Define the Just Qualified Candidate *(continued)*

Standard-setting training presentation

Practice Round: Selected-response standard-setting judgments

Break

Practice Round: Data Discussion

Lunch Break

Round 1: Selected-response standard-setting judgments

Break

Round 1: Selected-response standard-setting judgments *(continued)*

End of Day 2

AGENDA

Praxis® Mathematics (5165) Standard-Setting Study

Day 3

Overview of Day 3

Round 1 Feedback and Round 2 Judgments

Break

Round 1 Feedback and Round 2 Judgments *(continued)*

Break

Feedback on Round 2 Recommended Cut Score

Complete Final Evaluation

End of Study

APPENDIX C

JUST QUALIFIED CANDIDATE DESCRIPTION

Description of the Just Qualified Candidate[10]

A just qualified candidate…

Tasks of teaching mathematics across mathematical content areas

- Knows how to identify and reason about common mathematical misconceptions in student work
- Is familiar with identifying instructional items and examples that address a mathematical learning objective

Numbers & Quantity

1. Knows the structure and the basic operations and properties of the real and complex number systems.
2. Understands and is fluent with operations involving rational numbers
3. Understands how to determine the reasonableness of solutions within the context of a given problem
4. Understands ratios and proportions, especially in the context of dimensional analysis and estimation.
5. Knows properties of rational exponents and radicals as applied to number sets.

Algebra

6. Understands how to solve equations and inequalities using a variety of techniques such as graphical, algebraic, and tabular and understands how to justify the reasoning processes used.
7. Knows how varied techniques (e.g. graphical, algebraic, tabular) are used to solve systems of equations and inequalities
8. Knows how to find real and imaginary roots of common polynomials
9. Understands how to find and interpret the real and imaginary roots of quadratics
10. Understands how to rewrite algebraic expressions for specific purposes (e.g. factored form to find zeros, vertex form to find maxima or minima, point slope to slope intercept)
11. Knows how to model real world scenarios with algebraic expressions, including average rate of change

Functions

12. Understands how new functions are obtained from existing functions (e.g., compositions, transformations, and inverses)
13. Understands and can identify key characteristics of functions (e.g., domain, range, end behavior, increasing/decreasing/constant)
14. Understands how function behavior is analyzed using non-algebraic representations (e.g., graphs, mapping, and tables)
15. Understands how to solve basic trigonometric, logarithmic, and exponential equations
    Knows how to use basic trigonometric, logarithmic, and exponential expressions for modeling contextual situations.

[10] Description of the just qualified candidate focuses on the knowledge/skills that differentiate a *just* from a *not quite* qualified candidate.

Description of the Just Qualified Candidate (continued)

A just qualified candidate…

Calculus

16. Knows how to find the limit of a function numerically, algebraically or graphically.
17. Knows the derivative as a slope of a tangent line and as a rate of change
18. Is familiar with continuity and differentiability of functions.
19. Knows how and when to use standard differentiation and integration concepts

Geometry

20. Understands how trigonometry is applied to right triangles
21. Understands angle measurement in terms of radians and degrees.
22. Understands means for proving geometric properties (e.g., lines, angles, polygons, and their operations) using geometric and algebraic methods
23. Knows means for visualizing and reasoning algebraically among common 2D and 3D figures

Probability & Stats

24. Understands how to interpret a linear regression model (e.g., rate of change, intercepts, and correlation coefficient) in the context of the data
25. Understands and compute the concepts of interdependence and conditional probability (such as simple events, probabilities of compound events, conditional probabilities) and how to apply those concepts to data
26. Understands how to summarize, represent, and interpret common representations of qualitative and quantitative data
27. Knows how to use basic statistics to make inferences and informed decisions.
28. Is familiar with counting techniques such as permutations and combinations.

APPENDIX D

RESULTS

Table D1

Panel Member Demographics (by Panel)

	Panel 1		Panel 2	
	N	**%**	***N***	**%**
Current position				
Teacher	6	46	6	50
College faculty	4	31	4	33
Mathematics Specialist	1	8	1	8
Other	2	15	1	8
Race				
White	12	92	11	92
Black or African American	1	8	1	8
Gender				
Female	10	77	8	67
Male	3	23	4	33
Are you currently certified to teach mathematics in your state?				
Yes	12	92	10	83
No	1	8	2	17
Are you currently teaching mathematics in your state?				
Yes	11	85	8	67
No	2	15	4	33
Are you currently supervising or mentoring other mathematics teachers?				
Yes	12	92	11	92
No	1	8	1	8
At what K–12 grade level are you currently teaching this subject?				
Elementary (K–5 or K–6)	0	0	1	8
Middle school (6–8 or 7–9)	1	8	0	0
Middle and High school	0	0	1	8
High school (9–12 or 10–12)	6	46	4	33
All Grades	1	8	0	0
Not currently teaching at the K–12 level	5	38	6	50
Including this year, how many years of experience do you have teaching mathematics?				
3 years or less	0	0	0	0
4–7 years	0	0	0	0
8–11 years	3	23	4	33
12–15 years	1	8	2	17
16 years or more	9	69	6	50

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Table D1 (continued)

Panel Member Demographics (by Panel)

	Panel 1		Panel 2	
	N	**%**	***N***	**%**
Which best describes the location of your K–12 school?				
Urban	2	15	1	8
Suburban	5	38	2	17
Rural	1	8	3	25
Not currently working at the K–12 level	5	38	6	50
If you are college faculty, are you currently involved in the training/preparation of teacher candidates in mathematics?				
Yes	4	31	4	33
No	0	0	0	0
Not college faculty	9	69	8	67

Table D2
Passing Score Summary by Round of Judgments

	Panel 1		Panel 2	
Panelist	**Round 1**	**Round 2**	**Round 1**	**Round 2**
1	33.90	33.80	42.45	42.20
2	38.70	37.25	35.80	37.55
3	35.50	35.35	39.15	40.90
4	41.40	39.30	44.75	43.50
5	43.40	42.75	44.80	42.30
6	40.50	38.50	40.35	40.35
7	33.55	34.25	33.05	33.95
8	30.00	33.80	38.15	38.70
9	28.60	30.40	36.45	38.05
10	30.10	32.30	32.90	34.10
11	40.70	40.30	40.65	41.55
12	42.75	41.65	51.45	46.10
13	45.20	43.70		
Average	37.25	37.18	40.00	39.94
Lowest	28.60	30.40	32.90	33.95
Highest	45.20	43.70	51.45	46.10
SD	5.62	4.22	5.36	3.64
SEJ	1.56	1.17	1.55	1.05

Table D3
Final Evaluation: Panel 1

	Strongly agree		Agree		Disagree		Strongly disagree	
	N	**%**	***N***	**%**	***N***	**%**	***N***	**%**
• I understood the purpose of this study.	12	92	1	8	0	0	0	0
• The instructions and explanations provided by the facilitators were clear.	12	92	1	8	0	0	0	0
• The training in the standard-setting method was adequate to give me the information I needed to complete my assignment.	11	85	2	15	0	0	0	0
• The explanation of how the recommended passing score is computed was clear.	11	85	2	15	0	0	0	0
• The opportunity for feedback and discussion between rounds was helpful.	12	92	1	8	0	0	0	0
• The process of making the standard-setting judgments was easy to follow.	10	77	3	23	0	0	0	0

Table D3 (continued)

Final Evaluation: Panel 1

How influential was each of the following factors in guiding your standard-setting judgments?	Very influential		Somewhat influential		Not influential			
	N	%	*N*	%	*N*	%		
• The description of the just qualified candidate	12	92	1	8	0	0		
• The between-round discussions	10	77	3	23	0	0		
• The knowledge/skills required to answer each test item	11	85	2	15	0	0		
• The passing scores of other panel members	7	54	4	31	2	15		
• My own professional experience	10	77	3	23	0	0		
	Very comfortable		**Somewhat comfortable**		**Somewhat uncomfortable**		**Very uncomfortable**	
	N	%	*N*	%	*N*	%	*N*	%
• Overall, how comfortable are you with the panel's recommended passing score?	11	85	2	15	0	0	0	0
	Too low		**About right**		**Too high**			
	N	%	*N*	%	*N*	%		
• Overall, the recommended passing score is:	0	0	12	92	1	8		

Table D4
Final Evaluation: Panel 2

	Strongly agree		Agree		Disagree		Strongly disagree	
	N	%	*N*	%	*N*	%	*N*	%
• I understood the purpose of this study.	12	100	0	0	0	0	0	0
• The instructions and explanations provided by the facilitators were clear.	12	100	0	0	0	0	0	0
• The training in the standard-setting method was adequate to give me the information I needed to complete my assignment.	12	100	0	0	0	0	0	0
• The explanation of how the recommended passing score is computed was clear.	12	100	0	0	0	0	0	0
• The opportunity for feedback and discussion between rounds was helpful.	10	83	2	17	0	0	0	0
• The process of making the standard-setting judgments was easy to follow.	12	100	0	0	0	0	0	0

Table D4 (continued)

Final Evaluation: Panel 2

How influential was each of the following factors in guiding your standard-setting judgments?	Very influential		Somewhat influential		Not influential	
	N	%	*N*	%	*N*	%
• The description of the just qualified candidate	10	83	2	17	0	0
• The between-round discussions	8	67	4	33	0	0
• The knowledge/skills required to answer each test item	11	92	1	8	0	0
• The passing scores of other panel members	5	42	6	50	1	8
• My own professional experience	8	67	4	33	0	0

	Very comfortable		Somewhat comfortable		Somewhat uncomfortable		Very uncomfortable	
	N	%	*N*	%	*N*	%	*N*	%
• Overall, how comfortable are you with the panel's recommended passing score?	11	92	1	8	0	0	0	0

	Too low		About right		Too high	
	N	%	*N*	%	*N*	%
• Overall, the recommended passing score is:	0	0	12	100	0	0

TEST REVIEW WORKSHOP REPORT

Praxis Mathematics (5165) Test Review
Virginia
Tuesday, May 4, 2021

OVERVIEW

This meeting was conducted using Zoom. The participants and agenda followed are provided in ATTENDEES and AGENDA below. RESULTS provides the panelists' feedback—captured through online polls—about the test and a recommended passing score.

The results include:

- All five panelists agreed that the test is appropriate for licensure as a secondary mathematics teacher, with the sixth indicating that the test assesses content beyond (i.e., above) what would be expected.
- The panel's recommended passing score—the median of their individual recommendations—is 159 on the Praxis scale (39 out of 60 points on the form reviewed). Two panelists were "very comfortable" and threer "somewhat comfortable" with that recommendation.

ATTENDEES

- Malik McKinley (ETS)
- Eric Steinhauer (ETS)
- Titus Teodorescu (ETS)
- Maggie Clemmons (VA DOE)

Panelists

1. Brendon Albon
2. Tina Mazzacane
3. Amy Siepka
4. Toni Sorrell
5. Susan Stanbery

AGENDA

Activity	*Approx. time*
Whole panel ▪ Welcome and Introductions ▪ Overview of the test and its development ▪ Overview of standard setting process **Agreement to proceed.**	30 mins
Breakout rooms ▪ Review of the test form (BREAK 5 mins) ▪ Discussion: What is measured?	1:50
Whole panel ▪ Discussion: What is measured? **Poll: Test review feedback.**	20 mins
Whole panel ▪ Review of standard setting results **Poll: Passing score recommendation.** ▪ Review of panel recommendation **Poll: Feedback on panel recommendation.**	20 mins

RESULTS

I. Following an overview of (i) the test's structure and content (ii) the test's development and (iii) the standard setting process, panelists responded to a poll indicating their agreement to proceed. All agreed to these two statements

(A) I understand that, by proceeding, I agree not to keep or disclose (1) secure test material provided and/or (2) any information provided specific to secure tests and/or (3) details of panel discussions of secure material, including panel recommendation.

(B) I understand that, by proceeding, I agree not to take this Praxis test any time within the next year.

II. Following a review of the test form used in standard setting, panelists discussed among themselves what they saw being measured including (1) what content they expect to be particularly challenging for candidates and (2) what content is especially important for beginning practice. Then panelists provided feedback about the test, starting with two questions to evaluate the test:

(C) How important are the knowledge and skills being assessed for effective beginning practice as a secondary mathematics teacher?

○ Very important	3
○ Important	2
○ Moderately important	–
○ Of some importance	–
○ Of little importance	–
Total	*5*

(D) Based on the knowledge and skills being assessed, is the test appropriate for licensure as a secondary mathematics teacher?

○ Yes	5
○ No	–
Total	*5*

Provided with an optional open-ended question

(E) About the knowledge and skills being assessed: Please provide any further feedback about alignment to the knowledge and skills a secondary mathematics teacher needs?

Three panelists provided further feedback:

- *Knowledge and skills are indicative of mathematics content an entry level teacher should have mastery of and experience with.*
- *Some questions were easy to answer by elimination and could be answered without a good grasp of the content. Increase misconceptions/pedagogy questions. Increase higher level stats and decrease basic probability questions and basic stats (i.e. median of a box plot).*
- *I think it's important that teachers think around the math, (ie planning and delivery) regardless of their degree. Knowing math is essential...being able to teach it is more essential.*

III. The panel was provided an overview of the process and results of a multistate standard setting (MSSS) conducted for the test. *(Note: In advance of the meeting, panelists received a technical report describing the MSSS process and results in detail.)*

Results included:

- The recommended score value (RSV) from the study was 39 (out of 60 possible points) on the test form you reviewed
- The mean recommendation of 38.56 was rounded up to a whole number.
- The scale score associated with 39 raw points is 159.

Panelists were asked for their recommended passing score based on their review of the test, the panel's discussion of the test and the results of the MSSS. Choices presented were the MSSS RSV and raw scores ranging two conditional standard errors of measurement (CSEM) above and below the RSV:

(F) What passing score would you recommend [raw number correct

○ 47	Scale 177 (+2.0 CSEM)	–
○ 45		–
○ 43	Scale 168 (+1.0 CSEM)	1
○ 41		–
○ 39	Scale 159 (Recommended Value from MSSS)	3
○ 38		–
○ 36	Scale 152 (-1.0 CSEM)	1
○ 34		–
○ 32	Scale 143 (-2.0 CSEM)	–
Total		*5*

Note: One panelist had to leave the meeting immediately after providing a judgment regarding the recommended study value.

The panel was shown these results for a brief discussion and then asked for feedback on the panel's recommended passing score, identified as the median of the panelists' recommendations:

The panel's recommended passing score is 39, equivalent to a scaled score of 159.

Panelists provided feedback about this recommended passing score in answers to two questions:

(G) Overall, how comfortable are you with the panel's recommended cut score?

○ Very Comfortable	2
○ Somewhat Comfortable	3
○ Somewhat Uncomfortable	–
○ Very Uncomfortable	–
Total	*5*

(H) Overall, the panel's recommended cut score is:

○ Too high	1
○ About Right	4
○ Too Low	–
Total	*5*

Provided with an optional open-ended question

(I) Please provide any further comments about your recommendation

Three panelists provided further feedback:

— *Since Virginia was not involved in the standard setting process, it is difficult to ascertain the appropriate cut score without further review.*

- *I think the score is a little low but I also know that secondary math teachers are hard to come by, so overall, the cut score will allow just qualified teachers to get certified, who can grow as they gain experience in the profession.*
- *The time it takes to read and interpret the question is on average longer than expected. The questions that focus on mathematics pedagogy are important but challenging for pre-service teachers.*

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Mathematics (5165)

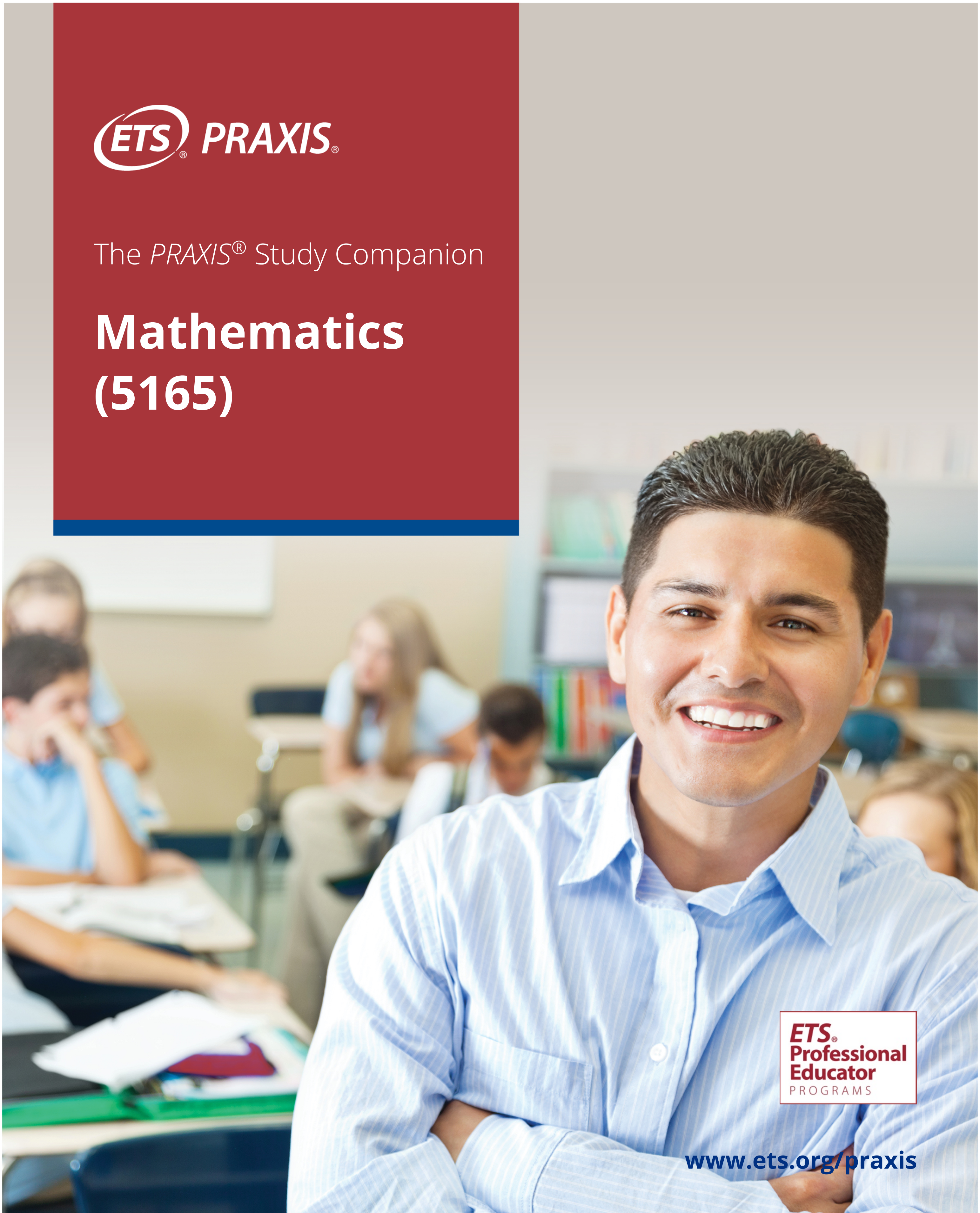

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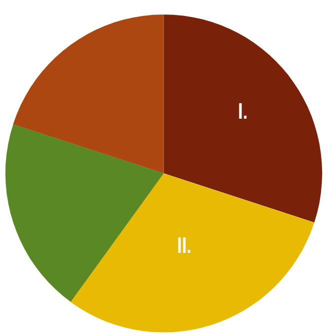
Table of Contents

Mathematics (5165)

Test at a Glance

The *Praxis*® Mathematics test is designed to measure knowledge and competencies important for safe and effective beginning practice as a secondary school mathematics teacher. Test-takers have typically completed a bachelor's degree program with appropriate coursework in mathematics and education.

Test Name	Mathematics
Test Code	5165
Time	180 minutes
Number of Questions	66 selected-response questions
Format	The test consists of a variety of selected-response questions, where you select one or more answer choices; questions where you enter a numeric answer in a box; and other types of questions. You can review the possible question types in Understanding Question Types.
Test Delivery	Computer Delivered



Content Categories	Approximate Number of Questions	Approximate Percentage of Examination
I. Number & Quantity and Algebra	20	30%
IA. Number & Quantity	7	10%
IB. Algebra	13	20%
II. Functions and Calculus	20	30%
IIA. Functions	13	20%
IIB. Calculus	7	10%
III. Geometry	13	20%
IV. Statistics & Probability	13	20%

All questions assess content from the Mathematics domains above. Approximately 25% of questions assess content applied to a Task of Teaching Mathematics.

About the Test

The Mathematics test content topics span the secondary mathematics curriculum including content related to (I) Number & Quantity and Algebra, (II) Functions and Calculus, (III) Geometry, and (IV) Statistics & Probability. A full list of the mathematics topics covered is provided in Content Topics.

Test takers will find that approximately 25 percent of the questions call for application of mathematics within a teaching scenario or an instructional task. Such questions—designed to measure applications of mathematics knowledge and skills to the kinds of decisions and evaluations a teacher must make during work with students, curriculum, and instruction—situate mathematics content questions in tasks that are critical for teaching. A full list of the teaching tasks covered, which have been identified based on research on mathematics instructions and are a routine part of mathematics instruction, is provided in Tasks of Teaching Mathematics.

Test takers have access to an on-screen graphing calculator. A list of notations, definitions, and formulas is available on the test's Help screen and is also provided in the Practice with Sample Test Questions section.

The assessment is designed and developed through work with practicing teachers and teacher educators to reflect the mathematics curriculum as well as state and national standards for mathematics, including the National Governors Association Center for Best Practices and the Council of Chief State School Officers *Common Core State Standards for Mathematics* (2010), the National Council of Teachers of Mathematics (NCTM) and the Council of the Accreditation of Educator Preparation (CAE) *NCTM CAEP Standards* (2012), and the NCTM *Principles and Standards for School Mathematics* (2000).

This test may contain some questions that will not count toward your score.

On-Screen Graphing Calculator

An on-screen graphing calculator is provided for the computer-delivered test. Please consult the Praxis Calculator Use web page (http://www.ets.org/praxis/test_day/policies/calculators/) for further information and for a link to download the calculator and view tutorials on using the calculator.

You are expected to know how and when to use the calculator since it will be helpful for some questions. The calculator is available as a free download for a 30-day trial period. You are expected to become familiar with its functionality before taking the test. The calculator may be used to perform calculations (e.g., division, exponents, roots, trigonometric values, logarithms, finding the mean of a data set), to graph and analyze functions, to find numerical solutions to equations, and to generate a table of values for a function.

Using Your Calculator

Take time to download the trial version of the calculator. View the tutorials on the website. Practice with the calculator so that you are comfortable using it on the test.

There are only some questions on the test for which a calculator is helpful or necessary. First, decide how you will solve a problem, then determine if you need a calculator. For many questions, there is more than one way to solve the problem. Don't use the calculator if you don't need to; you may waste time. Sometimes answer choices are rounded, so the answer that you get might not match the answer choices in the question. Since the answer choices are rounded, substituting the choices into the question might not produce an exact answer.

Don't round any intermediate calculations. For example, if the calculator produces a result for the first step of a solution, keep the result in the calculator and use it for the second step. If you round the result from the first step and the answer choices are close to each other, you might choose the incorrect answer.

Read the question carefully so that you know what you are being asked to do. Sometimes a result from the calculator is NOT the final answer. If an answer you get is not one of the choices in the question, it may be that you didn't answer the question being asked. Read the question again. It might also be that you rounded at an intermediate step in solving the problem.

Think about how you are going to solve the question before using the calculator. You may only need the calculator in the final step or two. Don't use it more than necessary.

Check the calculator modes (degree versus radian, floating decimal versus scientific notation) to see that these are correct for the question being asked. Make sure that you know how to perform the basic arithmetic operations and calculations (e.g., division, exponents, roots, trigonometric values, logarithms, finding the mean of a data set). Your test may involve questions that require you to do some of the following: graph functions and analyze the graphs, find zeros of functions, find points of intersection of graphs of functions, find minima/maxima of functions, find numerical solutions to equations, and generate a table of values for a function.

Content Topics

This list details the topics that may be included on the test. All test questions will cover one or more of these topics.

Discussion Questions

In this section, discussion questions are open-ended questions or statements intended to help test your knowledge of fundamental concepts and your ability to apply those concepts to classroom or real-world situations. **We do not provide answers for the discussion questions** but thinking about the answers will help improve your understanding of fundamental concepts and may help you answer a broad range of questions on the test. Most of the questions require you to combine several pieces of knowledge to formulate an integrated understanding and response. They are written to help you gain increased understanding and facility with the test's subject matter. You may want to discuss these questions and possible areas with a teacher or mentor.

I. Number & Quantity and Algebra

A. Number and Quantity

1. Understands the structure of the real number system and how the basic operations on numbers in this system are performed
   a. Represents and solves word problems involving addition, subtraction, multiplication, and division of real numbers
   b. Given operations on a number system, determines whether commutative, associative, and distributive properties hold
   c. Identifies whether the sum or product of rational and/or irrational numbers must be rational, must be irrational, or can be rational or irrational (e.g., the sum of two rational numbers must be rational, the product of two irrational numbers can be rational or irrational)
   d. Solves problems involving number theory properties (e.g., prime, composite, prime factorization, even, odd, factors, multiples)
   e. Uses proportional relationships to solve ratio, constant rate, and percent problems
2. Understands the properties of radicals and rational exponents
   a. Performs operations involving rational exponents
   b. Uses the properties of exponents to rewrite expressions that have radicals or expressions that have rational exponents
   c. Uses scientific notation to represent and compare numbers and to perform calculations
3. Understands how to reason quantitatively and use units to solve problems
   a. Chooses and interprets units consistently in formulas

b. Chooses and interprets the scale and the origin in graphs and data displays
c. Solves measurement, estimation, and conversion problems involving time, length, temperature, volume, and mass in standard measurement systems
d. Solves problems involving dimensional analysis (e.g., feet per second to miles per hour, feet per second to kilometers per hour)

4. Knows the structure of the complex number system and how basic operations with complex numbers are performed
   a. Performs operations with complex numbers, including conjugates
   b. Applies the commutative, associative, and distributive properties to complex numbers

B. Algebra

1. Understands how to write algebraic expressions in equivalent forms
   a. Uses the structure of a polynomial or exponential expression to identify ways to rewrite it in an equivalent form (e.g., differences of squares, factoring, changing bases)
   b. Understands how to rewrite algebraic expressions for specific purposes (e.g., factored form to find zeros, vertex form to find maxima or minima)
   c. Rearranges formulas to solve for a specified variable
   d. Adds, subtracts, multiplies, and divides polynomials
   e. Factors special polynomials over the complex numbers (e.g., $x^2 + y^2 = (x + yi)(x - yi)$)
2. Understands how to create equations and inequalities that describe relationships
   a. Creates equations and inequalities in one variable, uses them to solve problems, and graphs solutions on the number line
   b. Creates equations and inequalities in two or more variables, uses them to solve problems, and graphs the equations in two variables on the coordinate plane with appropriate labels and scales
   c. In a modeling context, represents constraints by systems of equations and/or inequalities and interprets solutions as viable or nonviable options
3. Understands how varied techniques (e.g., graphical, algebraic, tabular) are used to solve equations and inequalities

a. Solves linear equations and inequalities in one variable, including equations with variable coefficients

b. Solves quadratic equations with real coefficients that have complex solutions

c. Uses the method of completing the square to transform any quadratic equation in x into the equivalent form $(x-p)^2 = q$

d. Solves equations using a variety of methods (e.g., graphing, factoring, using the quadratic formula)

e. Uses different methods (e.g., discriminant analysis, graphical analysis) to determine the nature of the solutions of a quadratic equation

f. Graphs the solutions to a linear equation or inequality in two variables

g. Justifies each step in solving an equation or inequality

4. Understands how varied techniques (e.g., graphical, algebraic, tabular) are used to solve systems of equations and inequalities

   a. Solves a system consisting of two linear equations in two variables algebraically and graphically

   b. Solves a system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically

   c. Finds the solutions of $f(x) = g(x)$ approximately (e.g., uses technology to graph the functions, makes tables of values); includes cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, radical, or logarithmic functions

   d. Graphs the solution set to a system of linear inequalities in two variables

5. Understands the concept of rate of change of nonlinear functions

   a. Calculates and interprets the average rate of change of a function presented as a table of values, algebraically, or graphically over a given interval

6. Recognizes and is able to extract and interpret information about a linear equation when it is presented in various forms (e.g., slope-intercept, point-slope, standard)

   a. Calculates the intercepts of a line and interprets them in a modeling context

   b. Calculates the slope of a line presented as a table of values, algebraically, or graphically, and interprets it in a modeling context

7. Understands the relationship between zeros of polynomial functions (including their graphical representation) and factors of the related polynomial expressions
   a. Applies the remainder theorem to find factors of polynomials
   b. Uses factorization to identify zeros of polynomials
   c. Uses zeros and factorization of a polynomial to sketch a graph of the polynomial and uses the graph to determine the zeros and the factorization of the polynomial
   d. Uses a variety of techniques to find and analyze the zero or zeros (real and complex) of polynomial functions
8. Understands how to rewrite rational expressions
   a. Rewrites simple rational expressions in an equivalent form
   b. Adds, subtracts, multiplies, and divides rational expressions
9. Understands how to justify the reasoning process used to solve equations, accounting for potential extraneous solutions
   a. Solves simple rational and radical equations in one variable, accounting for potential extraneous solutions

Discussion areas: Number and Quantity

- Can you perform arithmetic operations on real numbers?
- Can you apply the order of operations in arithmetic computations?
- Can you solve word problems involving operations on real numbers?
- Can you identify the result of arithmetic operations on rational and irrational numbers as either rational or irrational?
- Can you compute or identify a ratio or rate?
- Can you solve problems involving primes, composites, factors, and multiples?
- Can you use proportional relationships to compute percent’s?
- Can you use the properties of exponents to simplify and rearrange expressions?
- Can you simplify expressions that contain radicals or rational exponents?
- Can you define and use negative exponents?
- Can you verify that radical expressions are equivalent numerically and analytically?
- Can you identify and represent very small and very large numbers in scientific notation?
- Can you do calculations involving scientific notation?
- Can you convert between units—for example, converting inches to meters?
- Can you solve problems using units to guide the solution?
- Can you solve measurement problems involving time, length, temperature, volume, and mass?

- Can you solve problems involving dimensional analysis—for example, converting feet per second to miles per hour, feet per second to kilometers per hour?
- Can you perform arithmetic operations on complex numbers?
- Can you recognize and use $a - bi$ as the conjugate of $a + bi$?
- Can you identify the associative, commutative, inverse, identity, and distributive properties from a given algebraic statement?

Discussion areas: Algebra

- Can you rewrite quadratic expressions to find zeros, complete the square, and find the relative extrema?
- Can you solve for the variable of interest in a formula?
- Can you add, subtract, divide, and multiply polynomials?
- Can you use linear equations or linear inequalities to model real-life problems?
- Can you solve linear equations and linear inequalities in one variable algebraically?
- Can you graph the solution of a linear inequality in one variable on the number line and the solution of a linear inequality in two variables on the coordinate plane?
- Can you extract and interpret information about a linear equation presented in slope-intercept form, point-slope form, or standard form?
- Can you solve quadratic equations with real solutions and complex solutions?
- Can you solve quadratic equations by completing the square, factoring, and using the quadratic formula?
- Can you use the discriminant to identify the types and multiplicities of roots of a quadratic equation?
- Can you solve a system consisting of two linear equations in two variables algebraically?
- Can you solve a system consisting of two linear equations in two variables by graphing?
- Can you solve a system consisting of two linear inequalities in two variables algebraically?
- Can you represent constraints by systems of equations or inequalities in a modeling context, and interpret the solutions as viable or nonviable?
- Can you solve a system consisting of a linear equation and a quadratic equation in two variables algebraically?
- Can you solve a system consisting of a linear equation and a quadratic equation in two variables by graphing?
- Can you find the intersection(s) of two curves algebraically or using technology?
- Can you calculate the average rate of change for functions?
- Can you calculate and interpret the intercepts and slope of a line?
- Can you use the remainder theorem and factor theorem for polynomials?
- Can you use the graph of a quadratic function to identify the types and multiplicities of the zeros of the function?
- Can you find and use zeros to sketch a graph of the function?
- Can you add, subtract, multiply, and divide rational expressions?
- Can you identify when extraneous solutions may occur in solving rational and radical equations?

II. Functions and Calculus

A. Functions

1. Understands functions and function notation
   a. Determines whether a relation is a function
   b. Evaluates functions and interprets statements that use function notation in terms of a context
   c. Determines the domain and range of a function from a function rule (e.g., $f(x)=2x+1$) graph, set of ordered pairs, or table
2. Understands how function behavior is analyzed using different representations (e.g., graphs, mappings, tables)
   a. For a function that models a relationship between two quantities, interprets key features of graphs and tables (e.g., increasing/ decreasing, maximum/ minimum, discontinuities, end-behavior) in terms of the quantities
   b. Given a verbal description of a function, sketches graphs that show key features of that function
   c. Graphs functions (e.g., linear, quadratic, exponential, piecewise, absolute value, step, radical, polynomial, rational, logarithmic, trigonometric) defined by an expression and identifies key features of the graph
   d. Writes a function that is defined by an expression in different but equivalent forms to reveal different properties of the function (e.g., zeros, extreme values, symmetry of the graph)
   e. Interprets the behavior of exponential functions (e.g., growth, decay)
   f. Determines whether a function is odd, even, or neither and whether the graph of the function has any symmetries
   g. Compares properties of two functions each represented in a different way (e.g., as a table of values, algebraically, graphically, or by verbal descriptions)
   h. Recognizes and is able to extract information about a quadratic function when it is presented in various forms (i.e., standard, vertex, factored)
   i. Converts among various forms of quadratic equations (i.e., standard, vertex, factored) using methods such as factoring and completing the square
3. Understands how functions and relations are used to model relationships between quantities

a. Writes a function that relates two quantities

b. Determines an explicit expression or a recursive process that builds a function from a context

c. Writes arithmetic and geometric sequences both recursively and with an explicit formula and uses them to model situations

d. Translates between recursive and explicit forms of arithmetic and geometric sequences

4. Understands how new functions are obtained from existing functions (e.g., compositions, transformations, inverses)

a. Describes how the graph of $g(x)$ is related to the graph of $f(x)$, where $g(x)=f(x)+k$, $g(x)=f(x+k)$, $g(x)=k\,f(x)$, or $g(x)=f(kx)$ for specific values of k (both positive and negative) and finds the value of k given the graphs

b. Given that a function f has an inverse, finds values of the inverse function from a graph or a table of f

c. Interprets the meaning of an inverse function in a modeling context

d. Given a noninvertible function, determines the largest possible domain of the function that produces an invertible function

e. Given a relation, finds its inverse and determines if its inverse is a function and writes an expression for the inverse function

f. Uses the inverse relationship between exponential and logarithmic functions to solve problems

g. Combines standard function types

h. Analyzes the domain of functions created by combining functions using arithmetic operations

i. Composes functions presented as tables of values, algebraically, or graphically

j. Analyzes the domain of functions resulting from composition

k. Uses composition to express the relationship between a function and its inverse

5. Understands differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems

a. Understands that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals
b. Identifies situations in which one quantity changes at a constant rate per unit interval relative to another
c. Identifies situations in which a quantity using arithmetic operations grows or decays by a constant percent rate per unit interval relative to another
d. Constructs linear and exponential functions, including arithmetic and geometric sequences, given a graph, table of values, a set of ordered pairs, or a description of a relationship
e. Observes that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratic ally, or (more generally) as a polynomial function
f. Interprets the parameters in a linear or exponential function in terms of a context (e.g., $A(t) = Pe^{rt}$)

6. Understands how to use logarithms to solve problems
    a. Applies the properties of logarithms to solve problems
    b. Expresses the solution to an exponential equation with base b as a logarithm (e.g., $3\,(2^{5t}) = 20$, $3\,(e^{5t}) = 20$)
    c. Uses technology to evaluate logarithms that have any base

7. Understands the relationship between points on the unit circle and the values of trigonometric functions for any given angle measure
    a. Converts between degree measure and radian measure
    b. Identifies the reference angle for a given angle and the relationship between the trigonometric values of an angle and its reference angle
    c. Finds the values of trigonometric functions of any angle
    d. Uses the unit circle to explain symmetry and periodicity of trigonometric functions

8. Understands how periodic phenomena are modeled using trigonometric functions
    a. Chooses trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline

b. Uses inverse functions to solve trigonometric equations that arise in modeling contexts and interprets them in terms of the context

9. Understands how to solve trigonometric, logarithmic, and exponential equations
   a. Solves trigonometric, logarithmic, and exponential equations

B. Calculus

1. Understands the meaning of a limit of a function and how to calculate limits of functions and conditions when the limit does not exist
   a. Solves limit problems using properties of limits, where all limits of the individual functions exist at the value that x is approaching
   b. Determines limits to a fixed value and interprets the 'limit graphically
   c. Determines one-sided limits to a fixed value, from both left and right, interprets the limit graphically, and uses it to determine if the limit to the value exists
   d. Computes limits to infinity or negative infinity and interprets the result graphically
   e. Identifies limits that do not exist for functions presented algebraically or graphically

2. Understands the derivative of a function as a limit, as the slope of a line tangent to a curve, and as a rate of change
   a. Given the graph of a function and a point on that graph, knows the relationship between the derivative of the function at that point, the slope of the tangent to the graph at that point, and the succession of slopes of secant lines connecting $(a, f(a))$ to $(x, f(x))$ as x approaches a

3. Understands what it means for a function to be continuous at a given point and knows the relationship between continuity and differentiability
   a. Applies the three steps (i.e., $f(a)$ exists, $\lim_{x\to a} f(x)$ exists, and $f(a)=\lim_{x\to a} f(x)$) that are part of the definition of what it means for a function to be continuous at $x=a$ to verify whether a given function is continuous at a given point
   b. Gives examples of functions that are continuous at $x=a$ but not differentiable at $x=a$ and explains why

4. Understands how and when to use standard differentiation and integration techniques
   a. Uses standard differentiation techniques

   b. Uses standard integration techniques, including both definite and indefinite integrals
   c. Uses the relationship between the position, velocity, and acceleration functions to solve problems

5. Understands how to analyze the behavior of a function (e.g., extrema, concavity, symmetry) and understands how to use integration to compute area
   a. Uses the first and second derivatives to analyze the graph of a function
   b. Matches graphs of functions with graphs of their derivatives or accumulations based on the second part of the fundamental theorem of calculus
   c. Uses integration techniques to compute area

Discussion areas: Functions

- Can you recognize function notation and understand that for each input, the function produces one and only one output?
- Can you determine whether a relation is a function numerically, algebraically, as a set of ordered pairs, and graphically?
- Can you recognize the domain as the set of valid inputs for a function and the range as the set of resulting outputs, and can you find these for a given function?
- Can you evaluate a function that is given algebraically or graphically?
- Can you find the zeros, extreme values, intervals of increasing or decreasing, and symmetry of a function given a graph, algebraic representation, or verbal description?
- Can you graph radical, piecewise, absolute value, polynomial, rational, logarithmic, exponential, and trigonometric functions?
- Can you determine whether an exponential function will grow or decay and at what rate?
- Can you determine whether a function is odd, even, or neither and whether the graph of the function has any symmetries?
- Can you create a function that models a relationship between two described quantities?
- Can you recognize and define sequences as recursive or explicit functions?
- Can you take one or more functions and create another function using functional operations, function composition, and transformations?
- Can you identify the domain and range of the sum, product, difference, quotient, or composition of two functions?
- Can you find the inverse of a given function?
- Can you determine whether two functions are inverses graphically and analytically?
- Can you determine if two functions, given as sets of ordered pairs, are inverse functions of each other?
- Can you interpret the meaning of an inverse function in a modeling context?
- Can you determine the type of function (linear, quadratic, exponential) that best fits a given scenario or situation?

- Can you use logarithms to solve problems with and without technology?
- Can you express exponential equations as logarithms and evaluate logarithms with any base using technology?
- Can you use the unit circle and special right triangles to determine the values of given trigonometric functions?
- Can you find the values of trigonometric functions for an angle given the value of one trigonometric function and the quadrant of the angle?
- Can you model situations that occur periodically using trigonometric functions?
- Can you recognize and use trigonometric identities?
- Can you solve exponential, logarithmic and trigonometric equations?

Discussion areas: Calculus

- Can you compute and analyze the limit of a function at a given point?
- Can you recognize the difference between the limit of a function at a point and the value of the function at the point?
- Can you compute limits to infinity or negative infinity and interpret the result graphically?
- Can you identify limits that do not exist for functions presented algebraically or graphically?
- Can you use the limit definition of the derivative to differentiate a function?
- Can you approximate derivatives given a table by using the slope of a secant line?
- Can you determine the continuity and differentiability of a function at a given point?
- Can you find the derivative of a function using the differentiation rules?
- Can you compute a definite integral using the fundamental theorem of calculus?
- Can you use differentiation and integration techniques to identify the relationship between position, velocity, and acceleration functions?
- Can you calculate the extrema of a function and determine when the function is increasing or decreasing using the first derivative?
- Can you find the concavity of a curve at a particular point and find any point(s) of inflection?
- Can you find the area between curves?

III. Geometry

A. Geometry

1. Knows the properties of lines (e.g., parallel, perpendicular, intersecting) and angles
    a. Solves problems involving parallel, perpendicular, and intersecting lines
    b. Applies angle relationships (e.g., supplementary, vertical, alternate interior) to solve problems
2. Knows the properties of triangles, quadrilaterals (e.g., parallelogram, rectangle, rhombus), and other polygons

a. Determines whether given side lengths or angle measures would produce a triangle (e.g., triangle inequality theorem) and classifies triangles by their sides or angles
b. Solves problems involving special triangles (e.g., isosceles, equilateral, right, $30° – 60° – 90°$)
c. Uses the definitions of median, midpoint, and altitude to solve problems involving triangles
d. Identifies geometric properties of various quadrilaterals and the relationships among them (e.g., parallelogram, rectangle, rhombus)
e. Solves problems involving sides, angles, or diagonals of polygons

3. Understands transformations in the plane
   a. Uses rigid motions (e.g., translations, rotations, reflections) to transform figures
   b. Uses dilations to transform figures
   c. Applies properties of rigid motions (e.g., rigid motions preserve distance and angle measure)
   d. Applies properties of dilation transformations (e.g., dilation transformations preserve angle measure but not distance)
   e. Identifies a sequence of transformations that maps a preimage onto an image
   f. Given a figure, describes the transformations that map the figure onto itself, including reflection over a line of symmetry
   g. Represents translations using vector notation

4. Understands congruence and similarity
   a. Determines whether two figures are congruent using triangle congruence theorems (e.g., ASA, SAS, SSS)
   b. Determines whether two figures are similar using triangle similarity theorems (e.g., AA criterion)
   c. Determines whether two figures are congruent by directly mapping one figure onto another using a sequence of one or more rigid motions
   d. Determines whether two figures are similar by directly mapping one figure onto another using a sequence of one or more transformations (dilations and/or rigid motions)

   e. Uses congruence and similarity to solve problems involving unknown side lengths or angle measurements in two-dimensional and three-dimensional figures

5. Knows how to prove geometric theorems such as those about lines, angles, triangles, and parallelograms
   a. Solves problems involving proofs of theorems about lines and angles (e.g., vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are equidistant from the segment's endpoints)
   b. Solves problems involving proofs of theorems about triangles (e.g., measures of interior angles of a triangle sum to $180°$; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point; a line parallel to one side of a triangle divides the other two sides proportionally; the Pythagorean theorem proved using triangle similarity)
   c. Solves problems involving proofs of theorems about parallelograms (e.g., opposite sides are congruent; opposite angles are congruent; the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals)
   d. Identifies whether geometric proofs are valid (e.g., direct proofs, counterexamples)

6. Understands how trigonometry is applied to triangles
   a. Uses the relationship between the sine and cosine of complementary angles to solve problems
   b. Uses trigonometric ratios and the Pythagorean theorem to solve for side lengths and angle measures of right triangles in geometric or applied problems
   c. Uses the values of trigonometric functions of special angles (e.g., $30°, 45°, 60°, 90°$) to solve problems
   d. Applies the Law of Sines and the Law of Cosines to find unknown measurements in triangles

7. Understands how to apply theorems about circles
   a. Solves problems involving circumference and area of a circle
   b. Solves problems involving lengths of arcs and areas of sectors
   c. Solves problems involving measures of inscribed angles, central angles, circumscribed angles, and arcs
   d. Uses properties of lines in a circle to solve problems (e.g., chords, secants, tangents, radii, diameters)
   e. Identifies and uses the geometric description of a circle as the set of points for which the distance from a point to a fixed point (the center) is constant
   f. Determines the equation of a circle given the center and radius of the circle
   g. Finds the center and radius of a circle given by an equation of the circle in any form
8. Understands how to use coordinate geometry to describe properties of geometric objects
   a. Uses coordinate geometry to represent and identify the properties of geometric shapes and to solve problems (e.g., Pythagorean theorem, perimeter of a polygon, area of a rectangle)
   b. Determines the distance between two points
   c. Finds the point on a directed line segment between two given points that partitions the segment in a given ratio
   d. Uses the slope criteria for parallel and perpendicular lines to solve geometric problems
9. Knows how to solve problems involving perimeter and area of polygons
   a. Calculates and interprets perimeter and area of polygons that can be composed of triangles and quadrilaterals
   b. Calculates changes in perimeter and area as the dimensions of a polygon change
10. Knows how to solve problems involving solids
    a. Calculates and interprets surface area and volume of solids (e.g., prisms, pyramids, cones, cylinders, spheres), including in real-world situations
    b. Calculates changes in surface area and volume as the dimensions of a solid change

c. Identifies the shapes of two-dimensional cross sections of three-dimensional objects and identifies three-dimensional objects generated by rotations of two-dimensional objects

d. Uses two-dimensional representations (e.g., nets) of three-dimensional objects to visualize and solve problems

Discussion areas: Geometry

- Can you use the definitions of angles, circles, line segments, perpendicular lines, and parallel lines?
- Can you use the relationships of the angles formed when parallel lines are cut by a transversal?
- Can you use the definitions and properties of special triangles (e.g., isosceles, equilateral, right, $30° - 60° - 90°$) and special polygons (e.g., regular polygon, parallelogram, trapezoid, rhombus, rectangle, square, and kite)?
- Can you prove and apply the theorems of supplementary angles, complementary angles, vertical angles, exterior angles, triangle sum, and base angles?
- Can you prove and apply theorems about angles, triangles, and parallelograms?
- Can you describe and use the properties of the median, altitude, and angle bisector of a triangle?
- Can you translate, reflect, rotate, and dilate figures in the plane and describe these transformations geometrically?
- Can you identify or describe a sequence of transformations that map a preimage onto an image?
- Can you identify the lines of symmetry of a polygon?
- Can you describe or prove congruence and similarity in terms of transformations?
- Can you apply triangle congruence and similarity criteria to solve problems or prove theorems?
- Can you recognize logical fallacies such as assuming the equivalence of a proposition and its converse?
- Can you describe some real-life applications that involve the Pythagorean Theorem and trigonometric ratios?
- Can you use the law of sines and the law of cosines to solve problems?
- Can you use trigonometric ratios to solve problems involving right triangles?
- Can you use the relationship between sine and cosine of complementary angles to solve problems?
- Can you use the definitions, properties, and theorems about circles, such as inscribed and central angles, radii, diameters, chords, arcs, tangents, secants, circumference, and area?
- Can you derive and use the formula for the arc length and the sector area of a circle?
- Can you use definitions and properties of the coordinate plane (e.g., quadrants)?
- Can you derive the equation of a circle given its graph in the coordinate plane?
- Can you find the center and radius of a circle from a given equation?
- Can you compute the perimeter of a polygon and the area of a triangle and a quadrilateral using coordinates?

- Can you solve perimeter and area problems involving plane figures either directly or by decomposing into familiar shapes?
- Can you calculate the changes in perimeter and area when the dimensions of a polygon are changed?
- Can you use coordinates to compute the length or the midpoint of a line segment?
- Can you use coordinates to find the point on a line segment that partitions the segment in a given ratio?
- Can you use coordinates to compute the slope of a line, given the end points?
- Can you use the slope criteria for parallel and perpendicular lines graphed in the coordinate plane?
- Can you apply the correct formula to compute the surface area and volume of prisms, cylinders, pyramids, cones, and spheres?
- Can you identify 2-dimensional cross sections of 3-dimensional shapes?
- Can you use 2-dimensional (nets) to represent 3-dimensional objects?
- Can you calculate the changes in surface area and volume when the dimensions of a solid are changed?

IV. Statistics & Probability

A. Statistics & Probability

1. Understands how to make inferences and justify conclusions from samples, experiments, and observational studies
   a. Uses statistics to make inferences about population parameters based on a sample from that population
   b. Identifies the purposes of and differences among sample surveys, experiments, and observational studies and explains how randomization relates to each
   c. Uses data from a sample survey to estimate a population mean or proportion
   d. Uses data from a randomized experiment to compare two treatments
2. Understands how to summarize, represent, and interpret data collected from measurements on a single variable (e.g., boxplots, dot plots, normal distributions)
   a. Represents and interprets data with plots on the real number line (e.g., dot plots, histograms, boxplots)
   b. Computes the center (e.g., median, mean) and spread (e.g., interquartile range, standard deviation) for a data set
   c. Uses statistics appropriate to the shape of the data distribution to compare center (e.g., median, mean) and spread (e.g., interquartile range, standard deviation) of two or more different data sets

   d. Interprets differences in shape, center, and spread in the context of the data sets, accounting for possible effects of outliers

3. Understands how to summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative (e.g., scatterplots, time series)
   a. Summarizes and interprets categorical data for two categories in two-way frequency tables (e.g., joint, marginal, conditional relative frequencies)
   b. Identifies possible associations and trends in the data
   c. Represents and interprets data for two quantitative variables on a scatterplot and describes how the variables are related

4. Understands how to create and interpret linear regression models (e.g., rate of change, intercepts, correlation coefficient)
   a. Uses technology to fit a function to data (i.e., linear regression) and determines a linear correlation coefficient
   b. Uses functions fitted to data to solve problems in the context of the data
   c. Assesses the fit of a function by plotting and analyzing residuals
   d. Interprets the slope and the intercept of a regression line in the context of the data
   e. Interprets a linear correlation coefficient
   f. Distinguishes between correlation and causation

5. Understands the concept of independence and understands how to compute probabilities of simple events, probabilities of compound events, and conditional probabilities
   a. Describes events as subsets of a sample space using characteristics of the outcomes or as unions, intersections, or complements of other events
   b. Determines and interprets when two events are independent
   c. Identifies and applies the concepts of conditional probability and independence
   d. Calculates probabilities of simple and compound events
   e. Constructs and interprets two-way frequency tables of data when two categories are associated with each object being classified; uses the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities

f. Applies the addition rule,
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
and interprets it in terms of a given model

g. Applies the general multiplication rule in a uniform probability model,
$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$
and interprets it in terms of a given model

6. Understands how to find probabilities involving finite sample spaces and independent trials
   a. Uses the fundamental counting principle to find probabilities involving finite sample spaces and independent trials
   b. Uses counting techniques (e.g., permutations, combinations) to solve problems
7. Knows how to make informed decisions using probabilities and expected values
   a. Interprets a probability distribution for a random variable, defined for a sample space in which theoretical probabilities can be calculated, and finds the expected value
   b. Interprets a probability distribution for a random variable, defined for a sample space in which probabilities are assigned empirically, and finds the expected value
   c. Weighs the possible outcomes of a decision by assigning probabilities to outcomes and finding expected values
8. Understands normal distributions
   a. Identifies whether data sets are normally distributed based on their shape
   b. Uses the mean and standard deviation of a normal distribution to interpret population percentages
   c. Estimates and interprets areas under the normal curve

Discussion areas: Probability and Statistics

- Can you recognize the purposes of and difference among samples, experiments, and observational studies?
- Can you create graphs such as histograms, line graphs, bar graphs, dot plots, circle graphs, scatterplots, stem-and-leaf plots, and boxplots from a given set of data?
- Can you understand and interpret simple diagrams of data sets presented in various forms, including tables, charts, histograms, line graphs, bar graphs, dot plots, circle graphs, scatterplots, stem-and-leaf plots, timelines, number lines, and boxplots?
- Can you determine measures of center (mean, median) and spread (interquartile range, standard deviation) for single-variable data presented in a variety of formats?
- Can you determine the differences between mean, median, and mode, including advantages and disadvantages of each?

- Can you identify possible effects of outliers on the shape, center, and spread of data sets?
- Can you calculate the interquartile range and standard deviation of a data set and use these values to compare the spread of two or more data sets?
- Can you analyze data presented in scatterplots and use this to predict associations or trends between two variables?
- Can you construct and interpret two-way frequency tables?
- Can you calculate the correlation coefficient between two variables and discuss the possibility of causation, causation by a third event, and coincidence?
- Can you use the correlation coefficient and explain what various values of that number mean?
- Can you use functions fitted to data to solve problems?
- Can you analyze and interpret unions, intersections, complements, and differences of sets given descriptions and/or Venn diagrams?
- Can you compute the probability of a single outcome occurring, one of multiple outcomes occurring, or an outcome occurring given certain conditions?
- Can you calculate conditional probabilities and understand the idea of independent events?
- Can you determine the expected gain or loss in a game of chance?
- Can you use counting techniques such as permutations and combinations to determine the number of outcomes in a given scenario or situation?
- Can you use appropriate counting principles to determine probabilities?
- Can you calculate a probability distribution and graph this distribution?
- Can you estimate areas under normal curves?

Tasks of Teaching Mathematics

This list includes instructional tasks that teachers engage in that are essential for effective teaching of secondary school mathematics. Approximately 25% of test questions will measure content knowledge by assessing how that content knowledge is applied in the context of one or more of these tasks.

Mathematical explanations, justifications, and definitions

1. Identifies valid explanations of mathematical concepts (e.g., explaining why a mathematical idea is considered to be true), procedures, representations, or models
2. Evaluates or compares explanations and justifications for their validity, generalizability, coherence, or precision, including identifying flaws in explanations and justifications
3. Determines the changes that would improve the validity, generalizability, coherence, and/or precision of an explanation or justification
4. Evaluates whether counterarguments address a critique of a given justification
5. Evaluates definitions or other mathematical language for validity, generalizability, precision, usefulness in a particular context, or support of key ideas

Mathematical problems, tasks, examples, and procedures

6. Identifies problems or tasks that fit a particular structure, address the same concept, demonstrate desired characteristics, or elicit particular student thinking
7. Identifies two or more problems that systematically vary in difficulty or complexity
8. Evaluates the usefulness of examples for introducing a concept, illustrating an idea, or demonstrating a strategy, procedure, or practice
9. Identifies examples that support particular strategies or address particular student questions, misconceptions, or challenges with content
10. Identifies no examples or counterexamples that highlight a mathematical distinction or demonstrate why a student conjecture is incorrect or partially incorrect
11. Evaluates procedures for working with mathematics content to identify special cases in which the procedure might be problematic or for validity, appropriateness, or robustness

Mathematical representations, models, manipulatives, and technology

12. Evaluates representations and models (e.g., concrete, pictorial) in terms of validity, generalizability, usefulness for supporting students' understanding, or fit to the concept, calculation, etc. to be represented
13. Evaluates how representations and models (e.g., concrete, pictorial) have been used to show particular ideas, relationships between ideas, processes, or strategies
14. Evaluates the use of technology (e.g., graphing tools, software) for its appropriateness or its support of key ideas

Students' mathematical reasoning

15. Identifies likely misconceptions about or partial understanding of particular mathematics content and practices
16. Identifies how new mathematics content and practices can build on or connect to students' prior knowledge, including misconceptions and errors
17. Evaluates or compares student work (e.g., solutions, explanations, justifications, representations) in terms of validity, generalizability, coherence, and/or precision
18. Evaluates student work to identify the use of a particular concept, idea, or strategy
19. Identifies how a student's reasoning would replicate across similar problems
20. Identifies different pieces of student work that demonstrate the same reasoning
21. Identifies situations in which student work that seems valid might mask incorrect thinking

Mathematics (5165) Sample Test Questions

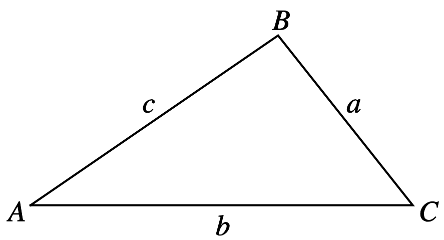
NOTATIONS

(a, b)	$\{x : a < x < b\}$
$[a, b)$	$\{x : a \le x < b\}$
$(a, b]$	$\{x : a < x \le b\}$
$[a, b]$	$\{x : a \le x \le b\}$
$\gcd(m, n)$	greatest common divisor of two integers *m* and *n*
$\operatorname{lcm}(m, n)$	least common multiple of two integers *m* and *n*
$[x]$	greatest integer *m* such that $m \le x$
$m \equiv k \pmod{n}$	*m* and *k* are congruent modulo *n* (*m* and *k* have the same remainder when divided by *n*, or equivalently, $m - k$ is a multiple of *n*)
f^{-1}	Inverse of an invertible function *f*; (not to be read as $\frac{1}{f}$)
$\lim\limits_{x \to a^+} f(x)$	right-hand limit of $f(x)$; limit of $f(x)$ as *x* approaches *a* from the right
$\lim\limits_{x \to a^-} f(x)$	left-hand limit of $f(x)$; limit of $f(x)$ as *x* approaches *a* from the left
$\varnothing$	the empty set
$x \in S$	*x* is an element of set *S*
$S \subset T$	set *S* is a proper subset of set *T*
$S \subseteq T$	either set *S* is a proper subset of set *T* or *S* = *T*
$\overline{S}$	complement of set *S*; the set of all elements not in *S* that are in some specified universal set
$T \setminus S$	relative complement of set *S* in set *T*, i.e., the set of all elements of *T* that are not elements of *S*
$S \cup T$	union of sets *S* and *T*
$S \cap T$	intersection of sets *S* and *T*

FORMULAS

Range of Inverse Trigonometric Functions

$\sin^{-1} x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$\left[0, \pi\right]$
$\tan^{-1} x$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

Volume

Sphere with radius *r*: $V = \frac{4}{3}\pi r^3$

Right circular cone with height *h* and base of radius *r*: $V = \frac{1}{3}\pi r^2 h$

Right circular cylinder with height *h* and base of radius *r*: $V = \pi r^2 h$

Pyramid with height *h* and base of area *B*: $V = \frac{1}{3}Bh$

Right prism with height *h* and base of area *B*: $V = Bh$

Surface Area

Sphere with radius r: $A = 4\pi r^2$

Right circular cone with radius r and slant height s: $A = \pi rs + \pi r^2$

Differentiation

$$\left(f(x)g(x)\right)' = f'(x)g(x) + f(x)g'(x)$$

$$\left(f(g(x))\right)' = f'(g(x))g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2} \text{ if } g(x) \neq 0$$

Sample Test Questions

The sample questions that follow are examples of the kinds of questions that are on the test. They are not, however, representative of the entire scope of the test in either content or difficulty. Answers with explanations follow the questions.

Directions: Select the best answer or answers for each question below.

1. If x and y are even numbers and $z = 2x^2 + 4y^2$, then the greatest even number that must be a divisor of z is

   (A) 2
   (B) 4
   (C) 8
   (D) 16

2. What is the units digit of 33^{408} ?

   (A) 1
   (B) 3
   (C) 7
   (D) 9

3. Jerry is 50 inches tall and is growing at the rate of $\frac{1}{24}$ inch per month. Adam is 47 inches tall and is growing at the rate of $\frac{1}{8}$ inch per month. If they each continue to grow at these rates for the next four years, after how many months will they be the same height?

   (A) 24
   (B) 30
   (C) 36
   (D) 42

4. **For the following question, select <u>all</u> the answer choices that apply.**

Three students found the correct solution to the equation $4(5x-11)=16$, but they used different methods to solve the equation.

Which of the following student methods are valid strategies for solving the equation?

Select **<u>ALL</u>** that apply.

(A)

$$\begin{aligned} 4(5x-11)&=16 \\ \frac{1}{\cancel{4}}\times\cancel{4}\times(5x-11)&=16\times\frac{1}{4} \\ 5x-11&=4 \\ 5x&=15 \\ x&=3 \end{aligned}$$

(B)

$$\begin{aligned} 4(5x-11)&=16 \\ \frac{20x}{20}-\frac{44}{20}&=\frac{16}{20} \\ \frac{20x}{20}-\frac{44}{20}+\frac{44}{20}&=\frac{16}{20}+\frac{44}{20} \\ x&=\frac{60}{20} \\ x&=3 \end{aligned}$$

(C)

$$\begin{aligned} 4(5x-11)&=16 \\ \frac{\cancel{4}}{\cancel{4}}\left(\frac{5x}{4}-\frac{11}{4}\right)&=\frac{16}{4} \\ \frac{5x}{4}-\frac{11}{4}+\frac{11}{4}&=\frac{16}{4}+\frac{11}{4} \\ \frac{5x}{4}&=4+\frac{11}{4} \\ \frac{5x}{4}&=\frac{15}{4} \\ \frac{4}{5}\times\frac{5x}{4}&=\frac{4}{5}\times\frac{15}{4} \\ x&=3 \end{aligned}$$

5. Ms. Quinn asked her students to solve the following quadratic equation.

$$3x^2 - 6x - 24 = 0$$

The following is Maurice's solution.

$$3x^2 - 6x = 24$$
$$x^2 - 2x = 8$$
$$x(x-2) = 8$$
$$x = 4 \text{ or } x = -2$$

When Ms. Quinn asked Maurice to explain his method, Maurice said, "The only pairs of numbers that are 2 apart and their product is 8 are 2 and 4, and -2 and -4. When you substitute these numbers in the last equation, you find out that only $x = 4$ and $x = -2$ work."

Which of the following statements best characterizes Maurice's approach to this problem?

(A) Maurice's method is wrong because you cannot solve an equation by factoring unless one side of the equation is equal to zero.
(B) Maurice's method is wrong because he should have first divided by 3 and then factored the left side of the equation.
(C) Maurice's method is correct, but his method often results in an equation that cannot be solved by inspection (that is, by reasoning about the factors of the constant term in the resulting equation).
(D) Maurice's method is correct, and his method always results in an equation that can be solved by inspection (that is, by reasoning about the factors of the constant term in the resulting equation).

6. **For the following question, select <u>all</u> the answer choices that apply.**

   Three students found the correct equation when asked to write an equation of the linear function represented in the following table, but they gave different explanations when describing their strategies to the class.

x	y
1	6
2	11
3	16
4	21

   Which of the following student explanations provide evidence of a mathematically valid strategy for finding an equation of the linear function?

   Select **<u>ALL</u>** that apply.

   (A) Each time the value of x goes up by 1, the value of y goes up by 5, so the slope is 5. And if x goes down by 1 from 1 to 0, then y will go down by 5 from 6 to 1, so the y-intercept is 1. That means the equation is $y = 5x + 1$.
   (B) The equation is $y = mx + b$. I just looked at the value of x and saw that it kept increasing by 1, and I looked at the value of y and saw that it kept increasing by 5, so $m = 5$. In the equation $y = 5x + b$, I substituted 1 for x and 6 for y and got that $b = 1$. The equation is $y = 5x + 1$.
   (C) For this function, I saw that you can always multiply the value of x by 5 and then add 1 to get the value of y, so the equation is $y = 5x + 1$.

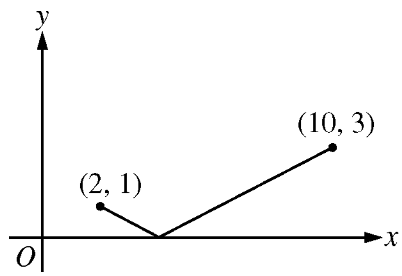
7. If $y = 5\sin x - 6$, what is the maximum value of y?

   (A) −6
   (B) −1
   (C) 1
   (D) 5

8. If $f(x) = 3x^2$, what are all real values of a and b for which the graph of $g(x) = ax^2 + b$ is below the graph of $f(x)$ for all values of x?

   (A) $a \geq 3$ and b is positive.
   (B) $a \leq 3$ and b is negative.
   (C) a is negative and b is positive.
   (D) a is any real number and b is negative.

9. **For the following question, enter your answer in the answer boxes.**



The graph of the function $f(x) = a|x - b|$ on the closed interval $[2, 10]$ is shown in the preceding *xy*-plane. What is the value of *a* ?

Give your answer as a fraction.

10. $$P(t) = 250 \cdot (3.04)^{\frac{t}{1.98}}$$

On January 1, 2010, the population of rabbits in a wooded area was 250. The preceding function was used to model the approximate population, *P*, of rabbits in the area *t* years after January 1, 2010. According to this model, which of the following best describes how the rabbit population changed in the area?

(A) The rabbit population doubled approximately every 4 months.
(B) The rabbit population tripled approximately every 6 months.
(C) The rabbit population doubled approximately every 36 months.
(D) The rabbit population tripled approximately every 24 months.

11. **For the following question, enter your answer in the answer box.**

If $3^{\log_3 2} + 5^{\log_5 9} = 8^{\log_8 x}$, what is the value of *x* ?

$x = \boxed{}$

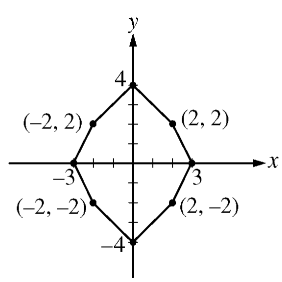
12. If $\lim_{x \to c} f(x) = 0$ and $\lim_{x \to c} g(x) = 0$, what can be concluded about the value of $\lim_{x \to c} \frac{f(x)}{g(x)}$?

(A) The value is not finite.
(B) The value is 0.
(C) The value is 1.
(D) The value cannot be determined from the information given.

13. In a certain chemical reaction, the number of grams, *N*, of a substance produced *t* hours after the reaction begins is given by $N(t) = 16t - 4t^2$, where $0 < t < 2$. At what instantaneous rate, in grams per hour, is the substance being produced 30 minutes after the reaction begins?

(A) 7
(B) 12
(C) 16
(D) 20

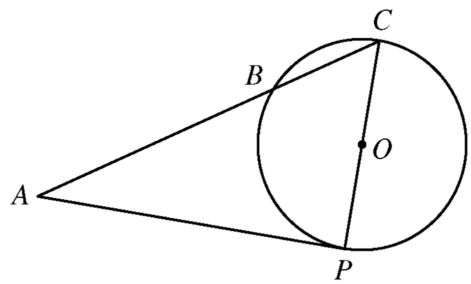
14.



For how many angles θ, where $0 < \theta \leq 2\pi$, will a rotation counterclockwise about the origin by angle θ map the octagon in the preceding figure onto itself?

(A) One
(B) Two
(C) Four
(D) Eight

15.



In the preceding circle with center O and radius 2, tangent $\overline{AP}$ has length 3 and is tangent to the circle at P. If $\overline{CP}$ is a diameter of the circle, what is the length of $\overline{BC}$?

(A) 1.25
(B) 2
(C) 3.2
(D) 5

16. The measures of the hand spans of ninth-grade students at Tyler High School are approximately normally distributed, with a mean of 7 inches and a standard deviation of 1 inch. Of the following, which group is expected to have the greatest percent of measures?

(A) The group of handspun measures that are less than 6 inches
(B) The group of handspun measures that are greater than 7 inches
(C) The group of handspun measures that are between 6 and 8 inches
(D) The group of handspun measures that are between 5 and 7 inches

17. A two-sided coin is unfairly weighted so that when it is tossed, the probability that heads will result is twice the probability that tails will result. If the coin is to be tossed 3 separate times, what is the probability that tails will result on exactly 2 of the tosses?

(A) $\frac{2}{9}$

(B) $\frac{3}{8}$

(C) $\frac{4}{9}$

(D) $\frac{2}{3}$

Sample Test Answers

1. The correct answer is (C). Since 2 is a divisor of both $2x^2$ and $4y^2$, it follows that 2 is a divisor of *z*. To find out if there is a greater even number that must be a divisor of *z*, consider the additional information given, which is that *x* and *y* are both even numbers. Since *x* and *y* are even numbers, they can be expressed as $x = 2m$ and $y = 2n$, respectively, where *m* and *n* can be either odd or even integers. Substituting these values for *x* and *y* into the expression for *z* yields $z = 2(2m)^2 + 4(2n)^2$. It follows then that $z = 8m^2 + 16n^2$ and that 8 is a divisor of *z*. The number 16 would also be a divisor of *z* if *m* is even but not if *m* is odd. Since *m* and *n* can be either even or odd and the question asks for the largest even number that must be a divisor of *z*, the correct answer is (C), 8.

Topic	I. Number & Quantity and Algebra
Subtopic	A. Number & Quantity

2. The correct answer is (A). To find the units digit of 33^{408}, it is helpful to find the first few integer powers of 33 and look for a pattern. For example,

$$33^1 = 33$$

$$33^2 = 1{,}089$$

$$33^3 = 35{,}937$$

$$33^4 = 1{,}185{,}921$$

$$33^5 = 39{,}135{,}393$$

$$33^6 = 1{,}291{,}467{,}969$$

The pattern in the units digit is 3, 9, 7, 1, 3, 9, ... The pattern will continue to repeat with every four integers of the exponent. Dividing 408 by 4 yields 102 with no remainder. Therefore, the units digit of 33^{408} will be the same as the units digit of 33^4, which is 1. So, the correct answer is (A).

Topic	I. Number & Quantity and Algebra
Subtopic	A. Number & Quantity

3. The correct answer is (C). The heights in this question can be expressed as two linear equations. Jerry's height in inches, *J*, can be expressed as $J = 50 + \frac{1}{24}m$, where *m* is the number of months from now. Adam's height in inches, *A*, can be expressed as $A = 47 + \frac{1}{8}m$. The question asks, "after how many months will they be the same height?" This is the same as asking, "for what value of *m* will $J = A$?" The solution can be found by solving $50 + \frac{1}{24}m = 47 + \frac{1}{8}m$ for *m*, as shown below.

$$\begin{aligned} 50 + \frac{1}{24}m &= 47 + \frac{1}{8}m \\ 50 - 47 &= \left(\frac{1}{8} - \frac{1}{24}\right)m \\ 3 &= \left(\frac{3}{24} - \frac{1}{24}\right)m \\ 3 &= \frac{1}{12}m \\ m &= 36 \end{aligned}$$

So the correct answer is (C), 36 months.

Topic	I. Number & Quantity and Algebra
Subtopic	B. Algebra

4. The correct answers are (A) and (B).
Choice (A): The student method is valid. The student first multiplies both sides of the equation by $\frac{1}{4}$ then adds 11 to both sides of the equation and then divides both sides of the equation by 5.
Choice (B): The student method is valid. The student correctly uses the distributive property on the left-hand side of the equation, then divides both sides of the equation by 20, and lastly adds $\frac{44}{20}$ to both sides of the equation.
Choice (C): The student method is not valid. There are two errors. The first error is in the transition from the first line to the second line, where the student incorrectly indicates that $\frac{4(5x-11)}{4}$ is equivalent to $\frac{4}{4}\left(\frac{5x}{4}-\frac{11}{4}\right)$. The second error is in the transition from the third line to the fourth line, where the student incorrectly indicates that $4+\frac{11}{4}$ is equivalent to $\frac{4+11}{4}$.

Task of Teaching	17. Evaluates or compares student work (e.g., solutions, explanations, justifications, representations) in terms of validity, generalizability, coherence, and/or precision
Topic	I. Number & Quantity and Algebra
Subtopic	B. Algebra

5. The correct answer is (C). All the steps in Maurice’s strategy are correct, so Maurice’s method is correct. However, his method often results in an equation that cannot be solved by reasoning about the factors of the constant term in the resulting equation. For example, if Ms. Quinn started with another equation, such as $3x^2-6x-21=0$, Maurice’s method would arrive at the equivalent equation $x(x-2)=7$, which cannot be solved by reasoning about the factors of 7.

Task of Teaching	17. Evaluates or compares student work (e.g., solutions, explanations, justifications, representations) in terms of validity, generalizability, coherence, and/or precision
Topic	I. Number & Quantity and Algebra
Subtopic	B. Algebra

6. The correct answers are (A), (B), and (C). The question presented to the students involves a table of x- and y-values, where the ratio of the change in y-values to the change in x-values is constant, which means there is a linear equation represented by the table of values that can be written in standard form, $y = mx + b$, where m is the slope and $(0, b)$ is the y-intercept. It remains to calculate the slope and the y-intercept, since neither is given directly.

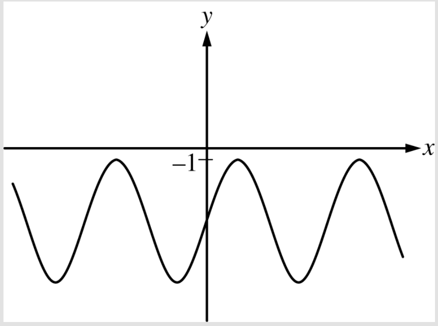
   Choice (A): This student correctly looks at the change in x and the change in y to find the slope of the line that passes through the points in the table. Since the ratio is a constant value of 5, the student concludes that this is the value for m in the standard form $y = mx + b$. Then the student determines the y-intercept by finding the value of y when x is equal to zero by subtracting 1 from the first x-value in the table and 5 from the corresponding y-value. The student concludes that the y-intercept is $(0, 1)$ and replaces b with 1. The explanation provides evidence of a mathematically valid strategy for finding an equation of the linear function.

   Choice (B): This student correctly looks at the change in x and the change in y to find the slope of the line that passes through the points in the table. Since the ratio is a constant value of 5, the student concludes that this is the value for m in the standard form $y = mx + b$ Then the student uses $(1, 6)$, the first pair of x- and y-values in the table; substitutes x with 1 and y with 6 in the equation $y = 5x + b$; and concludes that the value of b is 1. The explanation provides evidence of a mathematically valid strategy for finding an equation of the linear function.

   Choice (C): This student notices a consistent relationship between the values of x and y throughout the table and is able to find by inspection an equation that shows this relationship. Presumably, given another table for a different linear function, this student would also look for the pattern and would be able to come up with the appropriate equation. The explanation provides evidence of a mathematically valid strategy for finding an equation of the linear function.

Task of Teaching	17. Evaluates or compares student work (e.g., solutions, explanations, justifications, representations) in terms of validity, generalizability, coherence, and/or precision
Topic	I. Number & Quantity and Algebra
Subtopic	B. Algebra

7. The correct answer is (B). There are two ways to answer this question. The first solution is based on reasoning about the function $f(x) = \sin x$. First recall that the maximum value of $\sin x$ is 1, and, therefore, the maximum value of $5\sin x$ is 5. The maximum value of $y = 5\sin x - 6$ is then $5 - 6 = -1$. Alternatively, graph the function $y = 5\sin x - 6$ and find the maximum value of y from the graph.

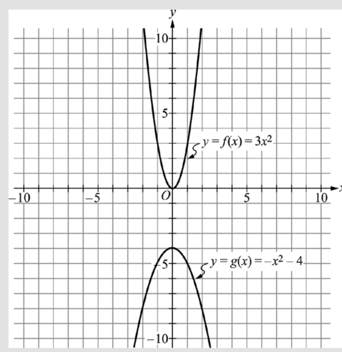


The maximum value is -1, and the correct answer is (B).

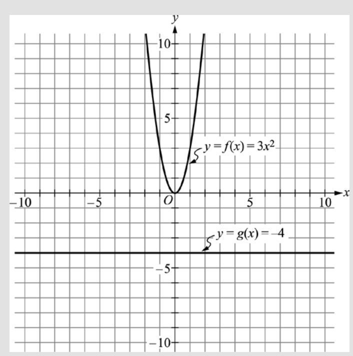
Topic	II. Functions and Calculus
Subtopic	A. Functions

8. The correct answer is (B). Since the graph of function *g* is below the graph of function *f* for all values of *x*, then $ax^2 + b < 3x^2$ for all values of *x*; that is, $(a-3)x^2 + b < 0$ for all values of *x*. In particular, substituting 0 for *x* in the last inequality gives $(a-3)(0) + b < 0$, or $b < 0$, so *b* is negative. If *a* were greater than 3, then the graph of $y = (a-3)x^2 + b$ would be a parabola that opens upward and there would be values of *x* that would make $(a-3)x^2 + b$ positive, which contradicts the information that $(a-3)x^2 + b < 0$ for all values of *x*; this contradiction means that $a \leq 3$ must be true. Therefore, the correct answer is (B). An alternate solution is as follows. Note that the graphs of *f* and *g* are parabolas in the *xy*-plane, with the *y*-axis as the common line of symmetry. (The graph of *g* degenerates to a horizontal line when $a = 0$.)

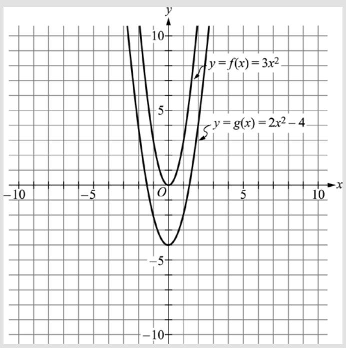
When *b* is negative, the vertex of the parabola $y = g(x)$ is lower than the vertex of the parabola $y = f(x)$. When *b* is negative and *a* is negative, the graph of *g* is a parabola in the third and fourth quadrants that opens downwards, so it's always under the graph of *f*, which is a parabola in the first and second quadrants that opens upwards. See the following graph of $f(x) = 3x^2$ and $g(x) = -x^2 - 4$.



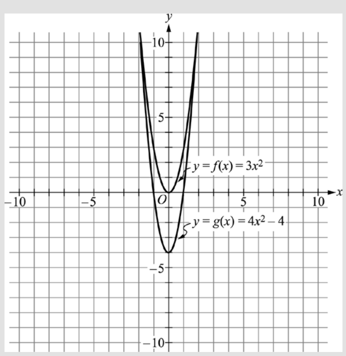
When *b* is negative and $a = 0$, the graph of *g* is a horizontal line in the third and fourth quadrants, so it's always under the graph of *f*, which is a parabola in the first and second quadrants that opens upwards. See the following graph of $f(x) = 3x^2$ and $g(x) = -4$.



When *b* is negative and $0 < a \leq 3$, the graph of *g* is a parabola that opens upwards and that grows wider horizontally more than or at the same rate as the parabola representing the graph of *f*; since the vertex for *g* is lower than the vertex for *f*, the two parabolas do not intersect and the graph of *g* is always below the graph of *f*. See the following graph of $f(x) = 3x^2$ and $g(x) = 2x^2 - 4$.



When *b* is negative and $a > 3$, the vertex for *g* is lower than the vertex for *f* and the parabola $y = f(x)$ grows wider horizontally than the parabola $y = g(x)$, so the two parabolas intersect at two points; therefore, the graph of *g* is not always below the graph of *f*. See the following graph of $f(x) = 3x^2$ and $g(x) = 4x^2 - 4$.



So, the correct answer is (B).

Topic	II. Functions and Calculus
Subtopic	A. Functions

9. The correct answer is $\frac{1}{2}$. The graph of the function *f* consists of two line segments that have a common endpoint at the point $(b, 0)$ on the *x*- axis. Since the graph of $y = f(x)$ lies on or above the *x*-axis, *a* is positive. The slope of the left line segment [with endpoints at $(2, 1)$ and $(b, 0)$] is $-a$, and the slope of the right line segment [with endpoints at $(b, 0)$ and $(10, 3)$] is *a*. Therefore,

$$a = -\frac{0-1}{b-2} = \frac{3-0}{10-b}$$

Solving for *b* gives $b = 4$, which implies that $a = \frac{1}{2}$

Topic	II. Functions and Calculus
Subtopic	A. Functions

10. The correct answer is (D). The question asks for a verbal description of the change in the rabbit population, based on the function given. Recall the meaning of the base (growth factor) and the exponent in an exponential growth model. Note that $P(t) = 250 \cdot (3.04)^{\frac{t}{1.98}} \approx 250 \cdot 3^{\frac{t}{2}}$. Observe from this approximation, with base 3 and exponent $\frac{t}{2}$, that the population tripled approximately every two years. In fact, $(3.04)^{\frac{2}{1.98}} \approx 3.07$, so the population tripled in a time period of a little less than 2 years. Thus, the correct answer is (D), "The rabbit population tripled approximately every 24 months."

Topic	II. Functions and Calculus
Subtopic	A. Functions

11. The correct answer is 11. Recall that logarithmic functions are the inverse functions of exponential functions and that, more specifically, we have the identity $a^{\log_a b} = b$ for any positive numbers a and b, where $a \neq 1$. Since $3^{\log_3 2} = 2$, $5^{\log_5 9} = 9$, and $8^{\log_8 x} = x$, the equation in the problem is equivalent to $2 + 9 = x$. Therefore, the value of x is 11.

Topic	II. Functions and Calculus
Subtopic	A. Functions

12. The correct answer is (D). Although both *f* and *g* have the limit 0 as $x \to 0$, one function might be approaching 0 more quickly than the other, which would affect the value of the limit of the quotient. For example, if one of the functions is x and the other is x^2, then the quotient is either x or $\frac{1}{x}$, and so the limit of the quotient is either 0 or nonexistent, respectively. In fact, the value of the limit can be any nonzero real number b, as shown by $\lim\limits_{x \to 0} \frac{bx}{x} = b$. Thus, answer choices (A), (B), and (C) are incorrect and the correct answer is (D).

Topic	II. Functions and Calculus
Subtopic	B. Calculus

13. The correct answer is (B). The instantaneous rate of change in the number of grams of substance produced 30 minutes after the reaction begins is the value of the first derivative of N evaluated at 30 minutes. First convert 30 minutes into hours, then evaluate the first derivative of N at that value of t. Since 30 minutes equals $\frac{1}{2}$ hour, you will need to evaluate $N'\left(\frac{1}{2}\right)$. Then find $N'(t)$.

$$N'(t) = 16 - 8t$$

Therefore, $N'\left(\frac{1}{2}\right) = 16 - 8\left(\frac{1}{2}\right) = 12$. The answer is 12 grams per hour, so the correct answer is (B).

Topic	II. Functions and Calculus
Subtopic	B. Calculus

14. The correct answer is (B). To begin, consider a single point on the octagon, such as the point $(0,4)$ at the top of the octagon in the figure. This point is 4 units from the origin, so any counterclockwise rotation about the origin that maps the octagon onto itself would need to map this point onto a point that is also 4 units from the origin. The only other point on the octagon that is 4 units from the origin is the point $(0,-4)$. A counterclockwise rotation about the origin of angle $\theta = \pi$ would map the point $(0,4)$ onto the point $(0,-4)$. The octagon is symmetric about the *x*- and *y*-axes, so a rotation of angle $\theta = \pi$ would map all of the points of the octagon onto corresponding points of the octagon.

Likewise, a counterclockwise rotation about the origin of angle $\theta = 2\pi$ would map the point $(0,4)$ onto itself (and map all other points of the octagon onto themselves). No other values of θ such that $0 < \theta \le 2\pi$ would map the octagon onto itself. Therefore, the correct answer is two, choice (B).

Topic	III. Geometry
Subtopic	A. Geometry

15. The correct answer is (C). To determine the length of $\overline{BC}$, it would be helpful to first label the figure with the information given. Since the circle has radius 2, then both $\overline{OC}$ and $\overline{OP}$ have length 2 and $\overline{CP}$ has length 4. $\overline{AP}$ is tangent to the circle at P, so angle APC is a right angle. The length of $\overline{AP}$ is given as 3. This means that triangle ACP is a 3-4-5 right triangle and $\overline{AC}$ has length 5. Notice that since $\overline{CP}$ is a diameter of the circle, angle CBP is also a right angle. Angle BCP is in both triangle ACP and triangle PCB, and, therefore, the two triangles are similar. Then find the length of $\overline{BC}$ by setting up a proportion between the corresponding parts of the similar triangles as follows:

$$\frac{CP}{AC} = \frac{BC}{PC}$$

$$\frac{4}{5} = \frac{BC}{4}$$

$$BC = \frac{16}{5} = 3.2$$

An alternate solution is to apply the tangent-secant theorem, which states that the square of the length of tangent $\overline{AP}$ equals the product of the lengths of the secants $\overline{AB}$ and $\overline{AC}$. That is, $AP^2 = AB \times AC$. It was already established that $AC = 5$. If x represents the length of $\overline{BC}$, then $AB = AC - BC = 5 - x$. Substituting $AP = 3$, $AB = 5 - x$, and $AC = 5$ in $AP^2 = AB \times AC$ results in the equation $3^2 = (5 - x)5$ with solution $x = 3.2$. The correct answer is (C), 3.2.

Topic	III. Geometry
Subtopic	A. Geometry

16. The correct answer is (C). Recall that approximately 68% of a normally distributed set of data lie within ± 1 standard deviation of the mean and approximately 95% of the data lie within ± 2 standard deviations of the mean. Evaluate each answer choice in order to determine which of the groups has the greatest percent. Choice (A): Since the mean handspun is 7 inches and the standard deviation is 1 inch, the group of handspun measures that are less than 6 inches is the group that is more than 1 standard deviation less than the mean. The group of handspun measures that are less than 7 inches includes 50% of the measures.

Approximately 34 percent ($\frac{1}{2}$ of 68 percent) of the measures are between 6 inches and 7 inches (within 1 standard deviation less than the mean). So, the group with handspun measures less than 6 inches would be approximately equal to $50\% - 34\%$, or 16% of the measures.

Choice (B): Since 7 inches is the mean, approximately 50% of the measures are greater than the mean. Choice (C): This is the group that is within ± 1 standard deviation of the mean. This group contains approximately 68% of the measures.

Choice (D): This group is between the mean and 2 standard deviations less than the mean. Approximately 47.5% ($\frac{1}{2}$ of 95%) of the measures are between 5 inches and 7 inches.

Of the answer choices given, the group described in (C) is expected to contain the greatest percent of the measures, approximately 68%, so (C) is the correct answer.

Topic	IV. Statistics & Probability
Subtopic	A. Statistics & Probability

17. The correct answer is (A). Because each toss of the coin is an independent event, the probability of tossing heads and then 2 tails, $P(HTT)$, is equal to $P(H) \cdot P(T) \cdot P(T)$, where $P(H)$ is the probability of tossing heads and $P(T)$ is the probability of tossing tails. The probability of tossing heads is twice the probability of tossing tails, so $P(H) = \frac{2}{3}$ and $P(T) = \frac{1}{3}$ Therefore, $P(HTT) = \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{2}{27}$ There are 3 ways in which exactly 2 of 3 tosses would be tails, and each of them has an equal probability of occurring:

$$P(THT) = P(TTH) = P(HTT) = \frac{2}{27}$$

Therefore, the total probability that tails will result exactly 2 times in 3 tosses is $3\left(\frac{2}{27}\right) = \frac{2}{9}$

An alternate solution is to use the binomial probability formula $P(k \text{ out of } n) = \binom{n}{k} p^k (1-p)^{n-k}$, which computes the probability of getting exactly k successes in n independent trials of a binomial experiment in which each trial has two possible outcomes (success and failure), the probability of success in each trial is p, and the probability of failure in each trial is $1-p$. In this problem, $n = 3$, $k = 2$, and $p = \frac{1}{3}$, so the probability that tails will result on exactly 2 of the 3 tosses is $P(2 \text{ out of } 3) = \binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1$, which is equivalent to $3 \times \frac{1}{9} \times \frac{2}{3}$ or $\frac{2}{9}$ Therefore, the correct answer is (A).

Topic	IV. Statistics & Probability
Subtopic	A. Statistics & Probability

Understanding Question Types

The *Praxis*® assessments include a variety of question types: constructed response (for which you write a response of your own); selected response, for which you select one or more answers from a list of choices or make another kind of selection (e.g., by selecting a sentence in a text or by selecting part of a graphic); and numeric entry, for which you enter a numeric value in an answer field. You may be familiar with these question formats from taking other standardized tests. If not, familiarize yourself with them so you don't spend time during the test figuring out how to answer them.

Understanding Selected-Response and Numeric-Entry Questions

For most questions, you respond by selecting an oval to select a single answer from a list of answer choices.

However, interactive question types may also ask you to respond by:

- Selecting more than one choice from a list of choices.
- Typing in a numeric-entry box. When the answer is a number, you may be asked to enter a numerical answer. Some questions may have more than one entry box to enter a response.
- Selecting parts of a graphic. In some questions, you will select your answers by selecting a location (or locations) on a graphic such as a map or chart, as opposed to choosing your answer from a list.
- Selecting sentences. In questions with reading passages, you may be asked to choose your answers by selecting a sentence (or sentences) within the reading passage.
- Dragging and dropping answer choices into targets on the screen. You may be asked to select answers from a list of choices and to drag your answers to the appropriate location in a table, paragraph of text or graphic.
- Selecting answer choices from a drop-down menu. You may be asked to choose answers by selecting choices from a drop-down menu (e.g., to complete a sentence).

Remember that with every question you will get clear instructions.

Understanding Constructed-Response Questions

Constructed-response questions require you to demonstrate your knowledge in a subject area by writing your own response to topics. Essays and short-answer questions are types of constructed-response questions.

For example, an essay question might present you with a topic and ask you to discuss the extent to which you agree or disagree with the opinion stated. You must support your position with specific reasons and examples from your own experience, observations, or reading.

Review a few sample essay topics:

- *Brown v. Board of Education of Topeka*

  "We come then to the question presented: Does segregation of children in public schools solely on the basis of race, even though the physical facilities and other 'tangible' factors may be equal, deprive the children of the minority group of equal educational opportunities? We believe that it does."

  A. What legal doctrine or principle, established in *Plessy v. Ferguson* (1896), did the Supreme Court reverse when it issued the 1954 ruling quoted above?

  B. What was the rationale given by the justices for their 1954 ruling?

- *In his self-analysis, Mr. Payton says that the better-performing students say small-group work is boring and that they learn more working alone or only with students like themselves. Assume that Mr. Payton wants to continue using cooperative learning groups because he believes they have value for all students.*

  - Describe **TWO** strategies he could use to address the concerns of the students who have complained.
  - Explain how each strategy suggested could provide an opportunity to improve the functioning of cooperative learning groups. Base your response on principles of effective instructional strategies.

- *"Minimum-wage jobs are a ticket to nowhere. They are boring and repetitive and teach employees little or nothing of value. Minimum-wage employers take advantage of people because they need a job."*

  - Discuss the extent to which you agree or disagree with this opinion. Support your views with specific reasons and examples from your own experience, observations, or reading.

Keep these things in mind when you respond to a constructed-response question:

1. **Answer the question accurately.** Analyze what each part of the question is asking you to do. If the question asks you to describe or discuss, you should provide more than just a list.
2. **Answer the question completely.** If a question asks you to do three distinct things in your response, you should cover all three things for the best score. Otherwise, no matter how well you write, you will not be awarded full credit.
3. **Answer the question that is asked.** Do not change the question or challenge the basis of the question. You will receive no credit or a low score if you answer another question or if you state, for example, that there is no possible answer.
4. **Give a thorough and detailed response.** You must demonstrate that you have a thorough understanding of the subject matter. However, your response should be straightforward and not filled with unnecessary information.
5. **Take notes on scratch paper** so that you don't miss any details. Then you'll be sure to have all the information you need to answer the question.
6. **Reread your response.** Check that you have written what you thought you wrote. Be sure not to leave sentences unfinished or omit clarifying information.

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