The purpose of the 2017 Mathematics SOL Institutes is to provide teachers with professional development focused on the instruction that supports implementation of the 2016 *Mathematics Standards of Learning*.

[**2016 Mathematics Standards of Learning Resources**](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/index.shtml)

* **2016 Standards of Learning**
* **2016 Curriculum Frameworks**
* **Crosswalks (Summary of Revisions 2009-2016)**
* **Summary of Revisions (Narrated PPT 2009-2016)**

**Professional Development Instructions**

A product of the 2017 Mathematics Institutes is a set of online professional development modules designed to be used by a group of teachers of a specific grade level or course, facilitated by a member of the team. Modifications could be made to adapt the professional development for more than one grade level/course or for large groups. Facilitators should review the PowerPoint, the Facilitator Guide, and the resources prior to facilitating this professional development.

| **Approximate Time**  | **Facilitator Instructions**  | **Links to Materials** |
| --- | --- | --- |
| **60 minutes total** | **Module 1: Revisions to Standards and Purpose****Essential Question: What are the new 2016 Standards of Learning and how might the VDOE documents support understanding of these standards? (PowerPoint Slides 5 – 21)** | * [EOC Session PowerPoint](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/eoc/sol-institute-ppt.pptx)
* 2009 to 2016 Crosswalks (Summary of Revisions)
	+ [Algebra I](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/algebra1_crosswalk.pdf)
	+ [Geometry](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/geometry_crosswalk.pdf)
	+ [Algebra II](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/algebra2_crosswalk.pdf)
* Narrated Crosswalk Presentations
	+ [Algebra I](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/a1-sol-2016.pptx)
	+ [Geometry](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/geom-sol-2016.pptx)
	+ [Algebra II](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/a2-sol-2016.pptx)
* [SOL G.2 Curriculum Framework](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/eoc/sol-G.2-cf.pdf)
* [2016 HS SOL Revision Activity & Key](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/eoc/hs-review-act-key.docx)
* [EOC Course Introductions Activity](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/eoc/course-intros.docx)
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|  | 1. (Slide 5) Lead with the essential question that will be the focus of Module 1.
2. (Slide 6) Highlight the five mathematical process goals – problem solving, connections, communication, representations, and reasoning – and emphasize their role in standards development and progression. Mathematical content understanding is achieved through the process goals.
3. (Slide 7) Introduce [NCTM’s *Principles to Actions* High Leverage Teaching Practices](https://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf) and consider the significance of each mathematics teaching practice and its implications for instruction. Participants will explore these principles in each of the following modules and do tasks that highlight specific practices.
	1. Activity – Have participants pick 2 strengths and 2 focus areas that could be improved. Create two posters, one titled “Strengths” and numbered from 1 to 8, and another titled “Focus” and numbered from 1 – 8. Participants will place two stickers on each poster. The chart can then be revisited during other activities.
4. (Slide 8) Explain changes to the EOC curriculum framework and invite participants to find examples of the changes in the SOL G.2 Curriculum Framework document. Discuss that the K-8 Curriculum Frameworks had previously been three columns but now have two in the 2016 standards. Point out the feature of mapping each Essential Knowledge and Skill back to the sub-bullets of a standard, when applicable.
5. (Slides 9 – 13) Introduce the Crosswalk document as a summary of the SOL revisions and explain the formatting: additions, deletions, parameter changes, and moves. Advise that a sample of the Algebra I Narrated PowerPoint is included and can serve as another resource. Have participants choose one component from each quadrant to share with a shoulder partner to ensure understanding of the structure.
6. (Slides 14 – 15) HS SOL Revision Activity: With a partner, use the Crosswalk documents to complete the scavenger hunt. Upon completion, partners should compare answers to the HS SOL Revision Activity Key (also included in the PowerPoint presentation). For item 6, reinforce the expectation is that students will simplify, for example, the cube root of negative 64. Good classroom dialogue could result from comparing the square root of -64 to the cube root of negative 64 and why the cube root has a real solution. For item 8, point out situations, using special right triangles, of when students might be asked to rationalize the denominator.
7. (Slide 16) Highlight the changes in Algebra I/II regarding expectations with the Properties of Real Numbers and Equality/Inequality, from *justifying* steps used in simplifying expressions and solving equations/inequalities to *applying* the properties to simplify and solve. What implications might this have on both instruction and assessment? Discuss how instruction that includes error analysis would allow students to determine if the properties of real numbers and equality/inequality were applied correctly (without necessarily naming the property).
8. (Slides 17 – 18) Discuss the progression of linear equations and inequalities from grade 6 through Algebra I. What is the difference between equations students would see in Math 8 and equations students would see in Algebra I? In small groups, ask participants to suggest components of Math 8 equations as compared to Algebra I equations. Each small group will share out to the whole group as the facilitator creates a master list on chart paper. Use slide 18 in the PowerPoint presentation that lists the Grade 8 EKS to annotate the chart paper with a check or a strike-through as each actual Grade 8 skill is revealed.
9. (Slide 19) EOC Course Introductions Activity - Form homogeneous groups of Algebra I, Geometry, and Algebra II teachers and ask them to read their respective course descriptions provided in the EOC Course Introductions Activity handout. In their groups, they should answer the following questions:
* What surprises you in the descriptions?
* What in these descriptions is already a component of your course? What will need to change?
* Which of these components is the hardest/easiest to implement?

Have each group report out while still in small groups, highlighting one or two take-aways from the discussion.1. (Slide 20) Have participants return to their original tables and talk with their colleagues about an overview of all of the changes:
* Now that you have examined the summary documents, what did you discover?
* Why do you think the changes were made?
1. (Slide 21) Revisit the essential question from Module 1 as a whole group.
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| **Approximate Time**  | **Facilitator Instructions**  | **Links to Materials** |
| --- | --- | --- |
| **3 hours 15 minutes total** | **Module 2: Emphasis on Specific Content**  | **Links for materials** |
| 45 minutes | **Part A: Transformations****Essential Questions:** What instructional strategies promote students’ understanding of transformations using technology? **(PowerPoint Slides 22 – 30)** | * [Grade Level/Course Cards](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/eoc/grade-course-cards.docx)
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|  | 1. (Slide 23) Begin with the essential question for Module 2a, that will help guide participant thinking throughout the module.
2. (Slides 24 – 25) Transformations Vocabulary sort: With a partner, as various vocabulary words are displayed in PowerPoint slide 24, raise the grade-level/course card (grades 3 – Algebra II) for the grade/course where the word is first introduced. Participants can keep track of the number they answered correctly. Give them 1 point if they guess within 1 grade level of the correct answer and 2 points if they guess the correct grade level.
	1. Vocabulary Sort KEY:
		* Congruent: Grade 3…SOL 3.13: The student will identify congruent and noncongruent figures.
		* Similar: Grade 7…SOL 7.5: solve problems involving similar quadrilaterals and triangles.
		* Transformations (Geometric context): Grade 5…SOL 5.14: recognize and apply transformations.
		* Translate (rotate, reflect - Geometric context): Grade 5…5.14: Apply transformations to polygons in order to determine congruence.
		* Dilation (Geometric context): Grade 8…SOL 8.7: Apply transformations to include dilations.

NOTE: Grade 5 transformations are in space; Grade 8 are in the coordinate plane. Grade 8 also includes isometric transformations. Geometry includes each of those transformations with additional parameters.* + - Transformations (Function context): Algebra I…SOL A.6: Describe transformations (of a linear function) defined by changes in the slope or *y*-intercept.
		- Parent Function: Algebra I…SOL A.6: Use the parent function *y* = *x*.
		- Translate (Function context): Algebra I…SOL A.6: Changes in the *y*-intercept may be described by transformations.
		- Reflect (and dilate – Function context): Algebra I…SOL A.6: Changes in the slope may be described by dilations.

AFDA.2—Use knowledge of transformations (horizontal and vertical translation; reflect over *x*- and *y*-axis; dilate by stretch and compress) to write an equation.Algebra II—AII.6: Identify the transformation of a function—absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic1. (Slide 26) As a whole group, explore the transformations progression from 3rd grade through Algebra I shown on the slide and how it relates to and provides a foundation for both the geometric and functional concept of transformations.
2. (Slide 27) Facilitate an Algebra II number talk about the graphs of *y* =2(*x* + *b*)2 *and y* = 2*x*2 + *b*. Give participants about 30 seconds to think about something they know about the functions. As a whole group, make a list of what they came up with. Facilitator may want to ask probing questions to clarify the observations provided by each participant.
3. (Slide 28) Demonstrate the robust exploration of transformations using the free online graphing tool [www.desmos.com](http://desmos/) (note that [learn.desmos.com](http://learn.desmos.com/) provides classroom activities that utilize Desmos. The link to a particular transformations lesson [Teacher Page](https://teacher.desmos.com/activitybuilder/custom/58509b6ae871657106f28ec3) is given (also included in the PowerPoint slide 28). The presenter should access this link and then select “create class code.” Participants should then access [student.desmos.com](https://student.desmos.com/) and enter the code that was created to act as students. Spend a few minutes walking through this activity. Discuss in table groups the benefits of such explorations using technology.
4. (Slide 29) Highlight the [NCTM’s *Principles to Actions* High Leverage Teaching Practices](https://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf) that are evident in the Transformations Desmos task, focusing on Practices #4 and 5. Also discuss how other activities in Module 2a incorporate these practices.
5. (Slide 30) Revisit the essential question for Module 2a.
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| 45 minutes | **Part B: Graphing Linear Functions****Essential Question: How do the K-8 progressions impact the way we teach graphing linear equations in Algebra I? (PowerPoint Slides 31 – 42)** | * [PFA Progression Sort Activity Strips](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/eoc/pfa-prog-sort.xlsx)
* [2016 Grade 5 – Algebra I PFA Progression](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/eoc/2016-gr5-a1-pfa-prog.docx)
* [Unpacking Standards Template & A.6c KEY](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/eoc/unpack-templ-sol-A.6c-key.docx)
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|  | 1. (Slide 32) Begin by reviewing the essential question for Module 2b.
2. (Slides 33 – 35) PFA Progression Sort Activity: Cut out the PFA Progression Sort Activity strips. Use sticky notes to create a number line using grade-level labels (Grades 5-Geometry) – see slide 34. With a partner, place each Essential Knowledge and Skills bullet (from the Sort Activity strips) with its appropriate grade level/subject. Upon completion of the work in pairs, ask all participants to stand. Use slide 35 in the PowerPoint to display the correct progression. Participant pairs will be asked to sit down when they have incorrectly sequenced a skill in their progression. Discuss and debrief the sort.
3. (Slides 36 – 37) As a whole group, explore the progression shown on slide 36: SOL 6.12 (Proportional Relationships), SOL 7.10 (Non-Proportional/Additive Relationships), SOL 8.16 (Linear Functions) and the progression shown on slide 37: SOL 6.12 (unit rates and ratio tables), SOL 7.10 (slope as rate of change, *y* = *mx* and *y* = *x* + *b,* SOL 8.16 (slope and *y*-intercept, *y* = m*x* + *b*. Discuss how this middle school progression leads to Algebra I and how it relates to and provides a foundation for linear function understanding.
4. (Slides 38 – 39) Table group discussion: Compare/contrast the 2009 version of SOL A.6 and the 2016 version shown on slide 38. There were minimal changes: Graph a linear equation became A.6c in 2016 instead of part of the standard in the 2009 version. Linear inequalities are now in A.5 in the 2016 standards. Have participants review the Grade 5 – Algebra I PFA Progression document. While the Algebra I expectations are the same, the foundational skills students experience in prior grades are very different. How should our instruction be adapted to build on these experiences?
5. (Slide 40) In table groups, unpack standard A.6 using the Unpacking Standards template. Discuss as a whole group.
6. (Slide 41) Highlight the [NCTM’s *Principles to Actions* High Leverage Teaching Practices](https://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf) that are evident in the Linear Equations progressions, focusing on Practices #1 and 6.
7. (Slide 42) Revisit the essential question for Module 2b.
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| 60 minutes | **Part C: Quadrilaterals and Proofs****Essential Questions: What instructional strategies promote students’ understanding of quadrilaterals? What is the role of proof in the Geometry curriculum? (Slides 43 – 53)** | * [Rectangle Task & Proof](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/eoc/rectang-task-proof.docx)
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|  | 1. (Slide 44) Begin with the essential question for Module 2c.
2. (Slides 45 – 47) Brainstorming activity:
	* In table groups, participants will record, using chart paper, what Essential Knowledge and Skills (EKS) regarding quadrilaterals participants think might be taught in Grades 4, 7, and Geometry. Consider types of quadrilaterals, vocabulary, angles, properties, and proofs. Participants may want to individually write down ideas on sticky notes to share before creating the list on chart paper.
	* Post the chart paper from each table group’s completed brainstorm. Ask participants to take part in a gallery walk to observe what other table groups have suggested.
	* Return to table groups and revise charts as desired.
	* View the Quadrilateral progression (slide 47) and have the whole group dialogue to compare/contrast with the created brainstorm charts. Note the following information regarding quadrilaterals in the 2016 *Mathematics Standards of Learning*:
		1. Quadrilaterals: The quadrilaterals addressed are the same in grades 4 and 7. Geometry adds the isosceles trapezoid. A kite is not included in Virginia’s SOL.
		2. Sides: In grade 4 students will address whether sides are parallel or perpendicular. Students also address whether sides are congruent. Grade 7 adds the diagonals and the fact that they bisect each other or may be congruent or perpendicular. Students will find unknown side lengths using properties. No new properties are added in Geometry other than the properties for isosceles trapezoids.
		3. Angles: Grade 4 includes solely right angles.Angle relationships are added in grade 7 with opposite angles and consecutive angles. Geometry adds that diagonals may bisect opposite angles.
		4. Level of Rigor: In grade 4, students are expected to develop definitions, classify, identify, and compare and contrast. In grade 7 students are expected to use properties to also solve problems involving unknown side and angle measures. In Geometry, students solve more complex problems and are asked to use proofs to justify.
3. (Slide 48) Individually complete the Rectangle Task using the document provided. Compare with others in table groups and refine conjectures and justifications. While participants are completing the problem, monitor their work and select a few to share. Sequence the order that answers are shared from simplest to most complex. For example, start with someone who put the rectangle in quadrant 1, then look for one who centered the rectangle on the origin, and finally choose someone whose rectangle was not parallel/perpendicular to the axes. If anyone started with a square, share that approach because the quadrilateral formed in the second step is also a square. Ask participants why they chose the starting points as the vertices. Then ask why the answers were shared in the particular order. Share the sample solution and proof available in the Rectangle Task Proof document. Discuss how the concept of thinking about proof can begin with a specific case, make a conjecture, verify the conjecture with the specific case, and then prove in general.
4. (Slide 49) Go to [www.geogebra.org](http://www.geogebra.org) and model the task using the GeoGebra dynamic geometry software to help solidify conjectures and justifications.
5. (Slide 50) Discuss the role of proof in the Rectangle Task and look through the provided proof. Use that discussion to introduce and frame a discussion of the significance of proof in the Geometry curriculum.
6. (Slide 51-52) Ask participants which [NCTM’s *Principles to Actions* High Leverage Teaching Practices](https://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf) are highlighted by the Rectangle Task. Focus the participants’ attention on Practice #7 *Support productive struggle in learning mathematics.* For participants who have a copy of NCTM’s book *Principles to Actions- Ensuring Mathematical Success for All*, discuss the Teacher and Student Actions found on page 52 that support productive struggle in learning mathematics.
7. (Slide 53) Revisit the essential question for Module 2C.
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| 45 minutes | **Part D: Multiple Representations of Functions****Essential Question: What instructional strategies promote students’ flexibility with multiple representations of functions? (Slides 54 – 62)** | * [Representation of Functions Sorting Activity and Key](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/eoc/repr-functions-sort-key.docx)
* [Multiple Representations of Functions Task & Key](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/eoc/mult-repr-funct-task-key.docx)
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|  | 1. (Slide 55) Begin with discussing the essential question for Module 2d.
2. (Slide 56) Representations of Functions Sorting Activity: Print and cut out the cards for the sorting activity. In small groups, have the participants create headers using sticky notes for each grade/subject: Elementary, grade 6, grade 7, grade 8, Algebra I, Geometry, Algebra II, Trigonometry, Math Analysis. Participants will sort the problems by the grade level/subject in which they think that students would encounter that type of problem. Discuss the groups’ answers and compare with the key. Which questions were difficult to sort and why?
3. (Slide 57) Notice how the wording in both the Algebra I and Algebra II SOLs (A.7 and AII.7 Essential Knowledge and Skill) shown on the slide is the same. What could multiple representations look like in each subject? Encourage dialogue among participants comparing and contrasting the two standards in regards to types of functions included and the complexity of the function components addressed.
4. (Slides 58-59) With a partner, participants should complete the Multiple Representations of Functions Task and then compare their answers with the key. As they are working, they should discuss with their partners:
* How does the use of multiple representations in this task inform you of student understanding?
* How does it demonstrate the need for multiple representations?
* How can you modify these problems to change the cognitive demand?
1. (Slides 60 – 61) Ask participants which [NCTM’s *Principles to Actions* High Leverage Teaching Practices](https://www.nctm.org/uploadedFiles/Standards_and_Positions/PtAExecutiveSummary.pdf) are highlighted by the Multiple Representations Task. Focus their attention on Practice #3 Use and connect mathematical representations. For participants who have a copy of NCTM’s book *Principles to Actions- Ensuring Mathematical Success for All*, discuss the Teacher and Student Actions found on page 29 that support the practice: use and connect mathematical representations.
2. (Slide 62) Have participants revisit the essential question for Module 2d.
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| **30 minutes total** | **Module 3: Support for Implementation****Essential Questions: In what ways can I support my colleagues in implementation of the 2016 Standards of Learning? What resources are available from VDOE to assist with implementation? (Slides 63-68)** | * [2016 Mathematics Standards of Learning Documents](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/index.shtml)
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|  | 1. (Slide 64) Review the essential question for Module 3.
2. (Slide 65) Show the resources available on the VDOE website and how to navigate the Mathematics page to find them (2016 SOLs and Curriculum Framework, Crosswalk, Grade Level/Course Videos, etc.) Discuss possible uses of the resources. Emphasize the roles all stakeholders have in implementation of the 2016 Standards.
3. (Slide 66) Describe the Implementation Timeline.
4. (Slide 67) Ask participants to reflect on their next steps as they return to their division, armed with the resources from the Institute:
	* On the Business card, individually record some action steps.
	* Visit with three other participants to hear their planned action steps, and record their plans in the remaining sections of the business card.
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