The purpose of the 2017 Mathematics SOL Institutes is to provide teachers with professional development focused on the instruction that supports implementation of the 2016 *Mathematics Standards of Learning*.

[**2016 Mathematics Standards of Learning Resources**](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/index.shtml)

* **2016 Standards of Learning**
* **2016 Curriculum Frameworks**
* **Crosswalks (Summary of Revisions 2009-2016)**
* **Summary of Revisions (Narrated PPT 2009-2016)**

**Professional Development Instructions**

A product of the 2017 Mathematics Institutes is a set of online professional development modules designed to be used by a group of teachers of a specific grade level or course, facilitated by a member of the team. Modifications could be made to adapt the professional development for more than one grade level/course or for large groups. Facilitators should review the PowerPoint, the Facilitator Guide, and the resources prior to facilitating this professional development.

| **Approximate Time** | **Facilitator Instructions** | **Links to Materials** |
| --- | --- | --- |
| **45 minutes total** | **Module 1: New 2016 Standards of Learning**  **Essential Question: What are the new 2016 Standards of Learning and how might the VDOE documents support understanding of these standards?** **(PowerPoint Slides 3-16)** |  |
|  | 1. Revisions to Standards and Purpose (PowerPoint Slides 3-16)   Highlight the Focus of Revisions evident in the2016 *Mathematics Standards of Learning:*   * + Improve Vertical Progression   + Improve Consistency of Math Language   + Clarify Teaching and Learning Expectations   + Increased Support for Teachers   + Ensure Proficiency with Computational Skills   + Ensure Developmental Appropriateness and Student Expectations  1. Review a problem that illustrates the vertical progression of the concept of ratios in SOL 6.1, 7.3, and 8.4. 2. Discuss SOL 6.6 that now includes integer operations. Discuss how this concept can be approached through a conceptual lens to ensure the developmental appropriateness of this content in grade 6. 3. (Slide 7) Five Strands Task: Assign a Strand Introductory statement to each group member. Have each group member read the statement and underline or highlight important ideas that will help them to make meaning of the passage. In round robin format, each group member will summarize their assigned statement for the group. As a small group, members will discuss each statement and the implications to mathematics instruction. 4. Introduce the Crosswalk documents (Slide 8) as a summary of the SOL revisions and explain the formatting: additions, deletions, parameter changes, and moves. (Slides 9-11) Provide a short excerpt from the Grade 7 Narrated Crosswalk Presentation and discuss how this might be used by teachers to further explore the changes to the standards. 5. Scavenger Hunt (Slides 12-13): With a partner, use the crosswalk documents to complete the scavenger hunt. Upon completion, partners should compare answers to the key in the PowerPoint presentation. 6. In table groups, unpack SOL 7.3 (Slide 14) using the provided template and the SOL 7.3 curriculum framework pages. Discuss as a whole group and review the completed template in the PowerPoint presentation. 7. Highlight the five process goals – problem solving, connections, communication, representations, and reasoning – and emphasize their role in standards development and progression. Mathematical content understanding is achieved through the process goals. 8. Introduce NCTM’s *Principles to Actions* (Slide 16) and consider the significance of each mathematics teaching practice and its implications for instruction. Participants will explore these principles in each of the following modules and complete tasks that highlight specific practices. 9. Reflection Module 1: Have participants talk with their colleagues about an overview of all of the changes:    * Now that you have examined the summary documents, what did you discover? Why do you think the changes were made?    * Revisit the essential question from Module 1 as a whole group. | * [Grade Band 6-8 Session PowerPoint](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/6-8-mathinstitute.pptx) * [Five Strands Task](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/five-strand-task.docx) * 2009 to 2016 Crosswalks (Summary of Revisions) * [Grade 6](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/grade6_crosswalk.pdf) * [Grade 7](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/grade7_crosswalk.pdf) * [Grade 8](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/grade8_crosswalk.pdf) * Narrated Crosswalk Presentations * [Grade 6](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/grade6-sol-2016.pptx) * [Grade 7](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/grade7-sol-2016.pptx) * [Grade 8](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/crosswalk/grade8-sol-2016.pptx) * [6-8 True False Scavenger Hunt & Key](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/t-f-scav-hunt-and-key.docx) * [SOL 7.3 Curriculum Framework](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/cf-sol7-3.docx) * [Unpacking Template - SOL 7.3](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/unpacking-sol7-3.docx) |

| **Approximate Time** | **Facilitator Instructions** | | **Links to Materials** | |
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| **180 minutes total** | | **Module 2: Emphasis on Specific Content** |  |
| 45 minutes  (Part A, B and C) | | **Part A: Equations**  **Essential Question: What are the limiting parameters for the types of algebraic equations presented at each grade level? (PowerPoint Slides 17-21)** |  |
|  | | 1. Equations Progression: As a whole group, participants will compare the standards related to equations (SOL 6.13, 7.12, 8.17). 2. Participants will be asked to compare/contrast in each grade: the number of steps to solve; application of properties of real numbers and equality; confirming solutions; writing expressions and equations from verbal statements; solving practical problems; and vocabulary. 3. Participants will be asked to complete a task, working in pairs, to identify the grade level at which students might be expected to solve each equation. Participants will compare the grade levels identified as a whole group and then be asked to note other parameters associated with each grade level when solving equations (e.g., unit fractions as coefficients in grade 6; rational numbers are coefficients and constants in grade 7; use of the distributive property to simplify on one side of an equation before solving in grade 8). 4. Revisit the essential question for Module 2a. | * [Grades 6-8 Equations Progression](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/equationprog.docx) * [Equations Task & KEY](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/eq-task-and-key.docx) |
|  | | **Part B: Inequalities**  **Essential Question: What are the limiting parameters for the types of algebraic inequalities presented at each grade level? (PowerPoint Slides 22-25)** |  |
|  | | 1. Inequalities Progression: As a whole group, participants will compare the standards related to inequalities (SOL 6.14, 7.13, 8.18). 2. Participants will be asked to compare/contrast in each grade: the number of steps to solve; application of properties of real numbers and equality (addition, subtraction properties of inequality for Grade 6; multiplication and division properties of inequality included in Grade 7); confirming solutions; solving practical problems (represent a practical situation in grade 6); representing solutions algebraically and graphically; and identifying solution sets. 3. Participants will be asked to complete a task, working in pairs, to identify the grade level at which students might be expected to solve each inequality. Participants will compare the grade levels identified as a whole group and then be asked to note other parameters associated with each grade level when solving equations (e.g., grade 6 apply addition and subtraction properties of inequality; grade 7 to include rational number coefficients; grade 8 to include variables on both sides and up to four steps to solve). 4. Revisit the essential question for Module 2b. | * [Grades 6 – 8 Inequalities Progression](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/inequalityprog.docx) * [Inequalities Task & KEY](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/ineq-task-and-key.docx) |
|  | | **Part C: Algebra (Proportional Reasoning) Progression**  **Essential Questions: What is the progression of instruction required for promoting proportional reasoning through the Patterns, Functions, and Algebra strand? What instructional ideas are prevalent throughout the grade levels? (PowerPoint Slides 26-29)** |  |
|  | | 1. Proportional Reasoning Progression: As a whole group, participants will compare SOL 6.12, 7.10, 8.16. Participants will be asked to note that algebra is studied through the lens of proportional relationships within all grade levels; multiplicative and additive relationships introduced in grade 6 and grade 7 to develop a better understanding of linear functions in Grade 8. 2. Participants will note the vertical Progression of representations used across grade levels to develop proportional reasoning (e.g., ratio tables and graphs are the primary tools used to solve and understand algebraic relationships in all grade levels; unit rates and recognizing proportional and non-proportional graphs are the primary focus in Grade 6; understanding slope as a rate of change is the primary focus in Grade 7; slope triangles are a visual tool to help students understand the rate of change; applying slope and y-intercept is the focus within grade 8; no abstract algebra formulas introduced in grade 6; grade 7 formulas introduced contextually and algebraically are y = mx and y = x + b; grade 8 formula introduced contextually and algebraically is y = mx + b).   Note that students will develop the concept of slope through patterns and slope triangles graphically and are not encouraged to utilize the slope formula until Algebra I.   1. Revisit the essential question for Module 2c. | * [Grades 6 – 8 Proportional Reasoning Progression](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/prop-reason-prog.docx) |
| 45 minutes | | **Part D: Proportional Reasoning and Unit Rates (Grade 6)**  **Essential Question: What instructional strategies can be used to promote the study of unit rates and proportional and nonproportional relationships? (PowerPoint Slides 30-33)** |  |
|  | | SOL 6.12 Task 1:   1. Participants will be asked to work in small groups to complete SOL 6.12 Task 1. Before beginning the task, ask participants how the concepts in SOL 6.1 and SOL 6.12 are connected. The concept of ratios in SOL 6.1 is needed as prior knowledge for the grade 6 Patterns, Functions, and Algebra strand. 2. Ask participants to compare the relationships in the two tables provided in the task. Discuss how the ordered pair (2, 6) appears in both tables but the relationships are not the same. Have participants use linking cubes to represent the information represented in the table showing Sarah’s costs. Suggest that they may wish to use certain colors to allow for patterns to be more apparent. Allow participants to find their own ways to represent the relationship without providing any other suggestions. Here are two examples of linking cube models (the second one shows the unit rate):      1. Ask participants to share their models with the whole group. A unit rate describes how many units of the first quantity of a ratio correspond to one unit of the second quantity. Have participants describe how the unit rate is represented in the model created by their team. Linking cubes are helpful in allowing students to visualize and understand the idea of a repeating unit ratio within a proportional relationship. 2. Discuss questions 1 – 7 in SOL 6.12 Task 1. Use the key and guiding questions provided to discuss the responses. 3. Ask participants to use the linking cubes to represent the information represented in the table showing Kayla’s costs. Ask participants how this model differs from the model for Sarah’s costs. Have participants share and explain their models with the group. Again, suggest that participants may wish to use certain colors to make patterns more apparent, but allow them to create without suggestions. 4. Discuss questions 8 – 10 from SOL 6.12 Task 1 as a whole group. Use the key and guiding questions provided to discuss the responses.   SOL 6.12 Task 2   1. Introduce SOL 6.12 Task 2 to the participants. Have them create tables to represent each of the graphs shown. Discuss how using ratio tables and graphs promote the understanding that proportional relationships must go through the origin while non-proportional relationships will have a non-zero y- intercept. 2. Ask the participants to work in small groups and use the linking cubes to model ratio pairs for Jerome’s earnings and Tory’s earnings. Ask the participants to describe their models to the whole group. Have participants extend their models to represent other relationships shown in the graphs for Jerome’s and Tory’s earnings. Discuss how the linking cubes can be used to represent the additive nature of the ratios within a non-proportional relationship, and that additive relationships are a concept discussed in grade 7. Here are some examples of linking cube models:     Jerome’s Earnings  Tori’s Earnings   1. Ask the participants to complete the questions for SOL 6.12 Task 2. Use the key and guiding questions provided to discuss the responses.   SOL 6.12 Task 3   1. Introduce SOL 6.12 Task 3. Ask participants to explain their thinking in completing the table representing the money Sam earns for completing chores. Discuss how proportional relationships are multiplicative, thus enabling doubling, halving, etc. in order to complete a ratio table. Students arrive from elementary grades with an intuitive understanding of doubling and halving. 2. Complete the questions for SOL 6.12 Task 3. Use the key and guiding questions provided to discuss the responses. 3. Revisit the essential question for Module 2d. 4. Reflection: Discuss NCTM’s *Principles to Actions* in the context of the three tasks just completed. Ask the participants to consider which of the mathematics teaching practice were evident in the tasks. Have the participants highlight the specific practices and discuss what evidence from the tasks promotes the practices designated. | * [SOL 6.12 Tasks 1-3](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/sol-6-12-tasks1-3.docx) * [SOL 6.12 Tasks 1-3 KEY](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/sol-6-12-key.docx) |
| 45 minutes | | **Part E: Slope as Rate of Change (Grade 7)**  **Essential Questions: What instructional strategies can be used to promote the study of slope as a rate of change, and the difference between multiplicative and additive relationships? (PowerPoint Slides 35-38)** |  |
|  | | SOL 7.10 Task 1   1. Prior to providing the handout for SOL 7.10 Task 1, discuss how a double number line is a useful representation to show the behavior of two quantities within a ratio acting as a composed unit (i.e. time and distance). Provide the context for the task regarding David walking at a constant rate and ask the participants to create a double number line to show the relationship between time and distance. Have the participants share their strategies for creating the double number line. 2. Pass out the handout for SOL 7.10 Task 1 and have the participants complete the table (question #2) and then discuss how their strategy for completing the table may have differed from that for the double number line. Ratio tables and double number lines allow students to determine the constant value that is multiplied to any x-value to arrive at its corresponding y-value. 3. Complete questions 3-6 for SOL 7.10 Task 1. Use the key and guiding questions provided to discuss the responses. Discuss how Proportional relationships can be confirmed with the equivalency of any two ratios from a ratio table. Reiterate that halving and doubling are intuitively understood by students. Discuss how unit rates can be used to determine any unknown quantity of a ratio within a proportion. And how unit rates relate the change in y compared to the change in x. 4. Display the blank graph for question #7 and question participants to create the graph that represents David’s distance walked over time. Discuss how slope triangles are a visual way for students to notice the rate of change (unit rate) between any two points on a graph. Discuss questions 8 – 19 on SOL 7.10 Task 1. Use the key and guiding questions provided to discuss the responses. Discuss how the rate of change (unit rate) between linear points represents slope. Be sure to point out that the rate of change, also known as slope, is represented as m in the equation y = mx.   SOL 7.10 Task 2   1. Provide the participants with SOL 7.10 Task 2. Have the participants read the context of the problem and complete the table of values. Have participants share with a shoulder partner whether or not they think that the situation of cost compared to the number of games played represents a proportional relationship and why. Have some pairs share out their responses to the whole group. Discuss how a non-proportional relationship could have an additive relationship between the *x*-value and its corresponding *y*-value. 2. Ask the participants to create a graph to represent the relationship (question #3). Ask the participants why the graph does not go through the origin. Discuss how the point at which a line intersects the *y*-axis is known as the *y*-intercept, b. Discuss that in grade 7 students will only be expected to example additive relationships represented by the equation *y* = *x* + *b*. Have participants complete questions 4 – 8 for SOL 7.10 Task 2. Use the key and guiding questions provided to discuss the responses. 3. Have participants work with a partner. Have one person create the graph for question #9 while the other person provides step by step directions for doing so. Discuss as a whole group how the each pair of participants went about explaining how to graph the line represented by y = x – 4. Discuss how the concept of Integer operations will be important for students to recognize that *y* = *x* - 4 is equivalent to *y* = *x* + (-4) when determining the *y*-intercept. Use the key and guiding questions provided to discuss the responses. 4. Revisit the essential question for Module 2e. 5. Reflection: Discuss *NCTM Principles to Actions* in the context of the two tasks just completed. Have the participants consider which of the mathematics teaching practice were evident in the tasks. Have the participants highlight the specific practices and discuss what evidence from the tasks promotes the practices designated. | * [SOL 7.10 Tasks 1 & 2](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/sol-7-10-tasks1-2.docx) * [SOL 7.10 Tasks 1 & 2 KEY](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/sol-7-10-key.docx) |
| 45 minutes | | **Part F: Linear Functions (Grade 8)**  **Essential Question: What instructional tasks can be used to promote the study of linear functions addressing the new idea of negative and zero slope and making connections between the various representations? (PowerPoint Slides 39-42)** |  |
|  | | SOL 8.16 Task 1   1. Provide the participants with SOL 8.16 Task 1. Have the participants work with a partner to complete questions #1 – 4. Use the key and guiding questions provided to discuss the responses. Discuss that grade 8 is the middle school students’ first look at lines with negative slope. It is helpful for students to understand the concept of a decreasing rate by examining it through the context of a practical problem and then connect to the graphical representation. 2. Have participants complete questions 7 – 10. Use the key and guiding questions provided to discuss the responses. Be sure to discuss how the y-intercept represents a unique relationship between the independent and dependent variables for contextual problems, but that all points on a line can be interpreted with meaning when given a contextual problem. When a line with a negative slope intersects the x-axis, this represents another unique relationship between the independent and dependent variable for contextual problems.   SOL 8.16 Task 2   1. Discuss the context for SOL 8.16 Task 2. Have the participants complete the table of values for each graph provided. Discuss questions 3 -5 with participants using the key and guiding questions to promote discourse. Encourage participants to use slope triangles in question #4 to find the slope. Students should not be encouraged to use the slope formula to determine slope until Algebra I. The parameters provided in the grade 8 standards are intended to promote a conceptual understanding of slope and linear functions. 2. Have participants work in pairs to complete questions 6 - 10. Use the key and guiding questions provided to discuss the responses. A horizontal line with zero-slope can be a misconception to students. The *y*- value does not change as the *x*-value changes may be incorrectly perceived by students as a constant rate of change. Exploring negative slope through tables in graphs provides a conceptual understanding without the need for the use of the slope formula. Undefined slope is not addressed until Algebra I when the slope formula will be used. 3. Revisit the essential question for Module 2F. 4. Reflection Mocule 2: Discuss the NCTM Principles to Actions in the context of the two tasks just completed. Have the participants consider which of the mathematics teaching practice were evident in the tasks. Have the participants highlight the specific practices and discuss what evidence from the tasks promotes the practices designated. | * [SOL 8.16 Task 1 & 2](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/sol-8-16-tasks1-2.docx) * [SOL 8.16 Task 1 & 2 KEY](http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2017/6-8/sol-8-16-key.docx) |

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| **30 minutes total** | **Module 3: Support for Implementation**  **Essential Questions: In what ways can I support my colleagues in implementation of the 2016 Standards of Learning? What resources are available from VDOE to assist with implementation? (PowerPoint Slides 43-46)** |  |
|  | 1. Discuss with participants that the VDOE has online resources to support the implementation of the 2016 Standards of Learning. Discuss that the standards documents, curriculum framework documents, and Crosswalk (Summary of Revisions) documents are all available online. There is also a video playlist available to provide an overview of the 2016 *Mathematics Standards of Learning* and information about the progression, implementation and resources. 2. Discuss that there are select progressions, narrated PowerPoint presentations summarizing the changes (grades K-8, Algebra I, Geometry, and Algebra II), and the resources for this Institute that will be available in late April on the VDOE 3. Emphasize the 2016 *Mathematics Standards of Learning* Implementation timeline (Slide 45).    * What does the curriculum development mean for the rest of the 2016-2017 school year?    * What does the Crossover Year mean for the 2017-2018 school year?    * What does Full Implementation Year mean for the 2018-2019 school year? 4. Closure – What are your next steps in preparing to implement the 2016 Mathematics Standards of Learning? | [2016 Mathematics Standards of Learning Documents](http://www.doe.virginia.gov/testing/sol/standards_docs/mathematics/2016/index.shtml) |