6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

Understanding the Standard

- A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities. Ratios are used in practical situations when there is a need to compare quantities.
- In the elementary grades, students are taught that fractions represent a part-to-whole relationship. However, fractions may also express a measurement, an operator (multiplication), a quotient, or a ratio. Examples of fraction interpretations include:
  - Fractions as parts of wholes: $\frac{3}{4}$ represents three parts of a whole, where the whole is separated into four equal parts.
  - Fractions as measurement: the notation $\frac{3}{4}$ can be interpreted as three one-fourths of a unit.
  - Fractions as an operator: $\frac{3}{4}$ represents a multiplier of three-fourths of the original magnitude.
  - Fractions as a quotient: $\frac{3}{4}$ represents the result obtained when three is divided by four.
  - Fractions as a ratio: $\frac{3}{4}$ is a comparison of 3 of a quantity to the whole quantity of 4.
- A ratio may be written using a colon (a:b), the word *to* (a to b), or fraction notation $\left(\frac{a}{b}\right)$.
- The order of the values in a ratio is directly related to the order in which the quantities are compared.
  - Example: In a certain class, there is a ratio of 3 girls to 4 boys (3:4). Another comparison that could represent the relationship between these quantities is the ratio of 4 boys to 3 girls (4:3). Both ratios give the same information about the number of girls and boys in the class, but they are distinct ratios. When you switch the order of comparison (girls to boys vs. boys to girls), there are different ratios being expressed.
- Fractions may be used when determining equivalent ratios.
  - Example: The ratio of girls to boys in a class is 3:4, this can be interpreted as:

$$\text{number of girls} = \frac{3}{4} \cdot \text{number of boys.}$$

In a class with 16 boys, number of girls = $\frac{3}{4} \cdot (16) = 12$ girls.

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Represent a relationship between two quantities using ratios.
- Represent a relationship in words that makes a comparison by using the notations $\frac{a}{b}$, *a*:*b*, and *a to b.*
- Create a relationship in words for a given ratio expressed symbolically.

6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.

Understanding the Standard	Essential Knowledge and Skills

- Example: A similar comparison could compare the ratio of boys to girls in the class as being 4:3, which can be interpreted as:

  $$\text{number of boys} = \frac{4}{3} \cdot \text{number of girls.}$$

  In a class with 12 girls, number of boys = $\frac{4}{3} \cdot (12) = 16$ boys.

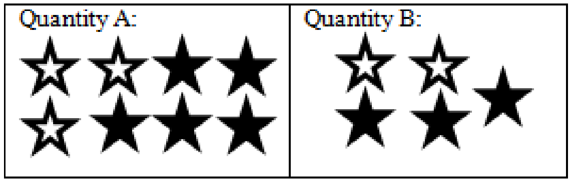
- A ratio can compare two real-world quantities (e.g., miles per gallon, unit rate, and circumference to diameter of a circle).
- Ratios may or may not be written in simplest form.
- A ratio can represent different comparisons within the same quantity or between different quantities.

Ratio	Comparison
part-to-whole (within the same quantity)	compare part of a whole to the entire whole
part-to-part (within the same quantity)	compare part of a whole to another part of the same whole
whole-to-whole (different quantities)	compare all of one whole to all of another whole
part-to-part (different quantities)	compare part of one whole to part of another whole

6.1 **The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*.**

Understanding the Standard	Essential Knowledge and Skills

- Examples: Given Quantity A and Quantity B, the following comparisons could be expressed.



Ratio	Example	Ratio Notation(s)
part-to-whole (within the same quantity)	compare the number of unfilled stars to the total number of stars in Quantity A	3:8; 3 to 8; or $\frac{3}{8}$
part-to-part [1] (within the same quantity)	compare the number of unfilled stars to the number of filled stars in Quantity A	3:5 or 3 to 5
whole-to-whole [1] (different quantities)	compare the number of stars in Quantity A to the number of stars in Quantity B	8:5 or 8 to 5
part-to-part [1] (different quantities)	compare the number of unfilled stars in Quantity A to the number of unfilled stars in Quantity B	3:2 or 3 to 2

[1] Part-to-part comparisons and whole-to-whole comparisons are ratios that are not typically represented in fraction notation except in certain contexts, such as determining whether two different ratios are equivalent.

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) compare and order numbers greater than zero written in scientific notation;*
c) compare and order rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• Negative exponents for powers of 10 are used to represent numbers between 0 and 1. (e.g., $10^{-3} = \frac{1}{10^3} = 0.001$). • Negative exponents for powers of 10 can be investigated through patterns such as: $10^{2} = 100$ $10^{1} = 10$ $10^{0} = 1$ $10^{-1} = \frac{1}{10^1} = \frac{1}{10} = 0.1$ $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$ • Percent means “per 100” or how many “out of 100”; percent is another name for hundredths. • A percent is a ratio in which the denominator is 100. A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100 (e.g., $\frac{3}{5} = \frac{60}{100} = 0.60 = 60\%$). • Scientific notation should be used whenever the situation calls for use of very large or very small numbers. • A number written in scientific notation is the product of two factors — a decimal greater than or equal to 1 but less than 10, and a power of 10 (e.g., $3.1 \times 10^5 = 310{,}000$ and $2.85 \times 10^{-4} = 0.000285$). • The set of integers includes the set of whole numbers and their opposites, {…-2, -1, 0, 1, 2…}. Zero has no opposite and is neither positive nor negative. • The opposite of a positive number is negative and the opposite of a negative number is positive.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize powers of 10 with negative exponents by examining patterns. (a) • Represent a power of 10 with a negative exponent in fraction and decimal form. (a) • Convert between numbers greater than 0 written in scientific notation and decimals. (b) • Compare and order no more than four numbers greater than 0 written in scientific notation. Ordering may be in ascending or descending order. (b) • Compare and order no more than four rational numbers expressed as integers, fractions (proper or improper), mixed numbers, decimals, and percents. Fractions and mixed numbers may be positive or negative. Decimals may be positive or negative and are limited to the thousandths place. Ordering may be in ascending or descending order. (c) • Identify the perfect squares from 0 to 400. (d) • Determine the positive square root of a perfect square from 0 to 400. (d) • Demonstrate absolute value using a number line. (e)

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) compare and order numbers greater than zero written in scientific notation;*
c) compare and order rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• The set of rational numbers includes the set of all numbers that can be expressed as fractions in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* does not equal zero. The decimal form of a rational number can be expressed as a terminating or repeating decimal. A few examples of rational numbers are: $\sqrt{25}$, $\frac{1}{4}$, -2.3, 82, 75%, $4.\overline{59}$. • Rational numbers may be expressed as positive and negative fractions or mixed numbers, positive and negative decimals, integers and percents. • Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). Fractions can be positive or negative. • Equivalent relationships among fractions, decimals, and percents may be determined by using concrete materials and pictorial representations (e.g., fraction bars, base ten blocks, fraction circles, colored counters, cubes, decimal squares, shaded figures, shaded grids, number lines and calculators). • Negative numbers lie to the left of zero and positive numbers lie to the right of zero on a number line. • Smaller numbers always lie to the left of larger numbers on the number line. • A perfect square is a whole number whose square root is an integer. Zero (a whole number) is a perfect square. (e.g., $36 = 6 \cdot 6 = 6^2$).	• Determine the absolute value of a rational number. (e) • Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle to solve practical problems. (e)

7.1 The student will

a) investigate and describe the concept of negative exponents for powers of ten;
b) compare and order numbers greater than zero written in scientific notation;*
c) compare and order rational numbers;*
d) determine square roots of perfect squares;* and
e) identify and describe absolute value of rational numbers.

*On the state assessment, items measuring this objective are assessed without the use of a calculator.

Understanding the Standard	Essential Knowledge and Skills
• A square root of a number is a number which, when multiplied by itself, produces the given number (e.g., $\sqrt{121}$ is 11 since $11 \cdot 11 = 121$). • The symbol $\sqrt{\ }$ may be used to represent a non-negative (principal) square root. Students in grade 8 mathematics will explore the negative square root of a number, denoted $-\sqrt{\ }$. • The square root of a number can be represented geometrically as the length of a side of a square. • Squaring a number and taking a square root are inverse operations. • The absolute value of a number is the distance from 0 on the number line regardless of direction. Distance is positive (e.g., $\left\lvert -\frac{1}{2} \right\rvert = \frac{1}{2}$). • The absolute value of zero is zero.	

8.1 The student will compare and order real numbers.

Understanding the Standard	Essential Knowledge and Skills
• Real numbers can be represented as integers, fractions (proper or improper), decimals, percents, numbers written in scientific notation, radicals, and π. It is often useful to convert numbers to be compared and/or ordered to one representation (e.g., fractions, decimals or percents). • Proper fractions, improper fractions, and mixed numbers are terms often used to describe fractions. A proper fraction is a fraction whose numerator is less than the denominator. An improper fraction is a fraction whose numerator is equal to or greater than the denominator. An improper fraction may be expressed as a mixed number. A mixed number is written with two parts: a whole number and a proper fraction (e.g., $3\frac{5}{8}$). Fractions can have a positive or negative value. • The density property states that between any two real numbers lies another real number. For example, between 3 and 5 we can find 4; between 4.0 and 4.2 we can find 4.16; between 4.16 and 4.17 we can find 4.165; between 4.165 and 4.166 we can find 4.1655, etc. Thus, we can always find another number between two numbers. Students are not expected to know the term *density property* but the concept allows for a deeper understanding of the set of real numbers. • Scientific notation is used to represent very large or very small numbers. • A number written in scientific notation is the product of two factors: a decimal greater than or equal to one but less than 10 multiplied by a power of 10 (e.g., $3.1 \times 10^5 = 310,000$ and $3.1 \times 10^{-5} = 0.000031$). • Any real number raised to the zero power is 1. The only exception to this rule is zero itself. Zero raised to the zero power is undefined.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and order no more than five real numbers expressed as integers, fractions (proper or improper), decimals, mixed numbers, percents, numbers written in scientific notation, radicals, and π. Radicals may include both positive and negative square roots of values from 0 to 400. Ordering may be in ascending or descending order. • Use rational approximations (to the nearest hundredth) of irrational numbers to compare and order, locating values on a number line. Radicals may include both positive and negative square roots of values from 0 to 400 yielding an irrational number.

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;

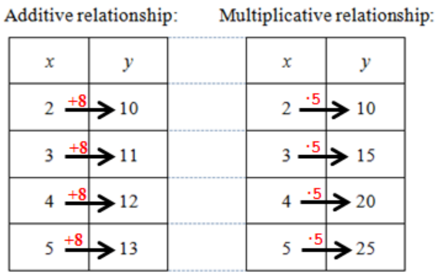
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

- A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities.
- Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 4:2 would be equivalent to the ratio 8:4, since each value in the first ratio could be multiplied by 2 to obtain the second ratio.
- A proportional relationship consists of two quantities where there exists a constant number (constant of proportionality) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.
- Proportional thinking requires students to thinking multiplicatively, versus additively. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, because context can help students to see the relationship. Students will explore algebraic representations of additive relationships in grade seven.
  - Example:



Additive relationship:

x	y
2 $\xrightarrow{+8}$	10
3 $\xrightarrow{+8}$	11
4 $\xrightarrow{+8}$	12
5 $\xrightarrow{+8}$	13

Multiplicative relationship:

x	y
2 $\xrightarrow{\cdot 5}$	10
3 $\xrightarrow{\cdot 5}$	15
4 $\xrightarrow{\cdot 5}$	20
5 $\xrightarrow{\cdot 5}$	25

Essential Knowledge and Skills

The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

- Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio. (a)
- Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a practical situation. (a)
- Identify the unit rate of a proportional relationship represented by a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b)
- Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate. Unit rates are limited to positive values. (b)
- Determine whether a proportional relationship exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (c)
- Determine whether a proportional relationship exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c)

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;

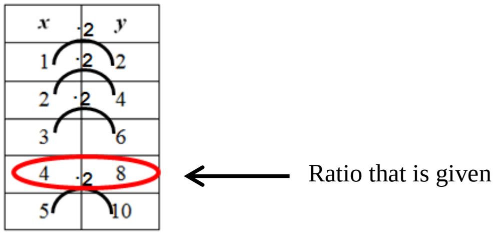
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

  - In the additive relationship, *y* is the result of adding 8 to *x.*
  - In the multiplicative relationship, *y* is the result of multiplying 5 times *x*.
  - The ordered pair (2, 10) is a quantity in both relationships; however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.
- Students have had experiences with tables of values (input/output tables that are additive and multiplicative) in elementary grades.
- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios. A constant exists that can be multiplied by the measure of one quantity to get the measure of the other quantity for every ratio pair. The same proportional relationship exists between each pair of quantities in a ratio table.
  - Example: Given that the ratio of *y* to *x* in a proportional relationship is 8:4, create a ratio table that includes three additional equivalent ratios.



x	y
1	2
2	4
3	6
4	8
5	10

Students have had experience with tables of values (input/output tables) in elementary grades and the concept of a ratio table should be connected to their prior knowledge of representing number patterns in tables.

Essential Knowledge and Skills

- Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values. (d)

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine whether a proportional relationship exists between two quantities; and
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

- A rate is a ratio that involves two different units and how they relate to each other. Relationships between two units of measure are also rates (e.g., inches per foot).
- A unit rate describes how many units of the first quantity of a ratio correspond to one unit of the second quantity.
  - Example: If it costs $10 for 5 items at a store (a ratio of 10:5 comparing cost to number of items), then the unit rate would be $2.00/per item (a ratio of 2:1 comparing cost to number of items).

Unit Rate
Given ratio

# of items (*x*)	1	2	5	10
Cost in $ (*y*)	$2.00	$4.00	$10.00	$20.00

- Any ratio can be converted into a unit rate by writing the ratio as a fraction and then dividing the numerator and denominator each by the value of the denominator. Example: It costs $8 for 16 gourmet cookies at a bake sale. What is the price per cookie (unit rate) represented by this situation?

$$\frac{8}{16} = \frac{8 \div 16}{16 \div 16} = \frac{0.5}{1}$$

So, it would cost $0.50 per cookie, which would be the unit rate.

  - Example: $\frac{8}{16}$ and 40 to 10 are ratios, but are not unit rates. However, $\frac{0.5}{1}$ and 4 to 1 are unit rates.
- Students in grade six should build a conceptual understanding of proportional relationships and unit rates before moving to more abstract representations and complex computations in higher grade levels. Students are not expected to use formal calculations for slope and unit rates (e.g., slope formula) in grade six.

Essential Knowledge and Skills

6.12 **The student will**

a) represent a proportional relationship between two quantities, including those arising from practical situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

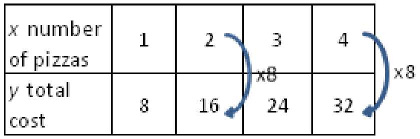
c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

- Example of a proportional relationship:

  Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This ratio table represents the cost (y) per number of pizzas ordered (x).



x number of pizzas	1	2	3	4
y total cost	8	16	24	32

  In this relationship, the ratio of y (cost in $) to x (number of pizzas) in each ordered pair is the same:

$$\frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4}$$

- Example of a non-proportional relationship:

  Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.

x number of pizzas	1	2	3	4
y total cost	10	17	24	31

  The ratios represented in the table above are not equivalent.

  In this relationship, the ratio of y to x in each ordered pair is not the same:

$$\frac{10}{1} \neq \frac{17}{2} \neq \frac{24}{3} \neq \frac{31}{4}$$

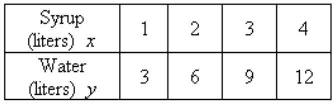
Essential Knowledge and Skills

6.12 The student will
- **a) represent a proportional relationship between two quantities, including those arising from practical situations;**
- **b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;**
- **c) determine whether a proportional relationship exists between two quantities; and**
- **d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.**

Understanding the Standard

Other non-proportional relationships will be studied in later mathematics courses.

- Proportional relationships can be described verbally using the phrases “for each,” “for every,” and “per.”
- Proportional relationships involve collections of pairs of equivalent ratios that may be graphed in the coordinate plane. The graph of a proportional relationship includes ordered pairs (*x*, *y*) that represent pairs of values that may be represented in a ratio table.
- Proportional relationships can be expressed using verbal descriptions, tables, and graphs.
  - Example: (verbal description) To make a drink, mix 1 liter of syrup with 3 liters of water. If *x* represents how many liters of syrup are in the mixture and *y* represents how many liters of water are in the mixture, this proportional relationship can be represented using a ratio table:



Syrup (liters) x	1	2	3	4
Water (liters) y	3	6	9	12

The ratio of the amount of water (*y*) to the amount of syrup (*x*) is 3:1. Additionally, the proportional relationship may be graphed using the ordered pairs in the table.

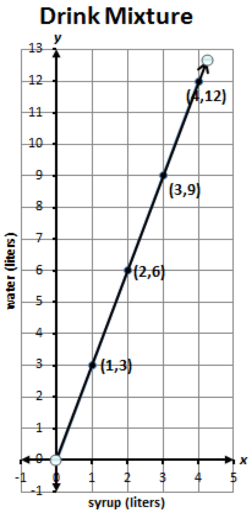
Essential Knowledge and Skills

6.12 The student will

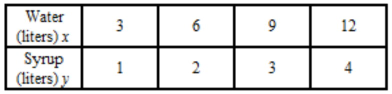
- **a) represent a proportional relationship between two quantities, including those arising from practical situations;**
- **b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;**
- **c) determine whether a proportional relationship exists between two quantities; and**
- **d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.**

Understanding the Standard	Essential Knowledge and Skills
(see below)	

Understanding the Standard



- The representation of the ratio between two quantities may depend upon the order in which the quantities are being compared.
  - Example: In the mixture example above, we could also compare the ratio of the liters of syrup per liters of water, as shown:



Water (liters) x	3	6	9	12
Syrup (liters) y	1	2	3	4

In this comparison, the ratio of the amount of syrup (y) to the amount of water (x) would be 1:3.

Essential Knowledge and Skills

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;

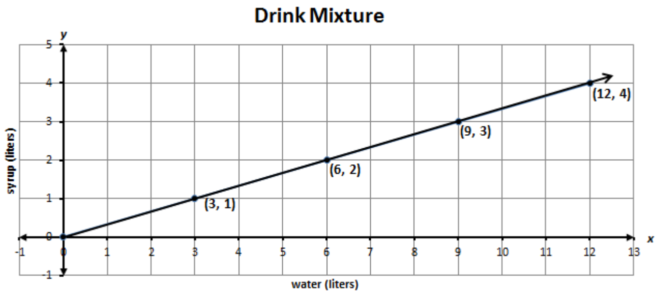
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

The graph of this relationship could be represented by:



Students should be aware of how the order in which quantities are compared affects the way in which the relationship is represented as a table of equivalent ratios or as a graph.

- Double number line diagrams can also be used to represent proportional relationships and create collections of pairs of equivalent ratios.
  - Example:

In this proportional relationship, there are three liters of water for each liter of syrup represented on the number lines.

Essential Knowledge and Skills

6.12 **The student will**

a) represent a proportional relationship between two quantities, including those arising from practical situations;

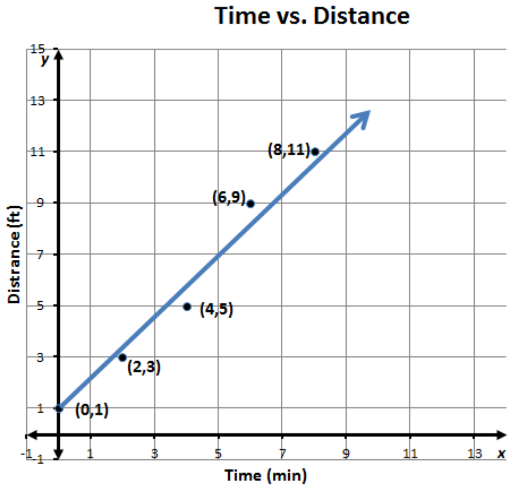
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard

- A graph representing a proportional relationship includes ordered pairs that lie in a straight line that, if extended, would pass through (0, 0), creating a pattern of horizontal and vertical increases. The context of the problem and the type of data being represented by the graph must be considered when determining whether the points are to be connected by a straight line on the graph.
  - Example of the graph of a non-proportional relationship:



The relationship of distance (*y*) to time (*x*) is non-proportional. The ratio of *y* to *x* for each ordered pair is not equivalent. That is,

$$\frac{11}{8} \neq \frac{9}{6} \neq \frac{5}{4} \neq \frac{3}{2} \neq \frac{1}{0}$$

Essential Knowledge and Skills

6.12 The student will

a) represent a proportional relationship between two quantities, including those arising from practical situations;

b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;

c) determine whether a proportional relationship exists between two quantities; and

d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Understanding the Standard	Essential Knowledge and Skills
The points of the graph do not lie in a straight line. Additionally, the line does not pass through the point (0, 0), thus the relationship of *y* to *x* cannot be considered proportional.	

- Practical situations that model proportional relationships can typically be represented by graphs in the first quadrant, since in most cases the values for *x* and *y* are not negative.
- Unit rates are not typically negative in practical situations involving proportional relationships.
- A unit rate could be used to find missing values in a ratio table.
  - Example: A store advertises a price of $25 for 5 DVDs. What would be the cost to purchase 2 DVDs? 3 DVDs? 4 DVDs?

# DVDs	1	2	3	4	5
Cost	$5	?	?	?	$25

The ratio of $25 per 5 DVDs is also equivalent to a ratio of $5 per 1 DVD, which would be the unit rate for this relationship. This unit rate could be used to find the missing value of the ratio table above. If we multiply the number of DVDs by a constant multiplier of 5 (the unit rate) we obtain the total cost. Thus, 2 DVDs would cost $10, 3 DVDs would cost $15, and 4 DVDs would cost $20.

7.10 The student will

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y* = *mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard	Essential Knowledge and Skills
• When two quantities, *x* and *y*, vary in such a way that one of them is a constant multiple of the other, the two quantities are “proportional”. A model for that situation is *y* = *mx* where *m* is the slope or rate of change. Slope may also represent the unit rate of a proportional relationship between two quantities, also referred to as the constant of proportionality or the constant ratio of *y* to *x*. • The slope of a proportional relationship can be determined by finding the unit rate. Example: The ordered pairs (4, 2) and (6, 3) make up points that could be included on the graph of a proportional relationship. Determine the slope, or rate of change, of a line passing through these points. Write an equation of the line representing this proportional relationship. *x* / *y*: 4, 2; 6, 3 The slope, or rate of change, would be $\frac{1}{2}$ or 0.5 since the *y*-coordinate of each ordered pair would result by multiplying $\frac{1}{2}$ times the *x*-coordinate. This would also be the unit rate of this proportional relationship. The ratio of *y* to *x* is the same for each ordered pair. That is, $\frac{y}{x} = \frac{2}{4} = \frac{3}{6} = \frac{1}{2} = 0.5$ The equation of a line representing this proportional relationship of *y* to *x* is $y = \frac{1}{2}x$ or *y* = 0.5*x*. • The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change of the line. slope = $\frac{change\ in\ y}{change\ in\ x} = \frac{vertical\ change}{horizontal\ change}$	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the slope, *m*, as rate of change in a proportional relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form *y* = *mx* to represent the relationship. Slope will be limited to positive values. (a) • Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, *m*, as rate of change. Slope will be limited to positive values. (b) • Graph a line representing a proportional relationship between two quantities given the equation of the line in the form *y* = *mx*, where *m* represents the slope as rate of change. Slope will be limited to positive values. (b) • Determine the *y*-intercept, *b*, in an additive relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form *y* = *x* + *b*, *b* ≠ 0, to represent the relationship. (c)

7.10 The student will

- **a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y = mx* to represent the relationship;**
- **b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y = mx* form where *m* represents the slope as rate of change.**
- **c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y = x + b* to represent the relationship;**
- **d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y = x + b*, where *b* represents the *y*-intercept; and**
- **e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard

- The graph of the line representing a proportional relationship will include the origin (0, 0).
- A proportional relationship between two quantities can be modeled given a practical situation. Representations may include verbal descriptions, tables, equations, or graphs. Students may benefit from an informal discussion about independent and dependent variables when modeling practical situations. Grade eight mathematics formally addresses identifying dependent and independent variables.
  - Example (using a table of values): Cecil walks 2 meters every second (verbal description). If *x* represents the number of seconds and *y* represents the number of meters he walks, this proportional relationship can be represented using a table of values:

x (seconds)	1	2	3	4
y (meters)	2	4	6	8

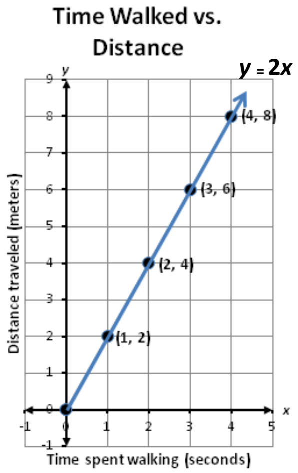
This proportional relationship could be represented using the equation $y = 2x$, since he walks 2 meters for each second of time. That is, $\frac{y}{x} = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = 2$, the unit rate (constant of proportionality) is 2 or $\frac{2}{1}$. The same constant ratio of *y* to *x* exists for every ordered pair. This proportional relationship could be represented by the following graph:

Essential Knowledge and Skills

- Graph a line representing an additive relationship ($y = x + b$, $b \neq 0$) between two quantities, given an ordered pair on the line and the *y*-intercept (*b*). The *y*-intercept (*b*) is limited to integer values and slope is limited to 1. (d)
- Graph a line representing an additive relationship between two quantities, given the equation in the form $y = x + b$, $b \neq 0$. The *y*-intercept (*b*) is limited to integer values and slope is limited to 1. (d)
- Make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs. (e)

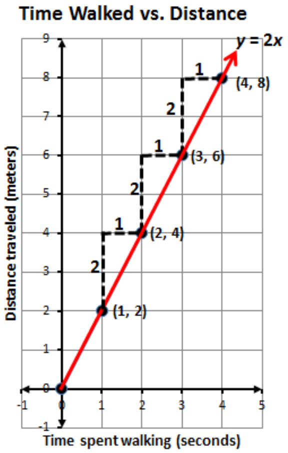
7.10 **The student will**

- **a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;**
- **b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y* = *mx* form where *m* represents the slope as rate of change.**
- **c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;**
- **d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and**
- **e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard	**Essential Knowledge and Skills**
 • A graph of a proportional relationship can be created by graphing ordered pairs generated in a table of values (as shown above), or by observing the rate of change or slope of the relationship and using slope triangles to graph ordered pairs that satisfy the relationship given. – Example (using slope triangles): Cecil walks 2 meters every second. If *x* represents the number of seconds and *y* represents the number of meters he walks, this proportional relationship can be represented graphically using slope triangles.	

7.10 **The student will**

- **a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;**
- **b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y* = *mx* form where *m* represents the slope as rate of change.**
- **c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;**
- **d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and**
- **e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard	Essential Knowledge and Skills
 The rate of change from (1, 2) to (2, 4) is 2 units up (the change in *y*) and 1 unit to the right (the change in *x*), $\frac{2}{1}$ or 2. Thus, the slope of this line is 2. Slope triangles can be used to generate points on a graph that satisfy this relationship.	

7.10 The student will

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y* = *mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b*, in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- Proportional thinking requires students to thinking multiplicatively. However, the relationship between two quantities is not always proportional. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, since context can help students to see the relationship.
  - Example:

Additive relationship:

x	y
2 $\xrightarrow{+8}$	10
3 $\xrightarrow{+8}$	11
4 $\xrightarrow{+8}$	12
5 $\xrightarrow{+8}$	13

Multiplicative relationship:

x	y
2 $\xrightarrow{\cdot 5}$	10
3 $\xrightarrow{\cdot 5}$	15
4 $\xrightarrow{\cdot 5}$	20
5 $\xrightarrow{\cdot 5}$	25

In the additive relationship, *y* is the result of adding 8 to *x.*
In the multiplicative relationship, *y* is the result of multiplying 5 times *x*.
The ordered pair (2, 10) is a quantity in both relationships, however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

Essential Knowledge and Skills

7.10 The student will

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y* = *mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- Two quantities, *x* and *y*, have an additive relationship when a constant value, *b*, exists where $y = x + b$, where $b \neq 0$. An additive relationship is not proportional and its graph does not pass through (0, 0). Note that *b* can be a positive value or a negative value. When *b* is negative, the right side of the equation could be written using a subtraction symbol (e.g., if *b* is −5, then the equation $y = x - 5$ could be used).
  - Example: Thomas is four years older than his sister, Amanda (verbal description). The following table shows the relationship between their ages at given points in time.

Amanda's Age	4	5	6	11
Thomas' Age	8	9	10	15
	+4	+4	+4	+4

The equation that represents the relationship between Thomas' age and Amanda's age is $y = x + 4$. A graph of the relationship between their ages is shown below:

Essential Knowledge and Skills

7.10 The student will

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;

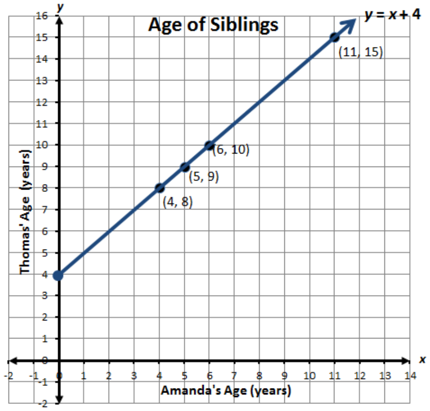
b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y* = *mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard



- Graphing a line given an equation can be addressed using different methods. One method involves determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. Another method involves using slope triangles to determine points on the line.
    - Example: Graph the equation $y = x - 1$.
      In order to graph the equation, we can create a table of values by substituting arbitrary values for x to determine coordinating values for y:

Essential Knowledge and Skills

7.10 The student will

a) determine the slope, *m*, as rate of change in a proportional relationship between two quantities and write an equation in the form *y* = *mx* to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in *y* = *mx* form where *m* represents the slope as rate of change.

c) determine the *y*-intercept, *b,* in an additive relationship between two quantities and write an equation in the form *y* = *x* + *b* to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the *y*-intercept and an ordered pair, or given the equation in the form *y* = *x* + *b*, where *b* represents the *y*-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

Understanding the Standard	Essential Knowledge and Skills

x	x - 1	y
-1	(-1) - 1	-2
0	(0) - 1	-1
1	(1) - 1	0
2	(2) - 1	1

These values can then be plotted as the points (-1, -2), (0, -1), (1, 0), and (2, 1) on a graph.

An equation written in *y* = *x* + *b* form provides information about the graph. If the equation is *y* = *x* – 1, then the slope, *m*, of the line is 1 or $\frac{1}{1}$ and the point where the line crosses the *y*-axis can be located at (0, -1). We also know,

slope = $m = \frac{change\ in\ y-value}{change\ in\ x-value} = \frac{+1}{+1}\ or\ \frac{-1}{-1}$

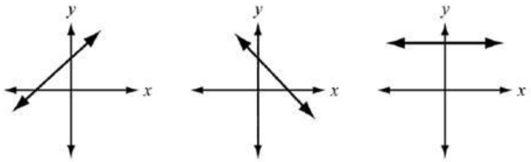
So we can plot some other points on the graph using this relationship between *y* and *x* values.

A table of values can be used to determine the graph of a line. The *y*-intercept is located on the *y*-axis which is where the *x*-coordinate is 0. The change in each *y*-value compared to the corresponding *x*-value can be verified by the patterns in the table of values.

	x	y	
+1	-1	-2	+1
+1	0	-1	+1
+1	1	0	+1
	2	1	

8.16 **The student will**

- **a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;**
- **b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;**
- **c) determine the independent and dependent variable, given a practical situation modeled by a linear function;**
- **d) graph a linear function given the equation in *y* = *mx* + *b* form; and**
- **e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.**

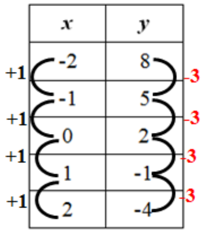
Understanding the Standard	Essential Knowledge and Skills
• A linear function is an equation in two variables whose graph is a straight line, a type of continuous function. • A linear function represents a situation with a constant rate. For example, when driving at a rate of 35 mph, the distance increases as the time increases, but the rate of speed remains the same. • Slope (*m*) represents the rate of change in a linear function or the "steepness" of the line. The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change. slope = $\frac{change\ in\ y}{change\ in\ x} = \frac{vertical\ change}{horizontal\ change}$ • A line is increasing if it rises from left to right. The slope is positive (i.e., $m > 0$). • A line is decreasing if it falls from left to right. The slope is negative (i.e., $m < 0$). • A horizontal line has zero slope (i.e., $m = 0$).  A line with a *positive slope* slants up to the right. A line with a *negative slope* slants down to the right. A line with a *slope of 0* is horizontal. • A discussion about lines with undefined slope (vertical lines) should occur with students in grade eight mathematics to compare undefined slope to lines with a defined slope. Further exploration of this concept will occur in Algebra I.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize and describe a line with a slope that is positive, negative, or zero (0). (a) • Given a table of values for a linear function, identify the slope and *y*-intercept. The table will include the coordinate of the *y*-intercept. (b) • Given a linear function in the form *y* = *mx* + *b*, identify the slope and *y*-intercept. (b) • Given the graph of a linear function, identify the slope and *y*-intercept. The value of the *y*-intercept will be limited to integers. The coordinates of the ordered pairs shown in the graph will be limited to integers. (b) • Identify the dependent and independent variable, given a practical situation modeled by a linear function. (c) • Given the equation of a linear function in the form *y* = *mx* + *b*, graph the function. The value of the *y*-intercept will be limited to integers. (d) • Write the equation of a linear function in the form *y* = *mx* + *b* given values for the slope, *m,* and the *y*-intercept or given a practical situation in which the slope, *m*, and *y*-intercept are described verbally.(e)

8.16 **The student will**

- **a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;**
- **b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;**
- **c) determine the independent and dependent variable, given a practical situation modeled by a linear function;**
- **d) graph a linear function given the equation in *y* = *mx* + *b* form; and**
- **e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard

- A linear function can be written in the form $y = mx + b$, where m represents the slope or rate of change in y compared to x, and b represents the y-intercept of the graph of the linear function. The y-intercept is the point at which the graph of the function intersects the y-axis and may be given as a single value, b, or as the location of a point $(0, b)$.
  - Example: Given the equation of the linear function $y = -3x + 2$, the slope is -3 or $\frac{-3}{1}$ and the y-intercept is 2 or (0, 2).
  - Example: The table of values represents a linear function.
    In the table, the point (0, 2) represents the y-intercept. The slope is determined by observing the change in each y-value compared to the corresponding change in the x-value.



x	*y*
-2	8
-1	5
0	2
1	-1
2	-4

slope = $m = \frac{change\ in\ y-value}{change\ in\ x-value} = \frac{-3}{+1} = -3$

- The slope, m, and y-intercept of a linear function can be determined given the graph of the function.

Essential Knowledge and Skills

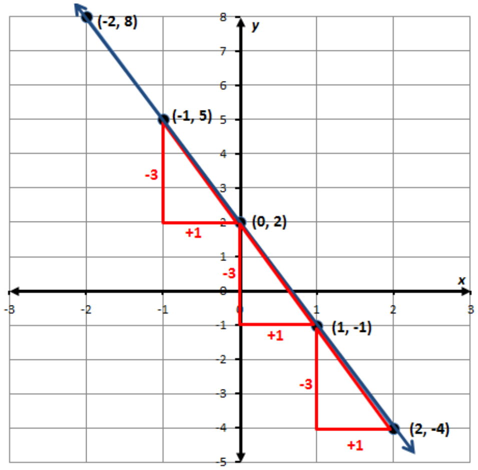
- Make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs. (e).

8.16 The student will

a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;
b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;
c) determine the independent and dependent variable, given a practical situation modeled by a linear function;
d) graph a linear function given the equation in *y* = *mx* + *b* form; and
e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- Example: Given the graph of the linear function, determine the slope and *y*-intercept.



Given the graph of a linear function, the *y*-intercept is found by determining where the line intersects the *y*-axis. The *y*-intercept would be 2 or located at the point (0, 2). The slope can be found by determining the change in each *y*-value compared to the change in each *x*-value. Here, we could use slope triangles to help visualize this:

slope = $m = \frac{change\ in\ y-value}{change\ in\ x-value} = \frac{-3}{+1} = -3$

- Graphing a linear function given an equation can be addressed using different methods. One method involves determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. Another method involves using slope triangles to

Essential Knowledge and Skills

8.16 **The student will**

- **a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;**
- **b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;**
- **c) determine the independent and dependent variable, given a practical situation modeled by a linear function;**
- **d) graph a linear function given the equation in *y* = *mx* + *b* form; and**
- **e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard

determine points on the line.

- Example: Graph the linear function whose equation is *y* = 5*x* − 1.
  In order to graph the linear function, we can create a table of values by substituting arbitrary values for *x* to determining coordinating values for *y*:

x	$5x - 1$	y
-1	5(-1) - 1	-6
0	5(0) - 1	-1
1	5(1) - 1	4
2	5(2) - 1	9

The values can then be plotted as points on a graph.

Knowing the equation of a linear function written in *y* = *mx* + *b* provides information about the slope and *y*-intercept of the function. If the equation is *y* = 5*x* − 1, then the slope, *m*, of the line is 5 or $\frac{5}{1}$ and the *y*-intercept is −1 and can be located at the point (0, −1). We can graph the line by first plotting the *y*-intercept. We also know,

slope = *m* = $\frac{change\ in\ y-value}{change\ in\ x-value} = \frac{+5}{+1}$

Other points can be plotted on the graph using the relationship between the *y* and *x* values.

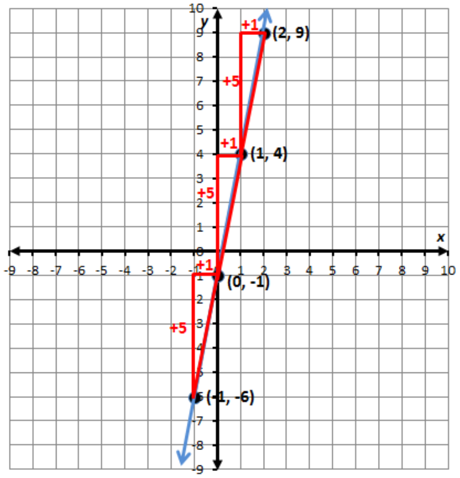
Slope triangles can be used to help locate the other points as shown in the graph below:

Essential Knowledge and Skills

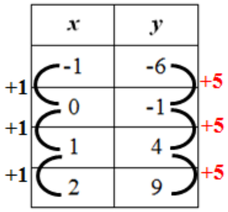
8.16 **The student will**

- **a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;**
- **b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;**
- **c) determine the independent and dependent variable, given a practical situation modeled by a linear function;**
- **d) graph a linear function given the equation in *y* = *mx* + *b* form; and**
- **e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.**

Understanding the Standard



- A table of values can be used in conjunction with using slope triangles to verify the graph of a linear function. The *y*-intercept is located on the *y*-axis which is where the *x*-coordinate is 0. The change in each *y*-value compared to the corresponding *x*-value can be verified by the patterns in the table of values.



x	*y*
-1	-6
0	-1
1	4
2	9

Essential Knowledge and Skills

8.16 The student will

a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;

b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in $y = mx + b$ form;

c) determine the independent and dependent variable, given a practical situation modeled by a linear function;

d) graph a linear function given the equation in $y = mx + b$ form; and

e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

Understanding the Standard

- The axes of a coordinate plane are generally labeled *x* and *y*; however, any letters may be used that are appropriate for the function.
- A function has values that represent the input (*x*) and values that represent the output (*y*). The independent variable is the input value.
- The dependent variable depends on the independent variable and is the output value.
- Below is a table of values for finding the approximate circumference of circles, $C = \pi d$, where the value of π is approximated as 3.14.

Diameter	Circumference
1 in.	3.14 in.
2 in.	6.28 in.
3 in.	9.42 in.
4 in.	12.56 in.

  - The independent variable, or input, is the diameter of the circle. The values for the diameter make up the domain.
  - The dependent variable, or output, is the circumference of the circle. The set of values for the circumference makes up the range.
- In a graph of a continuous function every point in the domain can be interpreted. Therefore, it is possible to connect the points on the graph with a continuous line because every point on the line answers the original question being asked.
- The context of a problem may determine whether it is appropriate for ordered pairs representing a linear relationship to be connected by a straight line. If the independent variable (*x*) represents a discrete quantity (e.g., number of people, number of tickets, etc.) then it is not appropriate to connect the ordered pairs with a straight line when graphing. If the independent variable (*x*)

Essential Knowledge and Skills

8.16 The student will

a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;

b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;

c) determine the independent and dependent variable, given a practical situation modeled by a linear function;

d) graph a linear function given the equation in *y* = *mx* + *b* form; and

e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

Understanding the Standard	Essential Knowledge and Skills
represents a continuous quantity (e.g., amount of time, temperature, etc.), then it is appropriate to connect the ordered pairs with a straight line when graphing. – Example: The function $y = 7x$ represents the cost in dollars (y) for x tickets to an event. The domain of this function would be discrete and would be represented by discrete points on a graph. Not all values for x could be represented and connecting the points would not be appropriate. – Example: The function $y = -2.5x + 20$ represents the number of gallons of water (y) remaining in a 20-gallon tank being drained for x number of minutes. The domain in this function would be continuous. There would be an x-value representing any point in time until the tank is drained so connecting the points to form a straight line would be appropriate (Note: the context of the problem limits the values that x can represent to positive values, since time cannot be negative.). • Functions can be represented as ordered pairs, tables, graphs, equations, physical models, or in words. Any given relationship can be represented using multiple representations. • The equation $y = mx + b$ defines a linear function whose graph (solution) is a straight line. The equation of a linear function can be determined given the slope, m, and the y-intercept, b. Verbal descriptions of practical situations that can be modeled by a linear function can also be represented using an equation. – Example: Write the equation of a linear function whose slope is $\frac{3}{4}$ and y-intercept is −4, or located at the point (0, −4). The equation of this line can be found by substituting the values for the slope, $m = \frac{3}{4}$, and the y-intercept, $b = -4$, into the general form of a linear function $y = mx + b$. Thus, the equation would be $y = \frac{3}{4}x - 4$.	

8.16 The student will

a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;

b) identify the slope and *y*-intercept of a linear function given a table of values, a graph, or an equation in *y* = *mx* + *b* form;

c) determine the independent and dependent variable, given a practical situation modeled by a linear function;

d) graph a linear function given the equation in *y* = *mx* + *b* form; and

e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

Understanding the Standard	Essential Knowledge and Skills
− Example: John charges a $30 flat fee to trouble shoot a personal watercraft that is not working properly and $50 per hour needed for any repairs. Write a linear function that represents the total cost, *y* of a personal watercraft repair, based on the number of hours, *x*, needed to repair it. Assume that there is no additional charge for parts. In this practical situation, the *y*-intercept, *b*, would be $30, to represent the initial flat fee to trouble shoot the watercraft. The slope, *m*, would be $50, since that would represent the rate per hour. The equation to represent this situation would be *y* = 50*x* + 30. • A proportional relationship between two variables can be represented by a linear function *y* = *mx* that passes through the point (0, 0) and thus has a *y*-intercept of 0. The variable *y* results from *x* being multiplied by *m,* the rate of change or slope. • The linear function *y* = *x* + *b* represents a linear function that is a non-proportional additive relationship. The variable *y* results from the value *b* being added to *x.* In this linear relationship, there is a y-intercept of *b,* and the constant rate of change or slope would be 1. In a linear function with a slope other than 1, there is a coefficient in front of the *x* term, which represents the constant rate of change, or slope. • Proportional relationships and additive relationships between two quantities are special cases of linear functions that are discussed in grade seven mathematics.	

6.7 The student will
a) derive π (pi);
b) solve problems, including practical problems, involving circumference and area of a circle; and
c) solve problems, including practical problems, involving area and perimeter of triangles and rectangles.

Understanding the Standard	Essential Knowledge and Skills
• The value of pi (π) is the ratio of the circumference of a circle to its diameter. Thus, the circumference of a circle is proportional to its diameter. • The calculation of determining area and circumference may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi (π) button on a calculator. • Experiences in deriving the formulas for area, perimeter, and volume using manipulatives such as tiles, one-inch cubes, graph paper, geoboards, or tracing paper, promote an understanding of the formulas and their use. • Perimeter is the path or distance around any plane figure. The perimeter of a circle is called the circumference. • The circumference of a circle is about three times the measure of its diameter. • The circumference of a circle is computed using $C = \pi d$ or $C = 2\pi r$, where d is the diameter and r is the radius of the circle. • The area of a closed curve is the number of nonoverlapping square units required to fill the region enclosed by the curve. • The area of a circle is computed using the formula $A = \pi r^2$, where r is the radius of the circle. • The perimeter of a square whose side measures s can be determined by multiplying 4 by s ($P = 4s$), and its area can be determined by squaring the length of one side ($A = s^2$). • The perimeter of a rectangle can be determined by computing the sum of twice the length and twice the width ($P = 2l + 2w$, or $P = 2(l + w)$), and its area can be determined by computing the product of the length and the width ($A = lw$). • The perimeter of a triangle can be determined by computing the sum of the side lengths ($P = a + b + c$), and its area can be determined by computing $\frac{1}{2}$ the product of base and the height ($A = \frac{1}{2}bh$).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Derive an approximation for pi (3.14 or $\frac{22}{7}$) by gathering data and comparing the circumference to the diameter of various circles, using concrete materials or computer models. (a) • Solve problems, including practical problems, involving circumference and area of a circle when given the length of the diameter or radius. (b) • Solve problems, including practical problems, involving area and perimeter of triangles and rectangles.(c)

7.4 The student will

a) describe and determine the volume and surface area of rectangular prisms and cylinders; and

b) solve problems, including practical problems, involving the volume and surface area of rectangular prisms and cylinders.

Understanding the Standard	Essential Knowledge and Skills
• A polyhedron is a solid figure whose faces are all polygons. • A rectangular prism is a polyhedron in which all six faces are rectangles. A rectangular prism has eight vertices and 12 edges. • A cylinder is a solid figure formed by two congruent parallel faces called bases joined by a curved surface. In this grade level, cylinders are limited to right circular cylinders. • A face is any flat surface of a solid figure. • The surface area of a prism is the sum of the areas of all 6 faces and is measured in square units. • The volume of a three-dimensional figure is a measure of capacity and is measured in cubic units. • Nets are two-dimensional representations of a three-dimensional figure that can be folded into a model of the three-dimensional figure. • A rectangular prism can be represented on a flat surface as a net that contains six rectangles — two that have measures of the length and width of the base, two others that have measures of the length and height, and two others that have measures of the width and height. The surface area of a rectangular prism is the sum of the areas of all six faces ($SA = 2lw + 2lh + 2wh$). • A cylinder can be represented on a flat surface as a net that contains two circles (the bases of the cylinder) and one rectangular region (the curved surface of the cylinder) whose length is the circumference of the circular base and whose width is the height of the cylinder. The surface area of the cylinder is the sum of the area of the two circles and the rectangle representing the curved surface ($SA = 2\pi r^2 + 2\pi rh$). • The volume of a rectangular prism is computed by multiplying the area of the base, B, (length times width) by the height of the prism ($V = lwh = Bh$). • The volume of a cylinder is computed by multiplying the area of the base, B, (πr^2) by the height of the cylinder ($V = \pi r^2 h = Bh$). • The calculation of determining surface area and volume may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi button on the calculator.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine the surface area of rectangular prisms and cylinders using concrete objects, nets, diagrams, and formulas. (a) • Determine the volume of rectangular prisms and cylinders using concrete objects, diagrams, and formulas. (a) • Determine if a practical problem involving a rectangular prism or cylinder represents the application of volume or surface area. (b) • Solve practical problems that require determining the surface area of rectangular prisms and cylinders. (b) • Solve practical problems that require determining the volume of rectangular prisms and cylinders. (b).

8.6 **The student will**

a) solve problems, including practical problems, involving volume and surface area of cones and square-based pyramids; and

b) describe how changing one measured attribute of a rectangular prism affects the volume and surface area.

Understanding the Standard	Essential Knowledge and Skills
• A polyhedron is a solid figure whose faces are all polygons. • Nets are two-dimensional representations of a three-dimensional figure that can be folded into a model of the three-dimensional figure. • Surface area of a solid figure is the sum of the areas of the surfaces of the figure. • Volume is the amount a container holds. • A rectangular prism is a polyhedron that has a congruent pair of parallel rectangular bases and four faces that are rectangles. A rectangular prism has eight vertices and twelve edges. In this course, prisms are limited to right prisms with bases that are rectangles. • The surface area of a rectangular prism is the sum of the areas of the faces and bases, found by using the formula $S.A. = 2lw + 2lh + 2wh$. All six faces are rectangles. • The volume of a rectangular prism is calculated by multiplying the length, width and height of the prism or by using the formula $V = lwh$. • A cube is a rectangular prism with six congruent, square faces. All edges are the same length. A cube has eight vertices and twelve edges. • A cone is a solid figure formed by a face called a base that is joined to a vertex (apex) by a curved surface. In this grade level, cones are limited to right circular cones. • The surface area of a right circular cone is found by using the formula, $S.A. = \pi r^2 + \pi rl$, where l represents the slant height of the cone. The area of the base of a circular cone is πr^2. • The volume of a cone is found by using $V = \frac{1}{3}\pi r^2 h$, where h is the height and πr^2 is the area of the base. • A square-based pyramid is a polyhedron with a square base and four faces that are triangles with a common vertex (apex) above the base. In this grade level, pyramids are limited to right regular pyramids with a square base. • The volume of a pyramid is $\frac{1}{3}Bh$, where B is the area of the base and h is the height.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Distinguish between situations that are applications of surface area and those that are applications of volume. (a) • Determine the surface area of cones and square-based pyramids by using concrete objects, nets, diagrams and formulas. (a) • Determine the volume of cones and square-based pyramids, using concrete objects, diagrams, and formulas. (a) • Solve practical problems involving volume and surface area of cones and square-based pyramids. (a) • Describe how the volume of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{4}, \frac{1}{3}, \frac{1}{2},$ 2, 3, or 4. (b) • Describe how the surface area of a rectangular prism is affected when one measured attribute is multiplied by a factor of $\frac{1}{2}$ or 2. (b)

8.6 The student will

a) solve problems, including practical problems, involving volume and surface area of cones and square-based pyramids; and

b) describe how changing one measured attribute of a rectangular prism affects the volume and surface area.

Understanding the Standard	Essential Knowledge and Skills
• The surface area of a pyramid is the sum of the areas of the triangular faces and the area of the base, found by using the formula $S.A. = \frac{1}{2}lp + B$ where l is the slant height, p is the perimeter of the base and B is the area of the base. • The volume of a pyramid is found by using the formula $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height. • The volume of prisms can be found by determining the area of the base and multiplying that by the height. • The formula for determining the volume of cones and cylinders are similar. For cones, you are determining $\frac{1}{3}$ of the volume of the cylinder with the same size base and height. The volume of a cone is found by using $V = \frac{1}{3}\pi r^2 h$. The volume of a cylinder is the area of the base of the cylinder multiplied by the height, found by using the formula, $V = \pi r^2 h$, where h is the height and πr^2 is the area of the base. • The calculation of determining surface area and volume may vary depending upon the approximation for pi. Common approximations for π include 3.14, $\frac{22}{7}$, or the pi button on the calculator. • When the measurement of one attribute of a rectangular prism is changed through multiplication or division the volume increases by the same factor by which the attribute increased. For example, if a prism has a volume of $2 \cdot 3 \cdot 4$, the volume is 24 cubic units. However, if one of the attributes is doubled, the volume doubles. That is, $2 \cdot 3 \cdot 8$, the volume is 48 cubic units or 24 doubled. • When one attribute of a rectangular prism is changed through multiplication or division, the surface area is affected differently than the volume. The formula for surface area of a rectangular prism is $2(lw) + 2(lh) + 2(wh)$ when the width is doubled then four faces are affected. For example, a rectangular prism with length = 7 in., width = 4 in., and height = 3 in. would have a surface area of $2(7 \cdot 4) + 2(7 \cdot 3) + 2(4 \cdot 3)$ or 122 square inches. If the height is doubled to 6 inches then the surface area would be found by $2(7 \cdot 4) + 2(7 \cdot 6) + 2(4 \cdot 6)$ or 188 square inches.	