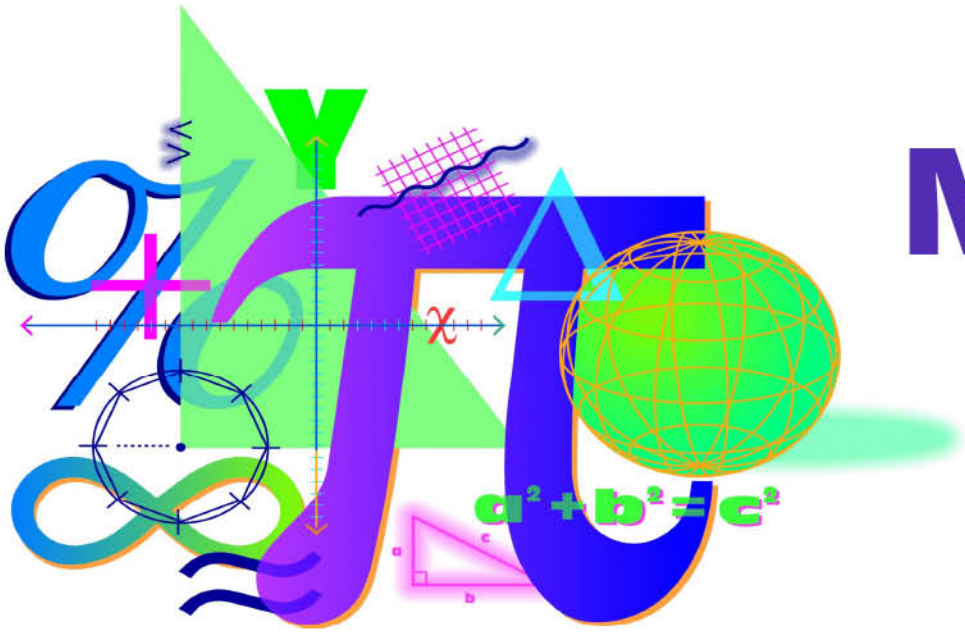

Mathematics
Standards of Learning

Curriculum Framework 2009

Grade 3

Board of Education
Commonwealth of Virginia

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The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia *Mathematics Standards of Learning* Curriculum Framework 2009
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships ("more than," "less than," and "equal to"), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

3.1 The student will

a) read and write six-digit numerals and identify the place value and value of each digit;

b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and

c) compare two whole numbers between 0 and 9,999, using symbols (>, <, or =) and words (*greater than, less than,* or *equal to*).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The structure of the Base-10 number system is based upon a simple pattern of tens, where each place is ten times the value of the place to its right. This is known as a ten-to-one place value relationship. • The structure of the Base-10 blocks is based on the ten-to-one place value relationship (e.g., 10 units make a long, 10 longs make a flat, 10 flats make a cube). • Place value refers to the value of each digit and depends upon the position of the digit in the number. In the number 7,864, the eight is in the hundreds place, and the value of the 8 is eight hundred. • Flexibility in thinking about numbers — or "decomposition" of numbers (e.g., 12,345 is 123 hundreds, 4 tens, and 5 ones) — is critical and supports understandings essential to multiplication and division. • Whole numbers may be written in a variety of formats: – Standard: 123,456; – Written: one hundred twenty-three thousand, four hundred fifty-six; and – Expanded: $(1 \times 100{,}000) + (2 \times 10{,}000) + (3 \times 1{,}000) + (4 \times 100) + (5 \times 10) + (6 \times 1)$.	**All students should** • Understand that knowledge of place value is essential when comparing numbers. • Understand the relationships in the place value system, where each place is ten times the value of the place to its right. • Understand that rounding gives an estimate to use when exact numbers are not needed for the situation. • Understand the relative magnitude of numbers by comparing numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and identify the place and value for each digit in a six-digit numeral, using Base-10 manipulatives (e.g., Base-10 blocks). • Use the patterns in the place value system to read and write numbers. • Read six-digit numerals orally. • Write six-digit numerals that are stated verbally or written in words. • Round a given whole number, 9,999 or less, to the nearest ten, hundred, and thousand. • Solve problems, using rounding of numbers, 9,999 or less, to the nearest ten, hundred, and thousand. • Determine which of two whole numbers between 0 and 9,999 is greater. • Determine which of two whole numbers between 0 and 9,999 is less. • Compare two whole numbers between 0 and 9,999, using the symbols >, <, or =. • Use the terms *greater than, less than*, and *equal to* when comparing two whole numbers.

3.1 The student will

a) read and write six-digit numerals and identify the place value and value of each digit;

b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and

c) compare two whole numbers between 0 and 9,999, using symbols (>, <, or =) and words (*greater than, less than,* or *equal to*).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Numbers are arranged into groups of three places called *periods* (ones, thousands, millions, and so on). Places within the periods repeat (hundreds, tens, ones). Commas are used to separate the periods. Knowing the place value and period of a number helps students find the value of a digit in any number as well as read and write numbers. • To read a whole number through the hundred thousands place, – read the digits to the first comma; – say the name of the period (e.g., "thousands"); then – read the last three digits, but do not say the name of the ones period. • Reading and writing large numbers should be related to numbers that have meanings (e.g., numbers found in the students' environment). Concrete materials, such as Base-10 blocks may be used to represent whole numbers through thousands. Larger numbers may be represented on place value charts. • Rounding is one of the estimation strategies that is often used to assess the reasonableness of a solution or to give an estimate of an amount. • Students should explore reasons for estimation, using practical experiences, and use rounding to solve practical situations.		

3.1 The student will

a) read and write six-digit numerals and identify the place value and value of each digit;

b) round whole numbers, 9,999 or less, to the nearest ten, hundred, and thousand; and

c) compare two whole numbers between 0 and 9,999, using symbols (>, <, or =) and words (*greater than, less than,* or *equal to*).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The concept of rounding may be introduced through the use of a number line. When given a number to round, locate it on the number line. Next, determine the multiple of ten, hundred, or thousand it is between. Then identify to which it is closer. • A procedure for rounding numbers to the nearest ten, hundred, or thousand is as follows: – Look one place to the right of the digit to which you wish to round. – If the digit is less than 5, leave the digit in the rounding place as it is, and change the digits to the right of the rounding place to zero. – If the digit is 5 or greater, add 1 to the digit in the rounding place, and change the digits to the right of the rounding place to zero. • A procedure for comparing two numbers by examining may include the following: – Line up the numbers by place value by lining up the ones. – Beginning at the left, find the first place value where the digits are different. – Compare the digits in this place value to determine which number is greater (or which is less). – Use the appropriate symbol > or < or the words *greater than* or *less than* to compare the numbers in the order in which they are presented. – If both numbers are the same, use the symbol = or the words *equal to*.		

3.2 The student will recognize and use the inverse relationships between addition/subtraction and multiplication/division to complete basic fact sentences. The student will use these relationships to solve problems.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition and subtraction are inverse operations, as are multiplication and division. • In building thinking strategies for subtraction, an emphasis is placed on connecting the subtraction fact to the related addition fact. The same is true for division, where the division fact is tied to the related multiplication fact. Building fact sentences helps strengthen this relationship. • Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. • Multiplication and division should be taught concurrently in order to develop understanding of the inverse relationship.	**All students should** • Understand how addition and subtraction are related. • Understand how multiplication and division are related.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use the inverse relationships between addition/subtraction and multiplication/division to solve related basic fact sentences. For example, $5 + 3 = 8$ and $8 - 3 = \_\_$; $4 \times 3 = 12$ and $12 \div 4 = \_\_$. • Write three related basic fact sentences when given one basic fact sentence for addition/subtraction and for multiplication/division. For example, given $3 \times 2 = 6$, solve the related facts $\_\_ \times 3 = 6$, $6 \div 3 = \_\_$, and $6 \div \_\_ = 3$.

3.3 The student will

a) name and write fractions (including mixed numbers) represented by a model;

b) model fractions (including mixed numbers) and write the fractions' names; and

c) compare fractions having like and unlike denominators, using words and symbols (>, <, or =).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction is a way of representing part of a whole (as in a region/area model or a length/measurement model) or part of a group (as in a set model). Fractions are used to name a part of one thing or a part of a collection of things. Models can include pattern blocks, fraction bars, rulers, number line, etc. • In each area/region and length/measurement model, the parts must be equal-sized (congruent). Wholes are divided or partitioned into equal-sized parts. In the set model, each member of the set is an equal part of the set. The members of the set do not have to be equal in size. • The denominator tells how many equal parts are in the whole or set. The numerator tells how many of those parts are being considered. • Provide opportunities to make connections among fraction representations by connecting concrete or pictorial representations with oral language and symbolic representations. • Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., *thirds* means "three equal parts of a whole," $\frac{1}{3}$ represents one of three equal-size parts when a pizza is shared among three students, or *three-fourths* means "three of four equal	**All students should** • Understand that the whole must be defined. • Understand that the denominator tells the number of equal parts that represent a whole. • Understand that the numerator is a counting number that tells how many equal size parts are being considered. • Understand that the value of a fraction is dependent on both the number of parts in a whole (denominator) and the number of those parts being considered (numerator). • Understand that a proper fraction is a fraction whose numerator is smaller than its denominator. • Understand that an improper fraction is a fraction whose numerator is greater than or equal to the denominator and is one or greater than one. • Understand that an improper fraction can be expressed as a whole number or a mixed number. • Understand that a mixed number is written as a whole number and a proper fraction.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Name and write fractions (including mixed numbers) represented by a model to include halves, thirds, fourths, eighths, tenths, and twelfths. • Use concrete materials and pictures to model at least halves, thirds, fourths, eighths, tenths, and twelfths. • Compare fractions using the terms greater than, less than, or equal to and the symbols (<, >, and =). Comparisons are made between fractions with both like and unlike denominators, using models, concrete materials and pictures.

3.3 The student will

a) name and write fractions (including mixed numbers) represented by a model;

b) model fractions (including mixed numbers) and write the fractions' names; and

c) compare fractions having like and unlike denominators, using words and symbols (>, <, or =).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
parts of a whole") furthers this development. • Comparing unit fractions (a fraction in which the numerator is one) builds a mental image of fractions and the understanding that as the number of pieces of a whole increases, the size of one single piece decreases (e.g., $\frac{1}{5}$ of a bar is smaller than $\frac{1}{4}$ of a bar). • Comparing fractions to a benchmark on a number line (e.g., close to 0, less than $\frac{1}{2}$, exactly $\frac{1}{2}$, greater than $\frac{1}{2}$, or close to 1) facilitates the comparison of fractions when using concrete materials or pictorial models.		

A variety of contexts are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from real-life experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have altogether?" or "How many cookies are left if I eat two?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, subtraction, multiplication, division and related facts;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as equal groupings of objects and sharing equally; and
- performing initial operations with fractions.

3.4 The student will estimate solutions to and solve single-step and multistep problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Addition is the combining of quantities; it uses the following terms: *addend* → 423 *addend* → + 246 *sum* → 669 • Subtraction is the inverse of addition; it yields the difference between two numbers and uses the following terms: *minuend* → 7,698 *subtrahend* → – 5,341 *difference* → 2,357 • An algorithm is a step-by-step method for computing. • An example of an approach to solving problems is Polya's four-step plan: – Understand: Retell the problem; read it twice; take notes; study the charts or diagrams; look up words and symbols that are new. – Plan: Decide what operation(s) and sequence of steps to use to solve the problem. – Solve: Follow the plan and work accurately. If the first attempt does not work, try another plan. – Look back: Does the answer make sense? • Knowing whether to find an exact answer or to make an estimate is learned through practical experiences in recognizing which is appropriate. • When an exact answer is required, opportunities to explore whether the answer can be determined mentally or must involve paper and pencil or calculators help students select the correct approach.	**All students should** • Understand that estimation skills are valuable, time-saving tools particularly in practical situations when exact answers are not required or needed. • Understand that estimation skills are also valuable in determining the reasonableness of the sum or difference when solving for the exact answer is needed. • Develop and use strategies to estimate whole number sums and differences to determine the reasonableness of an exact answer. • Develop flexible methods of adding whole numbers by combining numbers in a variety of ways, most depending on place values.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine whether an estimate or an exact answer is an appropriate solution for practical addition and subtraction problems situations involving single-step and multistep problems. • Determine whether to add or subtract in practical problem situations. • Estimate the sum or difference of two whole numbers, each 9,999 or less when an exact answer is not required. • Add or subtract two whole numbers, each 9,999 or less. • Solve practical problems involving the sum of two whole numbers, each 9,999 or less, with or without regrouping, using calculators, paper and pencil, or mental computation in practical problem situations. • Solve practical problems involving the difference of two whole numbers, each 9,999 or less, with or without regrouping, using calculators, paper and pencil, or mental computation in practical problem situations. • Solve single-step and multistep problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping.

3.4 The student will estimate solutions to and solve single-step and multistep problems involving the sum or difference of two whole numbers, each 9,999 or less, with or without regrouping.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Determining whether an estimate is appropriate and using a variety of strategies to estimate requires experiences with problem situations involving estimation. • There are a variety of mental mathematics strategies for each basic operation, and opportunities to practice these strategies give students the tools to use them at appropriate times. For example, with addition, mental mathematics strategies include – Adding 9: add 10 and subtract 1; and – Making 10: for column addition, look for numbers that group together to make 10. • Using Base-10 materials to model and stimulate discussion about a variety of problem situations helps students understand regrouping and enables them to move from the concrete to the abstract. Regrouping is used in addition and subtraction algorithms. • Conceptual understanding begins with concrete experiences. Next, the children must make connections that serve as a bridge to the symbolic. One strategy used to make connections is representations, such as drawings, diagrams, tally marks, graphs, or written comments.		

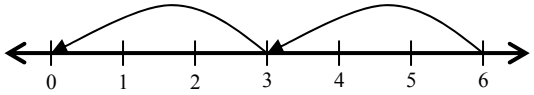
3.5 The student will recall multiplication facts through the twelves table, and the corresponding division facts.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The development of computational fluency relies on quick access to number facts. • A certain amount of practice is necessary to develop fluency with computational strategies; however, the practice must be motivating and systematic if students are to develop fluency in computation, whether mental, with manipulative materials, or with paper and pencil. • Strategies to learn the multiplication facts through the twelves table include an understanding of multiples/skip counting, properties of zero and one as factors, pattern of nines, commutative property, and related facts. • In order to develop and use strategies to learn the multiplication facts through the twelves table, students should use concrete materials, hundred chart, and mental mathematics. • To extend the understanding of multiplication, three models may be used: – The equal-sets or equal-groups model lends itself to sorting a variety of concrete objects into equal groups and reinforces repeated addition or skip counting. – The array model, consisting of rows and columns (e.g., 3 rows of 4 columns for a 3-by-4 array) helps build the commutative property. – The length model (e.g., a number line) also reinforces repeated addition or skip counting.	**All students should** • Develop fluency with number combinations for multiplication and division. • Understand that multiplication is repeated addition. • Understand that division is the inverse of multiplication. • Understand that patterns and relationships exist in the facts. • Understand that number relationships can be used to learn and retain the facts.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recall and state the multiplication and division facts through the twelves table. • Recall and write the multiplication and division facts through the twelves table.

3.6 The student will represent multiplication and division, using area, set, and number line models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The multiplication and division facts through the twelves tables should be modeled. • Multiplication is a shortcut for repeated addition. The terms associated with multiplication are listed below: *factor* → 54 *factor* → × 3 *product* → 162 • Creating real-life problems and solving them facilitates the connection between mathematics and everyday experiences (e.g., area problems). • The use of Base-10 blocks and repeated addition can serve as a model. For example, 4 × 12 is read as four sets consisting of one rod and two units. The sum is renamed as four rods and eight units or 48. This can be thought of as 12 + 12 + 12 + 12 = (SET) • The use of Base-10 blocks and the array model can be used to solve the same problem. A rectangle array that is one rod and two units long by four units wide is formed. The area of this array is represented by 4 rods and 8 units. • The number line model can be used to solve a multiplication problem such as 3 × 4. This is represented on the number line by three jumps of four. • The number line model can be used to solve a division problem such as 6 ÷ 3 and is represented on the number line by noting how many jumps of three go from 6 to 0.	**All students should** • Understand the meanings of multiplication and division. • Understand the models used to represent multiplying and dividing whole numbers.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model multiplication, using area, set, and number line models. • Model division, using area, set, and number line models. • Solve multiplication problems, using the multiplication algorithm, where one factor is 99 or less and the second factor is 5 or less. • Create and solve word problems involving multiplication, where one factor is 99 or less and the second factor is 5 or less.

3.6 The student will represent multiplication and division, using area, set, and number line models, and create and solve problems that involve multiplication of two whole numbers, one factor 99 or less and the second factor 5 or less.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
 The number of jumps (two) of a given length (three) is the answer to the question. • An algorithm is a step-by-step method for computing.		

3.7 The student will add and subtract proper fractions having like denominators of 12 or less.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A proper fraction is a fraction whose numerator is less than the denominator. A proper fraction is a fraction that is always less than one. • An improper fraction is a fraction whose numerator is greater than or equal to the denominator. An improper fraction is a fraction that is equal to or greater than one. • An improper fraction can be expressed as a mixed number. A mixed number is written as a whole number and a proper fraction. • The strategies of addition and subtraction applied to fractions are the same as the strategies applied to whole numbers. • Reasonable answers to problems involving addition and subtraction of fractions can be established by using benchmarks such as 0, $\frac{1}{2}$, and 1. For example, $\frac{3}{5}$ and $\frac{4}{5}$ are each greater than $\frac{1}{2}$, so their sum is greater than 1. • Concrete materials and pictorial models representing area/regions (circles, squares, and rectangles), length/measurements (fraction bars and strips), and sets (counters) can be used to add and subtract fractions having like denominators of 12 or less.	**All students should** • Understand that a proper fraction is a fraction whose numerator is smaller than its denominator. • Understand that an improper fraction is a fraction whose numerator is greater than or equal to the denominator and is one or greater than one. • Understand that an improper fraction can be expressed as a whole number or a mixed number. • Understand that a mixed number is written as a whole number and a proper fraction. A mixed number is the sum of a whole number and the proper fraction. • Understand that computation with fractions uses the same strategies as whole number computation.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Demonstrate a fractional part of a whole, using – region/area models (e.g., pie pieces, pattern blocks, geoboards, drawings); – set models (e.g., chips, counters, cubes, drawings); and – length/measurement models (e.g., nonstandard units such as rods, connecting cubes, and drawings). • Name and write fractions and mixed numbers represented by drawings or concrete materials. • Represent a given fraction or mixed number, using concrete materials, pictures, and symbols. For example, write the symbol for one-fourth and represent it with concrete materials and/or pictures. • Add and subtract with proper fractions having like denominators of 12 or less, using concrete materials and pictorial models representing area/regions (circles, squares, and rectangles), length/measurements (fraction bars and strips), and sets (counters).

Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to find measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

3.8 The student will determine, by counting, the value of a collection of bills and coins whose total value is $5.00 or less, compare the value of the bills and coins, and make change.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The value of a collection of coins and bills can be determined by counting on, beginning with the highest value, and/or by grouping the coins and bills. • A variety of skills can be used to determine the change after a purchase, including – counting on, using coins and bills, i.e., starting with the amount to be paid (purchase price), counting forward to the next dollar, and then counting forward by dollar bills to reach the amount from which to make change; and – mentally calculating the difference.	**All students should** • Understand that a collection of coins and bills has a value that can be counted. • Understand how to make change from $5.00 or less.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count the value of collections of coins and bills up to $5.00. • Compare the values of two sets of coins or bills, up to $5.00, using the terms *greater than*, *less than*, and *equal to*. • Make change from $5.00 or less.

3.9 The student will estimate and use U.S. Customary and metric units to measure

a) length to the nearest $\frac{1}{2}$ inch, inch, foot, yard, centimeter, and meter;

b) liquid volume in cups, pints, quarts, gallons, and liters;

c) weight/mass in ounces, pounds, grams, and kilograms; and

d) area and perimeter.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?"). • The concept of a standard measurement unit is one of the major ideas in understanding measurement. Familiarity with standard units is developed through hands-on experiences of comparing, estimating, measuring, and constructing. • Benchmarks of common objects need to be established for each of the specified units of measure (e.g., the mass of a mathematics book is about one kilogram). Practical experience measuring the mass of familiar objects helps to establish benchmarks and facilitates the student's ability to estimate measures. • One unit of measure may be more appropriate than another to measure an object, depending on the size of the object and the degree of accuracy desired. • Correct use of measurement tools is essential to understanding the concepts of measurement.	**All students should** • Understand how to estimate measures of length, liquid volume, weight/mass, area and perimeter. • Understand how to determine the actual measure of length, liquid volume, weight/mass, area and perimeter. • Understand that perimeter is a measure of the distance around a polygon. • Understand that area is a measure of square units needed to cover a surface.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Estimate and use U.S. Customary and metric units to measure lengths of objects to the nearest $\frac{1}{2}$ of an inch, inch, foot, yard, centimeter, and meter. • Determine the actual measure of length using U.S. Customary and metric units to measure objects to the nearest $\frac{1}{2}$ of an inch, foot, yard, centimeter, and meter. • Estimate and use U.S. Customary and metric units to measure liquid volume to the nearest cup, pint, quart, gallon, and liter. • Determine the actual measure of liquid volume using U.S. Customary and metric units to measure to the nearest cup, pint, quart, gallon, and liter. • Estimate and use U.S. Customary and metric units to measure the weight/mass of objects to the nearest ounce, pound, gram, and kilogram.

3.9 The student will estimate and use U.S. Customary and metric units to measure

a) length to the nearest $\frac{1}{2}$ inch, inch, foot, yard, centimeter, and meter;

b) liquid volume in cups, pints, quarts, gallons, and liters;

c) weight/mass in ounces, pounds, grams, and kilograms; and

d) area and perimeter.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Perimeter is the distance around any two-dimensional figure and is found by adding the measures of the sides. • Area is a two-dimensional measure and is therefore measured in square units. • Area is the number of square units needed to cover a figure, or more precisely, it is the measure in square units of the interior region of a two-dimensional figure.		• Determine the actual measure of weight/mass using U.S. Customary and metric units to measure the weight/mass of objects to the nearest ounce, pound, gram, and kilogram. • Estimate and use U.S. Customary and metric units to measure area and perimeter. • Determine the actual measure of area or perimeter using U.S. Customary and metric units.

3.10 The student will

a) measure the distance around a polygon in order to determine perimeter; and

b) count the number of square units needed to cover a given surface in order to determine area.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A polygon is a closed plane figure composed of at least three line segments that do not cross. None of the sides are curved. • Perimeter is a measure of the distance around a polygon and is found by adding the measures of the sides. • Area is the number of iterations of a two-dimensional unit needed to cover a surface. The two-dimensional unit is usually a square, but it could also be another shape such as a rectangle or an equilateral triangle. • Opportunities to explore the concepts of perimeter and area should involve hands-on experiences (e.g., placing tiles (units) around a polygon and counting the number of tiles to determine its perimeter and filling or covering a polygon with cubes (square units) and counting the cubes to determine its area).	**All students should** • Understand the meaning of a polygon as a closed figure with at least three sides. None of the sides are curved and there are no intersecting lines. • Understand that perimeter is a measure of the distance around a polygon. • Understand how to determine the perimeter by counting the number of units around a polygon. • Understand that area is a measure of square units needed to cover a surface. • Understand how to determine the area by counting the number of square units.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Measure each side of a variety of polygons and add the measures of the sides to determine the perimeter of each polygon. • Determine the area of a given surface by estimating and then counting the number of square units needed to cover the surface.

3.11 The student will

a) tell time to the nearest minute, using analog and digital clocks; and

b) determine elapsed time in one-hour increments over a 12-hour period.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• While digital clocks make reading time easy, it is necessary to ensure that students understand that there are sixty minutes in an hour. • Use of a demonstration clock with gears ensures that the positions of the hour hand and the minute hand are precise when time is read**.** • Students need to understand that time has passed or will pass. • Elapsed time is the amount of time that has passed between two given times. • Elapsed time should be modeled and demonstrated using geared analog clocks and timelines**.** • It is necessary to ensure that students understand that there are sixty minutes in an hour when using analog and digital clocks. • Elapsed time can be found by counting on from the beginning time to the finishing time. – Count the number of whole hours between the beginning time and the finishing time. For example, to find the elapsed time between 7 a.m. and 10 a.m., students can count on to find the difference between the times (7 and 10), so the total elapsed time is 3 hours.	**All students should** • Apply appropriate techniques to determine time to the nearest minute, using analog and digital clocks. • Understand how to determine elapsed time in one-hour increments over a 12-hour period.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Tell time to the nearest minute, using analog and digital clocks. • Match the times shown on analog and digital clocks to written times and to each other. • When given the beginning time and ending time, determine the elapsed time in one-hour increments within a 12-hour period (times do not cross between a.m. and p.m.). • Solve practical problems in relation to time that has elapsed.

3.12 The student will identify equivalent periods of time, including relationships among days, months, and years, as well as minutes and hours.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The knowledge that a year has 365 and $\frac{1}{4}$ days will help students understand the necessity of adding a full day every fourth year, called a *leap year*. • The use of a calendar facilitates the understanding of time relationships between days and months, days and weeks, days and years, and months and years. Recognize that students need to know the relationships, such as if there are 24 hours in one day, how many hours are in three days? If the date is January 6, what date would it be in two weeks? How many weeks are in March, April, and May? • The use of an analog clock facilitates the understanding of time relationships between minutes and hours and hours and days.	**All students should** • Understand the relationship that exists among periods of time, using calendars, and clocks.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify equivalent relationships observed in a calendar, including the number of days in a given month, the number of days in a week, the number of days in a year, and the number of months in a year. • Identify the number of minutes in an hour and the number of hours in a day.

3.13 The student will read temperature to the nearest degree from a Celsius thermometer and a Fahrenheit thermometer. Real thermometers and physical models of thermometers will be used.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Estimating and measuring temperatures in the environment in Fahrenheit and Celsius require the use of real thermometers. • A physical model can be used to represent the temperature determined by a real thermometer. • The symbols for degrees in Celsius (°C) and degrees in Fahrenheit (°F) should be used to write temperatures. • Celsius and Fahrenheit temperatures should be related to everyday occurrences by measuring the temperature of the classroom, the outside, liquids, body temperature, and other things found in the environment.	**All students should** • Understand how to measure temperature in Celsius and Fahrenheit with a thermometer.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Read temperature to the nearest degree from real Celsius and Fahrenheit thermometers and from physical models (including pictorial representations) of such thermometers.

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to these figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

3.14 The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. – **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons. – **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., "I know it's a rectangle because it looks like a door, and I know that the door is a rectangle."). – **Level 2: Analysis**. Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., "I know it's a rectangle because it's closed, it has four sides and four right angles, and opposite sides are parallel."). • A plane geometric figure is any two-dimensional closed figure. Circles and polygons are examples of plane geometric figures.	**All students should** • Understand how to identify and describe plane and solid geometric figures by using relevant characteristics. • Understand the similarities and differences between plane and solid figures.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify models and pictures of plane geometric figures (circle, square, rectangle, and triangle) and solid geometric figures (cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by name. • Identify and describe plane geometric figures by counting the number of sides and angles. • Identify and describe solid geometric figures by counting the number of angles, vertices, edges, and by the number and shape of faces. • Compare and contrast characteristics of plane and solid geometric figures (e.g., circle/sphere, square/cube, triangle/square pyramid, and rectangle/rectangular prism), by counting the number of sides, angles, vertices, edges, and the number and shape of faces. • Compare and contrast characteristics of solid geometric figures (i.e., cube, rectangular prism, square pyramid, sphere, cylinder, and cone) to similar objects in everyday life (e.g., a party hat is like a cone). • Identify characteristics of solid geometric figures (cylinder, cone, cube, square pyramid, and rectangular prism).

3.14 The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Three-dimensional figures are called solid figures or simply solids. Solids enclose a region of space. The interior of both plane and solid figures are not part of the figure. Solids are classified by the types of surfaces they have. These surfaces may be flat, curved, or both. • The study of geometric figures must be active, using visual images and concrete materials. • Access to a variety of concrete tools such as graph paper, pattern blocks, geoboards, and geometric solids is greatly enhanced by computer software tools that support exploration. • Opportunity must be provided for building and using geometric vocabulary to describe plane and solid figures. • A cube is a solid figure with six congruent square faces and with every edge the same length. A cube has 8 vertices and 12 edges. • A cylinder is a solid figure formed by two congruent parallel circles joined by a curved surface. • A cone is a solid, pointed figure that has a flat, round face (usually a circle) that is joined to a vertex by a curved surface. • A rectangular prism is a solid figure in which all six faces are rectangles with three pair of parallel congruent opposite faces. • A sphere is a solid figure with all of its points the same distance from its center.		

3.14 The student will identify, describe, compare, and contrast characteristics of plane and solid geometric figures (circle, square, rectangle, triangle, cube, rectangular prism, square pyramid, sphere, cone, and cylinder) by identifying relevant characteristics, including the number of angles, vertices, and edges, and the number and shape of faces, using concrete models.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A square pyramid is a solid figure with one square face and four triangular faces that share a common vertex. • A face is a polygon that serves as one side of a solid figure (e.g., a square is a face of a cube). • An angle is formed by two rays with a common endpoint. This endpoint is called the *vertex*. Angles are found wherever lines intersect. An angle can be named in three different ways by using – three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or – a number written inside the rays of the angle. • An edge is the line segment where two faces of a solid figure intersect. • A vertex is the point at which two lines, line segments, or rays meet to form an angle. It is also the point on a three dimensional figure where three or more faces intersect. • Students should be reminded that a solid geometric object is hollow rather than solid. The "solid" indicates a three-dimensional figure.		

3.15 The student will identify and draw representations of points, line segments, rays, angles, and lines.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A point is an exact location in space. It has no length or width. Usually, a point is named with a capital letter. • A line is a collection of points going on and on in both directions. It has no endpoints. When a line is drawn, at least two points on it can be marked and given capital letter names. The line can also be named with a single, lower-case letter. Arrows must be drawn to show that the line goes on in both directions infinitely. • A line segment is part of a line. It has two endpoints and includes all the points between those endpoints. The endpoints are used to name a line segment. • A ray is part of a line. It has one endpoint and continues on and on in one direction. • An angle is formed by two rays having a common endpoint. This endpoint is called the *vertex*. Angles are found wherever lines and line segments intersect. An angle can be named in three different ways by using – three letters to name, in this order, a point on one ray, the vertex, and a point on the other ray; – one letter at the vertex; or – a number written inside the rays of the angle. • Angle rulers may be particularly useful in developing the concept of an angle.	**All students should** • Understand that line segments and angles are fundamental components of plane polygons. • Understand that a line segment is a part of a line, has two end points, and contains all the points between those two endpoints. • Understand that points make up a line. • Understand that a line continues indefinitely in two opposite directions. • Understand that a ray is part of a line, has one endpoint, and continues indefinitely in only one direction. • Understand that an angle is formed by two rays having a common endpoint.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify examples of points, line segments, rays, angles, and lines. • Draw representations of points, line segments, rays, angles, and lines, using a ruler or straightedge.

3.16 The student will identify and describe congruent and noncongruent plane figures.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Congruent plane figures are figures having exactly the same size and shape. Noncongruent plane figures are figures that are not exactly the same size and shape. Opportunities for exploring figures that are congruent and/or noncongruent can best be accomplished by using physical models. • Have students identify figures that are congruent or noncongruent by using direct comparisons and/or tracing procedures.	**All students should** • Understand that congruent plane figures match exactly. • Understand that noncongruent plane figures do not match exactly. • Understand that congruent plane figures remain congruent even if they are in different spatial orientations. • Understand that noncongruent plane figures remain noncongruent even if they are in different spatial orientations.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify examples of congruent and noncongruent figures. Verify their congruence by laying one on top of the other using drawings or models. • Determine and explain why plane figures are congruent or noncongruent, using tracing procedures.

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

3.17 The student will

a) collect and organize data, using observations, measurements, surveys, or experiments;

b) construct a line plot, a picture graph, or a bar graph to represent the data; and

c) read and interpret the data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data.

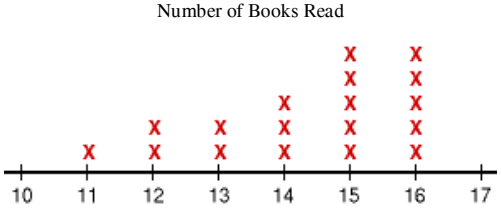
UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Investigations involving data should occur frequently and relate to students' experiences, interests, and environment. • Formulating questions for investigations is student-generated at this level. For example: What is the cafeteria lunch preferred by students in the class when four lunch menus are offered? • The purpose of a graph is to represent data gathered to answer a question. • Bar graphs are used to compare counts of different categories (categorical data). Using grid paper ensures more accurate graphs. – A bar graph uses parallel, horizontal or vertical bars to represent counts for categories. One bar is used for each category, with the length of the bar representing the count for that category. – There is space before, between, and after the bars. – The axis displaying the scale representing the count for the categories should extend one increment above the greatest recorded piece of data. Third grade students should collect data that are recorded in increments of whole numbers, usually multiples of 1, 2, 5, or 10. – Each axis should be labeled, and the graph should be given a title. – Statements representing an analysis and interpretation of the characteristics of the data in the graph (e.g., similarities and differences,	**All students should** • Understand how data can be collected and organized. • Understand that data can be displayed in different types of graphs depending on the data. • Understand how to construct a line plot, picture graph, or bar graph. • Understand that data sets can be interpreted and analyzed to draw conclusions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Formulate questions to investigate. • Design data investigations to answer formulated questions, limiting the number of categories for data collection to four. • Collect data, using surveys, polls, questionnaires, scientific experiments, and observations. • Organize data and construct a bar graph on grid paper representing 16 or fewer data points for no more than four categories. • Construct a line plot with no more than 30 data points. • Read, interpret and analyze information from line plots by writing at least one statement. • Label each axis on a bar graph and give the bar graph a title. Limit increments on the numerical axis to whole numbers representing multiples of 1, 2, 5, or 10. • Read the information presented on a simple bar or picture graph (e.g., the title, the categories, the description of the two axes). • Analyze and interpret information from picture and bar graphs, with up to 30 data points and up to 8 categories, by writing at least one sentence.

3.17 The student will

a) collect and organize data, using observations, measurements, surveys, or experiments;

b) construct a line plot, a picture graph, or a bar graph to represent the data; and

c) read and interpret the data represented in line plots, bar graphs, and picture graphs and write a sentence analyzing the data.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
least and greatest, the categories, and total number of responses) should be written. • A line plot shows the frequency of data on a number line. Line plots are used to show the spread of the data and quickly identify the range, mode, and any outliers.  Each x represents one student • When data are displayed in an organized manner, the results of the investigations can be described and the posed question answered. • Recognition of appropriate and inappropriate statements begins at this level with graph interpretations.		• Describe the categories of data and the data as a whole (e.g., data were collected on four ways to cook or prepare eggs — scrambled, fried, hard boiled, and egg salad — eaten by students). • Identify parts of the data that have special characteristics, including categories with the greatest, the least, or the same (e.g., most students prefer scrambled eggs). • Select a correct interpretation of a graph from a set of interpretations of the graph, where one is correct and the remaining are incorrect. For example, a bar graph containing data on four ways to cook or prepare eggs — eaten by students show that more students prefer scrambled eggs. A correct answer response, if given, would be that more students prefer scrambled eggs than any other way to cook or prepare eggs.

3.18 The student will investigate and describe the concept of probability as chance and list possible results of a given situation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A spirit of investigation and experimentation should permeate probability instruction, where students are actively engaged in explorations and have opportunities to use manipulatives. • Investigation of experimental probability is continued at this level through informal activities using two-colored counters, spinners, and random number generators (number cubes). • Probability is the chance of an event occurring. • The probability of an event occurring is the ratio of desired outcomes to the total number of possible outcomes. If all the outcomes of an event are equally likely to occur, the probability of the event = $\frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$. • The probability of an event occurring is represented by a ratio between 0 and 1. An event is "impossible" if it has a probability of 0 (e.g., the probability that the month of April will have 31 days). An event is "certain" if it has a probability of 1 (e.g., the probability that the sun will rise tomorrow morning). • When a probability experiment has very few trials, the results can be misleading. The more times an experiment is done, the closer the experimental probability comes to the theoretical probability (e.g., a coin lands heads up half of the time). • Students should have opportunities to describe in informal terms (i.e., *impossible*, *unlikely*, *as likely as*, *equally likely*, *likely*, and *certain*) the degree of likelihood of an event occurring. Activities should include real-life examples.	**All students should** • Investigate, understand, and apply basic concepts of probability. • Understand that probability is the chance of an event happening.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Define probability as the chance that an event will happen. • List all possible outcomes for a given situation (e.g., heads and tails are the two possible outcomes of flipping a coin). • Identify the degree of likelihood of an outcome occurring using terms such as *impossible*, *unlikely*, *as likely as*, *equally likely*, *likely*, and *certain*.

3.18 The student will investigate and describe the concept of probability as chance and list possible results of a given situation.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• For any event, such as flipping a coin, the equally likely things that can happen are called *outcomes*. For example, there are two equally likely outcomes when flipping a coin: the coin can land heads up, or the coin can land tails up. • A sample space represents all possible outcomes of an experiment. The sample space may be organized in a list, table, or chart.		

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. These students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.

3.19 The student will recognize and describe a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Exploring patterns requires active physical and mental involvement. • The use of materials to extend patterns permits experimentation or trial-and-error approaches that are almost impossible without them. • Reproduction of a given pattern in a different representation, using symbols and objects, lays the foundation for writing numbers symbolically or algebraically. • The simplest types of patterns are repeating patterns. In each case, students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience. • Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats. • Create an arithmetic number pattern. Sample numeric patterns include – 6, 9, 12, 15, 18, …(growing pattern); – 1, 2, 4, 7, 11, 16, …(growing pattern); – 20, 18, 16, 14,…(growing pattern); and – 1, 3, 5, 1, 3, 5, 1, 3, 5 …(repeating pattern).	**All students should** • Understand that numeric and geometric patterns can be expressed in words or symbols. • Understand the structure of a pattern and how it grows or changes. • Understand that mathematical relationships exist in patterns. • Understand that patterns can be translated from one representation to another.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize repeating and growing numeric and geometric patterns (e.g., skip counting, addition tables, and multiplication tables). • Describe repeating and growing numeric and geometric patterns formed using numbers, tables, and/or pictures, using the same or different forms. • Extend repeating and growing patterns of numbers or figures using concrete objects, numbers, tables, and/or pictures.

3.19 The student will recognize and describe a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation is the result of turning a figure around a point or a vertex, and reflection is the result of flipping a figure over a line. • Sample geometric patterns include – O Δ O O Δ Δ O O O Δ Δ Δ …; and – □□ □ □□ □ …. • A table of values can be analyzed to determine the pattern that has been used, and that pattern can then be used to find the next value.		

3.20 The student will

a) investigate the identity and the commutative properties for addition and multiplication; and

b) identify examples of the identity and commutative properties for addition and multiplication.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Investigating arithmetic operations with whole numbers helps students learn about several different properties of arithmetic relationships. These relationships remain true regardless of the numbers. • The commutative property for addition states that changing the order of the addends does not affect the sum (e.g., $4 + 3 = 3 + 4$). Similarly, the commutative property for multiplication states that changing the order of the factors does not affect the product (e.g., $2 \times 3 = 3 \times 2$). • The identity property for addition states that if zero is added to a given number, the sum is the same as the given number. The identity property of multiplication states that if a given number is multiplied by one, the product is the same as the given number. • A number sentence is an equation with numbers (e.g., $6 + 3 = 9$; or $6 + 3 = 4 + 5$).	**All students should** • Understand that mathematical relationships can be expressed using number sentences. • Understand the identity property for addition. • Understand the identity property for multiplication. • Understand the commutative property of addition. • Understand the commutative property of multiplication. • Understand that quantities on both sides of an equals sign must be equal. • Understand that quantities on both sides of the not equal sign are not equal.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate the identity property for addition and determine that when the number zero is added to another number or another number is added to the number zero, that number remains unchanged. Examples of the identity property for addition are $0 + 2 = 2$; $5 + 0 = 5$. • Investigate the identity property for multiplication and determine that when the number one is multiplied by another number or another number is multiplied by the number one, that number remains unchanged. Examples of the identity property for multiplication are $1 \times 3 = 3$; $6 \times 1 = 6$. • Recognize that the commutative property for addition is an order property. Changing the order of the addends does not change the sum ($5 + 4 = 9$ and $4 + 5 = 9$). • Recognize that the commutative property for multiplication is an order property. Changing the order of the factors does not change the product ($2 \times 3 = 3 \times 2$). • Write number sentences to represent equivalent mathematical relationships (e.g., $4 \times 3 = 14 - 2$). • Identify examples of the identity and commutative properties for addition and multiplication.