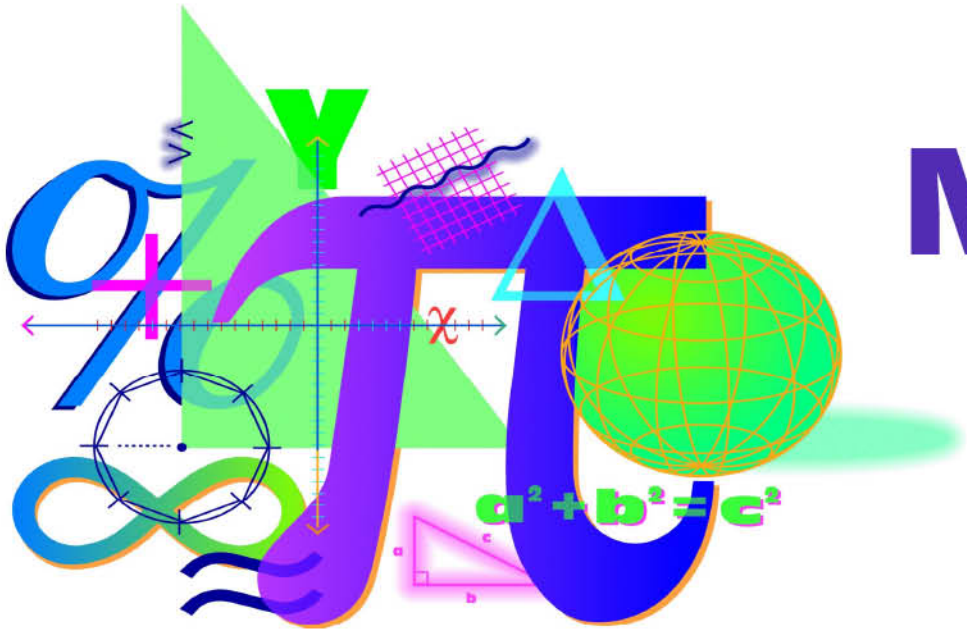

Mathematics
Standards of Learning

Curriculum Framework 2009

Grade 1

Board of Education
Commonwealth of Virginia

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Virginia *Mathematics Standards of Learning* Curriculum Framework 2009
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into three columns: Understanding the Standard; Essential Understandings; and Essential Knowledge and Skills. The purpose of each column is explained below.

Understanding the Standard
This section includes background information for the teacher (K-8). It contains content that may extend the teachers' knowledge of the standard beyond the current grade level. This section may also contain suggestions and resources that will help teachers plan lessons focusing on the standard.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning. In Grades 6-8, these essential understandings are presented as questions to facilitate teacher planning.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

Students in grades K–3 have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships ("more than," "less than," and "equal to"), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

1.1 The student will

a) count from 0 to 100 and write the corresponding numerals; and

b) group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• There are three developmental levels of counting: – rote sequence; – one-to-one correspondence; and – the cardinality of numbers. • Counting involves two separate skills: verbalizing the list of standard number words in order ("one, two, three, …") and connecting this sequence with the items in the set being counted, using one-to-one correspondence. Association of number words with collections of objects is achieved by moving, touching, or pointing to objects as the number words are spoken. • The last number stated represents the number of objects in the set. This is known as the cardinality of the set. • Rote counting is a prerequisite skill for the understanding of addition, subtraction, and the ten-to-one concept of place value. • Articulating the characteristics of each numeral when writing numbers has been found to reduce the amount of time it takes to learn to write numerals. • The number system is based on a pattern of tens where each place has ten times the value of the place to its right. This is known as the ten-to-one concept of place value.	**All students should** • Associate oral number names with the correct numeral and set of objects. • Understand that 1 and 10 are special units of numbers (e.g., 10 is 10 ones, but it is also 1 ten). • Understand the ten-to-one relationship of ones and tens (10 ones equals 1 ten). • Understand that numbers are written to show how many tens and how many ones are in the number. • Understand that groups of tens and ones can be used to tell how many.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count by rote from 0 to 100, using the correct name for each numeral. • Use the correct oral counting sequence to tell how many objects are in a set. • Write numerals correctly. • Write each numeral from 0 to 100. • Read two-digit numbers when shown a numeral, a Base-10 model of the number, or a pictorial representation of the number. • Identify the place value (ones, tens) of each digit in a two-digit numeral (e.g., The place value of the 2 in the number 23 is tens. The value of the 2 in the number 23 is 20). • Group a collection of objects into sets of tens and ones. Write the numeral that corresponds to the total number of objects in a given collection of objects that have been grouped into sets of tens and ones.

1.1 The student will

a) count from 0 to 100 and write the corresponding numerals; and

b) group a collection of up to 100 objects into tens and ones and write the corresponding numeral to develop an understanding of place value.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Opportunities to experience the relationships among tens and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place value concept of our number system and to understanding the value of each digit in a two-digit number. Ten-to-one trading activities with manipulatives on place value mats provide excellent experiences for developing the understanding of the places in the Base-10 system. • Models that clearly illustrate the relationships among tens and ones are physically proportional (e.g., the tens piece is ten times larger than the ones piece). • Providing students with opportunities to model two-digit numbers expressed with groups of ones and tens will help students understand the ideas of trading, regrouping, and equality. • Recording the numeral when using physical and pictorial models leads to an understanding that the position of each digit in a numeral determines the size of the group it represents.		

1.2 The student will count forward by ones, twos, fives, and tens to 100 and backward by ones from 30.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time telling, and multiplication. Powerful models for developing these concepts include counters, number line, hundred chart, and calculators. • Skip counting by twos supports the development of the concept of even numbers. • Skip counting by fives lays the foundation for reading a clock effectively and telling time to the nearest five minutes, counting money, and developing the multiplication facts for five. • Skip counting by tens is a precursor for use of place value, addition, counting money, and multiplying by multiples of 10. • Counting backward by rote lays the foundation for subtraction. Students should count backward beginning with 30, 29, 28, … through …3, 2, 1, 0. • Calculators can be used to reinforce skip counting.	**All students should** • Understand that collections of objects can be grouped and skip counting can be used to count the collection. • Describe patterns in counting by ones (both forward and backward) and skip counting and use those patterns to predict the next number in the counting sequence.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Count by ones, twos, fives, and tens to 100, using concrete objects, such as counters, connecting cubes, pennies, nickels, and dimes. • Demonstrate a one-to-one correspondence when counting by ones with concrete objects or representations. • Skip count orally by twos, fives and tens to 100 starting at various multiples of 2, 5, or 10. • Count backward by ones from 30.

1.3 The student will identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A fraction is a way of representing part of a whole set or a whole region. • In a set fraction model, students need opportunities to make fair shares. For example, when sharing a set of 12 markers with three friends, each person would have one-third of the whole set. Also, each element of the set, no matter its size, is considered to be an equal share of the whole. • In a region/area fraction model, a whole is broken into equal-size parts and reassembled into one whole. • The words *denominator* and *numerator* are not required at this grade, but the concepts of part and whole are required for understanding of a fraction. • At this level, students are expected to first understand the part-whole relationship (e.g., three out of four equal parts) before being expected to recognize or use symbolic representations for fractions (e.g., $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$). • Students should have opportunities to make connections and comparisons among fraction representations by connecting concrete or pictorial representations with spoken representations (e.g., "one-half," "one out of two equal parts, "or "two-thirds," "two out of three equal parts," and "one half is more than one fourth of the same whole").	**All students should** • Understand that a fraction represents a part of a whole. • Understand that fractional parts are equal shares of a whole. • Understand that the fraction name (*half*, *third*, *fourth*) tells the number of equal parts in the whole.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Represent a whole to show it having two equal parts and identify one-half ($\frac{1}{2}$), and two halves ($\frac{2}{2}$). • Represent a whole to show it having three equal parts and identify one-third ($\frac{1}{3}$), two-thirds ($\frac{2}{3}$) and three-thirds ($\frac{3}{3}$). • Represent a whole to show it having four equal parts and identify one-fourth ($\frac{1}{4}$), two-fourths ($\frac{2}{4}$), three-fourths ($\frac{3}{4}$) and four-fourths ($\frac{4}{4}$). • Identify and model halves, thirds, and fourths of a whole, using the set model (e.g., connecting cubes and counters), and region/area models (e.g., pie pieces, pattern blocks, geoboards, paper folding, and drawings). • Name and write fractions represented by drawings or concrete materials for halves, thirds, and fourths. • Represent a given fraction using concrete materials, pictures, and symbols for halves, thirds, and fourths. For example, write the symbol for one-fourth, and represent it with concrete materials and pictures.

1.3 The student will identify the parts of a set and/or region that represent fractions for halves, thirds, and fourths and write the fractions.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions (e.g., *thirds* means "three equal parts of a whole") furthers this development.		

A variety of contexts are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from practical experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as "How many books do we have altogether?" or "How many cookies are left if I eat two?" or "I have three more candies than you do." Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In grades K–3, computation and estimation instruction focuses on

- relating the mathematical language and symbolism of operations to problem situations;
- understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
- developing proficiency with basic addition, subtraction, multiplication, division and related facts;
- gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
- developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
- choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
- recognizing whether numerical solutions are reasonable;
- experiencing situations that lead to multiplication and division, such as equal groupings of objects and sharing equally; and
- performing initial operations with fractions.

1.4 The student, given a familiar problem situation involving magnitude, will

a) select a reasonable order of magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, 500); and

b) explain the reasonableness of the choice.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Magnitude refers to the size of a set. • Exploring ways to estimate the number of objects in a set, based on appearance, (e.g., clustering, grouping, comparing) enhances the development of number sense. • To estimate means to find a number that is close to the exact amount. When asking for an estimate, teachers might ask, "*about* how much?" or "*about* how many?" or "Is this about 10 or about 50?" • Students should be provided opportunities to estimate a quantity, given a benchmark of 10 and/or 100 objects.	**All students should** • Develop an understanding of the order of magnitude (size) of whole numbers and use this knowledge to estimate quantities.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Select a reasonable order of magnitude for a given set from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, and 500 jelly beans in jars) in a familiar problem situation. • Given a familiar problem situation involving magnitude, explain why a particular estimate was chosen as the most reasonable from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral.

1.5 The student will recall basic addition facts with sums to 18 or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Associate the terms *addition*, *adding*, and *sum* with the concept of joining or combining. • Associate the terms *subtraction*, *subtracting*, *minus*, and *difference* with the process of taking away or separating (i.e., removing a set of objects from the given set of objects, finding the difference between two numbers, or comparing two numbers). • Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include – counting back; – "one-more-than," "two-more-than" facts; – "one-less-than," "two-less-than" facts; – "doubles" to recall addition facts (e.g., 2 + 2 =__; 3 + 3 =__); – "near doubles" [e.g., 3 + 4 = (3 + 3) + 1 = __]; – "make-ten" facts (e.g., at least one addend of 8 or 9); – "think addition for subtraction" (e.g., for 9 – 5 = __, think "5 and what number makes 9?"); – use of the commutative property, without naming the property (e.g., 4 +3 is the same as 3 + 4); – use of related facts (e.g., 4 + 3 = 7 , 3 + 4 = 7, 7 – 4 = 3, and 7 – 3 = 4); – use of the additive identity property (e.g., 4 + 0 = 4), without naming the property but saying, "When you add zero to a number, you always get the original number."; and – use patterns to make sums (e.g., 0 + 5 = 5, 1 + 4 = 5, 2 + 3 = 5, etc.).	**All students should** • Develop an understanding of the addition and subtraction relationship. • Develop addition and subtraction strategies for fact recall. • Develop fluency with basic number combinations for addition and subtraction.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify + as a symbol for addition, – as a symbol for subtraction, and = as a symbol for equality. • Recall and state orally the basic addition facts for sums with two addends to 18 or less and the corresponding subtraction facts. • Recall and write the basic addition facts for sums to 18 or less and the corresponding subtraction facts, when addition or subtraction problems are presented in either horizontal or vertical written format.

1.5 The student will recall basic addition facts with sums to 18 or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Manipulatives should be used to develop an understanding of addition and subtraction facts. Automaticity of facts can be achieved through constant practice which may include games, hands-on activities, flash cards, and paper and pencil. • Students should first master facts to 10 and then master facts to 18.		

1.6 The student will create and solve one-step story and picture problems using basic addition facts with sums to 18 or less and the corresponding subtraction facts.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The problem solving process is enhanced when students – create their own story problems; and – model word problems, using manipulatives, representations, or number sentences/equations. • Students should experience a variety of problem types related to addition and subtraction, including – join and separate problems (action involved); join (for example: Sam had 8 pennies. Tom gave him 3 more. How many pennies does Sam have now? separate (for example: Sam had 11 pennies. He gave 3 to Tom. How many pennies does Sam have now?) – part-part-whole problems (no action involved); missing part (for example: There are 8 marbles. Five are shown. How many are missing?) – classification problems (for example: Jane had 12 hats. Only 3 of the hats are blue. How many are not blue?) – comparison problems (for example: Bill is 7 years old. Alice is 4 years old. How much younger is Alice than Bill?).	**All students should** • Understand various meanings of addition and subtraction in a variety of situations. • Understand that creating and solving problems involves the use of addition and/or subtraction.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Interpret and solve oral or written story and picture problems involving one-step solutions, using basic addition and subtraction facts (sums to18 or less and the corresponding subtraction facts). • Identify a correct number sentence to solve an oral or written story and picture problem, selecting from among basic addition and subtraction facts.

Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in grades K–3 should encounter measurement in many normal situations, from their daily use of the calendar and from science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight/mass, capacity, time, temperature, area, perimeter, volume, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply both techniques and nonstandard and standard tools to find measurements and to develop an understanding of the use of simple U.S. Customary and metric units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

1.7 The student will

a) identify the number of pennies equivalent to a nickel, a dime, and a quarter; and

b) determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Many experiences with coins help students develop an understanding of money, such as – drawing pennies to show the value of a given coin (e.g., a nickel, a dime, or a quarter); – playing store and purchasing classroom objects, using play money (pennies); – representing the value of a nickel, a dime, and a quarter, using pennies; and – trading the equivalent value of pennies for a nickel, a dime, and a quarter, using play money. • Counting money helps students gain an awareness of consumer skills and the use of money in everyday life. • A variety of classroom experiences in which students manipulate physical models of money and count forward to determine the value of a collection of coins are important activities to ensure competence with counting money. • Establishing a one-to-one correspondence between the number names and the items in a set of coins (pennies, nickels, or dimes) is essential for an accurate count. • The last number stated represents the value of a collection of coins being counted. This is known as the cardinality of the set.	**All students should** • Develop an understanding of exchanging the appropriate number of pennies for a nickel, a dime, or a quarter. • Develop an understanding of place value by skip counting a collection of coins by ones, fives, and tens.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the value of a nickel, a dime, and a quarter in terms of pennies. • Recognize the characteristics of pennies, nickels, and dimes (e.g., color, size). • Count by ones to determine the value of a collection of pennies whose total value is 100 cents or less. • Count by fives to determine the value of a collection of nickels whose total value is 100 cents or less. • Count by tens to determine the value of a collection of dimes whose total value is 100 cents or less. • Count by ones, fives, and tens to determine the value of a collection of pennies and nickels, pennies and dimes, and nickels and dimes whose total value is 100 cents or less. • Count by ones, fives, and tens to determine the value of a collection of pennies, nickels, and dimes whose total value is 100 cents or less.

1.8 The student will tell time to the half-hour, using analog and digital clocks.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Many experiences using clocks help students develop an understanding of the telling of time to the half-hour, including – identifying the parts of an analog clock (minute and hour hands); – demonstrating a given time to the half-hour, using a model clock; – writing correct digital time to the half-hour; and – relating time on the half-hour to daily routines and school schedules (e.g., the times of TV programs, bedtime, resource time, lunch time, recess time).	**All students should** • Understand how to tell time to the half-hour, using an analog and digital clock. • Understand the concepts of a.m., p.m., minutes, and hours. • Understand that there are sixty minutes in an hour.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Tell time shown on an analog clock to the half-hour. • Tell time shown on a digital clock to the half-hour. • Match a written time to the time shown on a digital and analog clock to the half-hour.

1.9 The student will use nonstandard units to measure length, weight/mass, and volume.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The process of measurement involves selecting a unit of measure, comparing the unit to the object to be measured, counting the number of times the unit is used to measure the object, and arriving at an approximate total number of units. • Premature use of instruments or formulas leaves children without the understanding necessary for solving measurement problems. • When children's initial explorations of length, weight/mass, and volume involve the use of nonstandard units, they develop some understanding about the need for standard measurement units for length, weight/mass, and volume especially when they communicate about these measures.	**All students should** • Understand that measurement involves comparing an attribute of an object to the same attribute of the unit of measurement (e.g., the length of a cube measures the length of a book. The weight/mass of the cube measures the weight/mass of the book. The volume of the cube measures the volume of a book). • Understand how to measure length, weight/mass, and volume using various nonstandard units of measurement.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Measure the length of objects, using various nonstandard units (e.g., connecting cubes, paper clips, erasers). • Measure the weight/mass of objects, using a balance scale with various nonstandard units (e.g., paper clips, bean bags, cubes). • Measure the volume of objects, using various nonstandard units (e.g., connecting cubes, blocks, rice, water).

1.10 The student will compare, using the concepts of more, less, and equivalent,
a) the volumes of two given containers; and
b) the weight/mass of two objects, using a balance scale.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Estimation is a commonly used strategy to compare the volumes of two containers. • Determining the volume of a container by counting the number of nonstandard units (e.g., a spoonful, or scoopful of concrete material, such as jelly beans, sand, water, or rice) that can be held by the container is a precursor to comparing volumes. • A variety of activities that focus on directly comparing the volume of objects leads to an understanding of volume. • The level of difficulty in measuring volume can be increased by varying and mixing the sizes of the containers (e.g., using short, wide containers as well as tall, narrow containers). • Weight and mass are different. Mass is the amount of matter in an object. Weight is determined by the pull of gravity on the mass of an object. The mass of an object remains the same regardless of its location. The weight of an object changes dependent on the gravitational pull at its location. In everyday life, most people are actually interested in determining an object's mass, although they use the term *weight* (e.g., "How much does it weigh?" versus "What is its mass?").	**All students should** • Understand how to fill containers with objects to determine their volume and compare the volumes of two containers. • Understand that a balance scale can be used to compare the weights of two objects using the terms more, less, or equivalent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other, using nonstandard units of measure (e.g., a spoonful or scoopful). • Compare the volumes of two containers to determine if the volume of one is more, less, or equivalent to the other by pouring the contents of one container into the other. • Compare the weight/mass of two objects, using the terms *lighter*, *heavier*, or *the same*, using a balance scale. The pan containing less weight/mass will rise and the pan containing more weight/mass will fall. If the objects are of equivalent weight/mass, the two pans will balance.

1.10 The student will compare, using the concepts of more, less, and equivalent,
a) the volumes of two given containers; and
b) the weight/mass of two objects, using a balance scale.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Balance scales are instruments used for comparing weight/mass. A balance scale usually has a beam that is supported in the center. On each side of the beam are two identical trays. When the trays hold equal weights, the beam is level, and the scale is "balanced." • Physically measuring the weights of objects, using a balance scale, helps students develop an intuitive idea of what it means to say something is "lighter," "heavier," or "the same." • Experience estimating the weights of two objects (one in each hand) using the terms lighter, heavier, or the same promotes an understanding of the concept of balance.		

1.11 The student will use calendar language appropriately (e.g., names of the months, *today, yesterday, next week, last week*).

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Practical situations are appropriate to develop a sense of the interval of time between events (e.g., Boy Scout meetings occur every week on Monday: there is a week between meetings). • The calendar is a way to represent units of time (e.g., days, weeks, and months). • Using a calendar develops the concept of day as a 24-hour period rather than a period of time from sunrise to sunset.	**All students should** • Understand how to use a calendar as a way to measure time.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Read a calendar to locate a given day or date. • Identify the months of the year. • Identify the seven days in a week. • Determine the days/dates before and after a given day/date (e.g., yesterday, today, tomorrow). • Determine the date that is a specific number of days or weeks in the past or in the future from a given date, using a calendar. • Identify specific dates (e.g., the third Monday in a given month).

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

- observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
- sorting objects and ordering them directly by comparing them one to the other;
- describing, comparing, contrasting, sorting, and classifying figures; and
- exploring symmetry, congruence, and transformation.

In the primary grades, children begin to develop basic vocabulary related to these figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

- **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
- **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during grades K and 1.)
- **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades 2 and 3.)

1.12 The student will identify and trace, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and right angles.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A plane geometric figure is any two-dimensional closed figure. Circles and polygons are examples of plane geometric figures. • The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding. – **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons. – **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same (e.g., "I know it's a rectangle because it looks like a door, and I know that a door is a rectangle"). – **Level 2: Analysis.** Properties are perceived, but are isolated and unrelated. Students should recognize and name properties of geometric figures (e.g., "I know it's a rectangle because it is closed; it has four sides and four right angles, and opposite sides are parallel.").	**All students should** • Develop strategies to sort and/or group plane geometric figures and refine the vocabulary used to explain their strategies.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Describe a circle. • Trace triangles, squares, rectangles, and circles. • Describe triangles, squares, and rectangles by the number of sides, vertices, and right angles. • Sort plane geometric figures into appropriate subsets (categories) based on characteristics (number of sides, vertices, angles, curved, etc.). • Identify the name of the geometric figure when given information about the number of sides, vertices, and right angles.

1.12 The student will identify and trace, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and right angles.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Students should have experiences with various plane geometric polygons. – Triangles could be equilateral, right, obtuse, acute, etc. – Quadrilaterals could be rectangles, squares, trapezoids, rhombi, etc. • A vertex is the point at which two line segments, lines, or rays meet to form an angle. • A polygon is a plane geometric figure which – has sides that are line segments; – is simple (its sides do not cross); – is closed; and – is two-dimensional (lies in a plane). • A triangle is a polygon with three angles and three sides. • A quadrilateral is a polygon with four sides. • A rectangle is a quadrilateral with four right angles. • A square is a rectangle with four sides of equal length. • A circle is a closed curve with all points in one plane and the same distance from a fixed point (the center). • Transformations (translations, reflections, and rotation) can be used to change the location of objects.		

1.12 The student will identify and trace, describe, and sort plane geometric figures (triangle, square, rectangle, and circle) according to number of sides, vertices, and right angles.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students do not develop the common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed.		

1.13 The student will construct, model, and describe objects in the environment as geometric shapes (triangle, rectangle, square, and circle) and explain the reasonableness of each choice.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Representations of circles, squares, rectangles, and triangles can be found in the students' environment at school and at home. Students should have opportunities to identify/classify things in the~~ir~~ environment by the type of figure those things represent. • A common misconception students have when a figure such as a square is rotated is they will frequently refer to the rotated square as a diamond. Clarification needs to be ongoing — i.e., a square is a square regardless of its location in space; there is no such geometric figure as a diamond. • Building geometric and spatial capabilities fosters enthusiasm for mathematics while providing a context to develop spatial sense. • Polygons can be constructed using other polygons (e.g., six equilateral triangles can be used to construct a hexagon, a triangle can be added to a rectangle to create a pentagon, etc.).	**All students should** • Understand that geometric figures are integral parts of the environment. • Use familiarity with the figure, structure, and location to develop spatial reasoning.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Construct plane geometric figures. • Identify models of representations of circles, squares, rectangles, and triangles in the environment at school and home and tell why they represent those figures. • Describe representations of circles, squares, rectangles, and triangles in the environment and explain the reasonableness of the choice.

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. They experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible*, *unlikely*, *equally likely*, *more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

1.14 The student will investigate, identify, and describe various forms of data collection (e.g., recording daily temperature, lunch count, attendance, favorite ice cream), using tables, picture graphs, and object graphs.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Students' questions about the physical world can often be answered by collecting data and observing the results. • Data are information collected about people or things. The primary purpose of collecting data is to answer questions. • After generating questions, students decide what information is needed and how it can be collected. • The collection of the data often leads to new questions to be investigated. • The entire process broadens children's views of mathematics and its usefulness. • Data collection could involve voting, informal surveys, tallying, and charts. • Surveys, which are data-collecting tools that list choices, should have a limited number of questions at the primary grades. • Tallying is a method for gathering information. Tally marks are used to show how often something happens or occurs. Each tally mark represents one occurrence. Tally marks are clustered into groups of five, with four vertical marks representing the first four occurrences and the fifth mark crossing the first four on a diagonal to represent the fifth occurrence.	**All students should** • Understand how data can be collected and presented in an organized manner. • Understand that data gathered and analyzed from observations and surveys can have an impact on our everyday lives.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate various forms of data collection, including counting and tallying, informal surveys, observations, and voting. • Identify and describe various forms of data collection in practical situations (e.g., recording daily temperature, lunch count, attendance, and favorite ice cream.)

1.15 The student will interpret information displayed in a picture or object graph, using the vocabulary *more, less, fewer, greater than, less than,* and *equal to*.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Statistics is used to describe and interpret with numbers the world around us; it is a tool for problem solving. • Students' questions about everyday life can often be answered by collecting and interpreting data. • Organized data provides a clearer picture for interpretation. Data may be described in object graphs and picture graphs. • Picture graphs are graphs that use pictures to show and compare information. • Object graphs are graphs that use concrete materials to represent the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student). • Interpretation of the data could lead to additional questions to be investigated.	**All students should** • Understand that picture graphs use pictures to represent and compare data while object graphs use concrete objects to represent and compare data. • Understand that data can be analyzed and interpreted, using the terms *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare one category to another in a graph, indicating which has more or which has less, or which is equal to. • Interpret information displayed in object graphs and picture graphs, using the words *more*, *less*, *fewer*, *greater than*, *less than*, and *equal to*. • Find answers to questions, using graphs (e.g., "Which category has more?", "How many more?", and "How many in all?").

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children's mathematical development.

Students in kindergarten through third grade develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

- sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
- identifying, analyzing, and extending patterns;
- creating repetitive patterns and communicating about these patterns in their own language;
- analyzing simple patterns and making predictions about them;
- recognizing the same pattern in different representations;
- describing how both repeating and growing patterns are generated; and
- repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. These students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.

1.16 The student will sort and classify concrete objects according to one or more attributes, including color, size, shape, and thickness.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Sorting, classifying, and ordering objects facilitate work with patterns, geometric shapes, and data. • To classify is to arrange or organize a set of materials according to a category or attribute (a quality or characteristic). • General similarities and differences among items are easily observed by primary students, who can begin to focus on more than one attribute at a time. During the primary grades, the teacher's task is to move students toward a more sophisticated understanding of classification in which two or more attributes connect or differentiate sets, such as those found in nature (e.g., leaves with different colors and different shapes).	**All students should** • Understand the same set of objects can be sorted and classified in different ways.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Sort and classify objects into appropriate subsets (categories) based on one or two attributes, such as size, shape, color, or thickness.

1.17 The student will recognize, describe, extend, and create a wide variety of growing and repeating patterns.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Patterns allow children to recognize order, generalize, and predict. • Patterning should include – reproducing a given pattern, using manipulatives; – recording a pattern with pictures or symbols; – transferring a pattern into a different form or different representation (e.g., blue–blue–red to an AAB repeating pattern); and – analyzing patterns in practical situations (e.g., calendar, seasons, days of the week). • The simplest types of patterns are repeating patterns. The part of the pattern that repeats is the core. The patterns can be oral, such as the refrain in "Old MacDonald's Farm" ("e-i-e-i-o"), or physical with clapping and snapping patterns, or combinations of both, such as is found in songs like the "Hokey Pokey." In each case, students need to identify the basic unit of the pattern and repeat it. Opportunities to create, recognize, describe, and extend repeating patterns are essential to the primary school experience. • Growing patterns are more difficult for students to understand than repeating patterns because not only must they determine what comes next, they must also begin the process of generalization. Students need experiences with growing patterns in both arithmetic and geometric formats.	**All students should** • Understand that patterns are a way to recognize order, to organize their world, and to predict what comes next in an arrangement. • Recognize and state the core of a pattern. • Analyze how both repeating and growing patterns are generated.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Recognize the pattern in a given rhythmic, color, geometric figure, or numerical sequence. • Describe the pattern in a given rhythmic, color, geometric figure, or numerical sequence in terms of the core (the part of the sequence that repeats). • Extend a pattern, using manipulatives, geometric figures, numbers, or calculators. • Transfer a pattern from one form to another. • Create a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators (e.g., the growing patterns 2, 3, 2, 4, 2, 5, 2, 6, 2, …).

1.17 The student will recognize, describe, extend, and create a wide variety of growing and repeating patterns.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Create an arithmetic number pattern. Sample numeric patterns include – 6, 9, 12, 15, 18, …(growing pattern); – 20, 18, 16, 14, … (growing pattern); – 1, 2, 4, 7, 11, 16, … (growing pattern); and – 1, 3, 5, 1, 3, 5, 1, 3, 5… (repeating pattern). • In geometric patterns, students must often recognize transformations of a figure, particularly rotation or reflection. Rotation is the result of turning a figure around a point or a vertex, and reflection is the result of flipping a figure over a line.		

1.18 The student will demonstrate an understanding of equality through the use of the equal sign.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Equality can be shown by measuring with a balance scale or a number balance. • Manipulatives such as rods, connecting cubes, and counters can be used to model equations. • An expression is a representation of a quantity. It is made up of numbers, variables, and/or computational symbols. It does not have an equal sign (e.g., $x + 3$). • An equation is a mathematical statement that two expressions are the same. Equations are written with an equal sign. • Equations have expressions of equal value on both sides (e.g., $5 + 3 = 8$, $8 = 5 + 3$ and $4 + 3 = 9 - 2$). • An equation can be represented using balance scales. There must be the same amount on each side of an equal sign (e.g., $5 + 3 = 3 + 5$). • A common misunderstanding is that the equal sign always means the answer. The equal sign can represent an equality. • Equations should be shown in many different forms (e.g., $6 = 6$, $4 + 2 = 6$, $5 + 1 = 4 + 2$, $6 = 4 + 2$, $4 = 6 - 2$). • Inequalities such as $5 < 4 + 3$ are not equations. Equations must have the equal sign. • It is important for children to understand that the expression $3 + 5$ is another representation of eight.	**All students should** • Understand that the equal sign means "is the same as" or "another name for" or "equal in value". • Understand that equality represents a balance concept. Both sides of the equation balance because they are equal (they have the same value).	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the equality (=) symbol. • Recognize that the equations $4 + 2 = 2 + 4$ and $6 + 1 = 4 + 3$ represent the relationship between two expressions of equal value. • Model an equation that represents the relationship of two expressions of equal value. • Identify equivalent values (e.g., $3 = 3$, $4 + 3 = 8 - 1$, $7 = 2 + 5$, etc.).

1.18 The student will demonstrate an understanding of equality through the use of the equal sign.

UNDERSTANDING THE STANDARD (Background Information for Instructor Use Only)	ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The equal sign is used when two representations name the same number, 5 + 3 = 10 - 2. These two expressions in the equation represent the same number, eight. • Equations should be routinely modeled in conjunction with story problems. • Solving missing addend problems and stories helps with the understanding of equality and the equal sign (e.g., There are 4 red birds in the tree. Some black birds fly to the tree. Now there are six birds in the tree. How many black birds flew to the tree? 4+__ =6)		