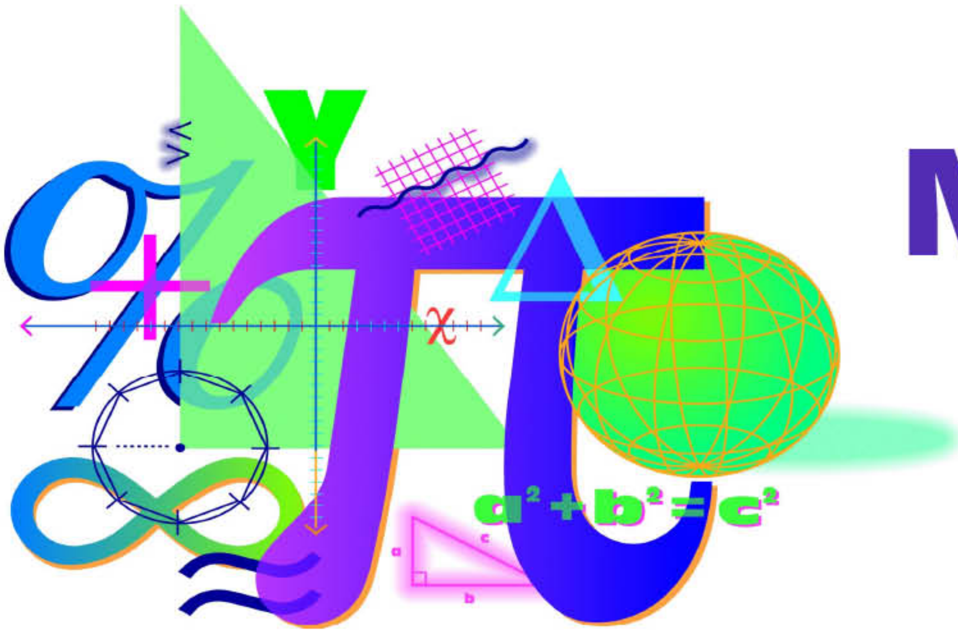

Mathematics

Standards of Learning

Curriculum Framework 2009

Discrete Mathematics

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Commonwealth of Virginia

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The 2009 *Mathematics Curriculum Framework* can be found in PDF and Microsoft Word file formats on the Virginia Department of Education's Web site at http://www.doe.virginia.gov.

Virginia *Mathematics Standards of Learning* Curriculum Framework 2009
Introduction

The 2009 *Mathematics Standards of Learning* Curriculum Framework is a companion document to the 2009 *Mathematics Standards of Learning* and amplifies the *Mathematics Standards of Learning* by defining the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The Curriculum Framework provides additional guidance to school divisions and their teachers as they develop an instructional program appropriate for their students. It assists teachers in their lesson planning by identifying essential understandings, defining essential content knowledge, and describing the intellectual skills students need to use. This supplemental framework delineates in greater specificity the content that all teachers should teach and all students should learn.

Each topic in the *Mathematics Standards of Learning* Curriculum Framework is developed around the Standards of Learning. The format of the Curriculum Framework facilitates teacher planning by identifying the key concepts, knowledge and skills that should be the focus of instruction for each standard. The Curriculum Framework is divided into two columns: Essential Understandings and Essential Knowledge and Skills. The purpose of each column is explained below.

Essential Understandings
This section delineates the key concepts, ideas and mathematical relationships that all students should grasp to demonstrate an understanding of the Standards of Learning.

Essential Knowledge and Skills
Each standard is expanded in the Essential Knowledge and Skills column. What each student should know and be able to do in each standard is outlined. This is not meant to be an exhaustive list nor a list that limits what is taught in the classroom. It is meant to be the key knowledge and skills that define the standard.

The Curriculum Framework serves as a guide for Standards of Learning assessment development. Assessment items may not and should not be a verbatim reflection of the information presented in the Curriculum Framework. Students are expected to continue to apply knowledge and skills from Standards of Learning presented in previous grades as they build mathematical expertise.

TOPIC: GRAPHS

DISCRETE MATHEMATICS
STANDARD DM.1

The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A tournament is a digraph that results from giving directions to the edges of a complete graph. • Adjacent vertices are connected by an edge. • In a connected graph, every pair of vertices is adjacent.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find the valence of each vertex in a graph. • Use graphs to model situations in which the vertices represent objects, and edges (drawn between vertices) represent a particular relationship between objects. • Represent the vertices and edges of a graph as an adjacency matrix, and use the matrix to solve problems. • Investigate and describe valence and connectedness. • Determine whether a graph is planar or nonplanar. • Use directed graphs (digraphs) to represent situations with restrictions in traversal possibilities.

TOPIC: GRAPHS

DISCRETE MATHEMATICS
STANDARD DM.2

The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Euler's Theorem states: If *G* is a connected graph and all its valences are even, then *G* has an Euler Circuit. • Pairs of routes (circuits) correspond to the same Hamilton Circuit because one route can be obtained from the other by traversing the vertices in reverse order. There are $\frac{(n-1)!}{2}$ Hamilton Circuits. • A multigraph is connected if there is a path between every pair of vertices.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine if a graph has an Euler Circuit or Path, and find it. • Determine if a graph has a Hamilton Circuit or Path, and find it. • Count the number of Hamilton Circuits for a complete graph with *n* vertices. • Use the Euler Circuit algorithm to solve optimization problems.

TOPIC: GRAPHS

DISCRETE MATHEMATICS
STANDARD DM.3

The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. Graph coloring and chromatic number will be used.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Every planar graph has a chromatic number that is less than or equal to four (the four-color-map theorem). • A graph can be colored with two colors if and only if it contains no cycle of odd length. • The chromatic number of a graph cannot exceed one more than the maximum number of degrees of the vertices of the graph.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model projects consisting of several subtasks, using a graph. • Use graphs to resolve conflicts that arise in scheduling.

TOPIC: GRAPHS

DISCRETE MATHEMATICS
STANDARD DM.4

The student will apply algorithms, such as Kruskal’s, Prim’s, or Dijkstra’s, relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• A spanning tree of a connected graph *G* is a tree that is a subgraph of *G* and contains every vertex of *G*.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use Kruskal’s Algorithm to find the shortest spanning tree of a connected graph. • Use Prim’s Algorithm to find the shortest spanning tree of a connected graph. • Use Dijkstra’s Algorithm to find the shortest spanning tree of a connected graph.

TOPIC: ELECTION THEORY AND FAIR DIVISION

DISCRETE MATHEMATICS
STANDARD DM.7

The student will analyze and describe the issue of fair division (e.g., cake cutting, estate division). Algorithms for continuous and discrete cases will be applied.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Group decision making combines the wishes of many to yield a single fair result. • A fair division problem may be discrete or continuous. • The success of the estate division algorithm requires that each heir be capable of placing a value on each object in the estate. • A fair division problem consists of *n* individuals (players) who must partition some set of goods, *s*, into *n* disjoint sets.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Investigate and describe situations involving discrete division (e.g., estate division). • Use an algorithm for fair division for a group of indivisible objects. • Investigate and describe situations involving continuous division of an infinitely divisible set (e.g., cake cutting). • Use an algorithm for fair division of an infinitely divisible set.

TOPIC: ELECTION THEORY AND FAIR DIVISION

DISCRETE MATHEMATICS
STANDARD DM.8

The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Historically, popular voting methods have often led to counterintuitive results. • A candidate who wins over every other candidate in a one-on-one ballot is a Condorcet winner. • A Borda count assigns points in descending order to each voter's subsequent ranking and then adds these points to arrive at a group's final ranking. • To select a voting system is to compromise between the shortcomings inherent in each system.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Determine in how many different ways a voter can rank choices. • Investigate and describe the following voting procedures: – weighted voting; – plurality; – majority; – sequential (winners run off); – sequential (losers are eliminated); – Borda count; and – Condorcet winner. • Compare and contrast different voting procedures. • Describe the possible effects of approval voting, insincere and sincere voting, a preference schedule, and strategic voting on the election outcome.

TOPIC: ELECTION THEORY AND FAIR DIVISION

DISCRETE MATHEMATICS
STANDARD DM.9

The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The apportionment of Congressional representatives is based on the latest census.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Compare and contrast the Hamilton and Jefferson methods of political apportionment with the Hill-Huntington method (currently in use in the U.S. House of Representatives) and the Webster-Willcox method. • Solve allocation problems, using apportionment methods. • Investigate and describe how salary caps affect apportionment.

TOPIC: COMPUTER MATHEMATICS

DISCRETE MATHEMATICS
STANDARD DM.11

The student will describe and apply sorting algorithms and coding algorithms used in sorting, processing, and communicating information. These will include

a) bubble sort, merge sort, and network sort; and
b) ISBN, UPC, Zip, and banking codes.

<table>
<tr><th>ESSENTIAL UNDERSTANDINGS</th><th>ESSENTIAL KNOWLEDGE AND SKILLS</th></tr>
<tr><td>

A bubble sort orders elements of an array by comparing adjacent elements.
A merge sort combines two sorted lists into a single sorted list.
Coding algorithms must account for the number of possible codes within the constraints of the coding system.

</td><td>
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to

Select and apply a sorting algorithm, such as a
– bubble sort;
– merge sort; and
– network sort.
Describe and apply a coding algorithm, such as
– ISBN numbers;
– UPC codes;
– Zip codes; and
– banking codes.

</td></tr>
</table>

TOPIC: COMPUTER MATHEMATICS

DISCRETE MATHEMATICS
STANDARD DM.12

The student will select, justify, and apply an appropriate technique to solve a logic problem. Techniques will include Venn diagrams, truth tables, and matrices.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Two-valued (Boolean) algebra serves as a workable method for interpreting the logical truth and falsity of compound statements. • Venn diagrams provide pictures of topics in set theory, such as intersection and union, mutually exclusive sets, and the empty set.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Generate truth tables that encode the truth and falsity of two or more statements. • Use Venn diagrams to codify and solve logic problems. • Use matrices as arrays of data to solve logic problems.

TOPIC: RECURSION AND OPTIMIZATION

DISCRETE MATHEMATICS
STANDARD DM.5

The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Critical path scheduling sometimes yields optimal solutions.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Specify in a digraph the order in which tests are to be performed. • Identify the critical path to determine the earliest completion time (minimum project time). • Use the list-processing algorithm to determine an optimal schedule. • Create and test scheduling algorithms.

TOPIC: RECURSION AND OPTIMIZATION

DISCRETE MATHEMATICS
STANDARD DM.6

The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Linear programming models an optimization process. • A linear programming model consists of a system of constraints and an objective quantity that can be maximized or minimized. • Any maximum or minimum value for a system of inequalities will occur at a corner point of a feasible region.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Model real-world problems with systems of linear inequalities. • Identify the feasibility region of a system of linear inequalities with no more than four constraints. • Identify the coordinates of the corner points of a feasibility region. • Find the maximum or minimum value of the system. • Describe the meaning of the maximum or minimum value in terms of the original problem.

TOPIC: RECURSION AND OPTIMIZATION

DISCRETE MATHEMATICS
STANDARD DM.10

The student will use the recursive process and difference equations with the aid of appropriate technology to generate

- **a) compound interest;**
- **b) sequences and series;**
- **c) fractals;**
- **d) population growth models; and**
- **e) the Fibonacci sequence.**

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• Recursion is a process that creates new objects from existing objects that were created by the same process. • A fractal is a figure whose dimension is not a whole number. • Fractals are self-similar.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Use finite differences and recursion to model compound interest and population growth situations. • Model arithmetic and geometric sequences and series recursively. • Compare and contrast the recursive process, and create fractals. • Compare and contrast the recursive process and the Fibonacci sequence. • Find a recursive relationship that generates the Fibonacci sequence.

TOPIC: RECURSION AND OPTIMIZATION

DISCRETE MATHEMATICS
STANDARD DM.13

The student will apply the formulas of combinatorics in the areas of

a) the Fundamental (Basic) Counting Principle;
b) knapsack and bin-packing problems;
c) permutations and combinations; and
d) the pigeonhole principle.

ESSENTIAL UNDERSTANDINGS	ESSENTIAL KNOWLEDGE AND SKILLS
• The branch of mathematics that addresses the number of ways objects can be arranged or combined is combinatorics. • If n and r are positive integers and $n \geq r$, $$n\,P\,r = \frac{n!}{(n-r)!} \quad \text{and} \quad n\,C\,r = \frac{n!}{r!\,(n-r)!}.$$ • A bin-packing problem determines the minimum number of containers of fixed volume (bins) required to hold a set of objects. • A knapsack problem determines the most valuable set of objects that fit into a container (knapsack) of fixed volume. • Bin packing and knapsack packing are optimization techniques.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Find the number of combinations possible when subsets of r elements are selected from a set of n elements without regard to order. • Use the Fundamental (Basic) Counting Principle to determine the number of possible outcomes of an event. • Use the knapsack and bin-packing algorithms to solve real-world problems. • Find the number of permutations possible when r objects selected from n objects are ordered. • Use the pigeonhole principle to solve packing problems to facilitate proofs.