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**NOTICE**

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**Virginia 2016 *Mathematics Standards of Learning* *Curriculum Framework***

**Introduction**

The 2016 *Mathematics Standards of Learning* *Curriculum Framework*, a companion document to the 2016 *Mathematics Standards of Learning*, amplifies the *Mathematics Standards of Learning* and further defines the content knowledge, skills, and understandings that are measured by the Standards of Learning assessments. The standards and *Curriculum Framework* are not intended to encompass the entire curriculum for a given grade level or course. School divisions are encouraged to incorporate the standards and *Curriculum Framework* into a broader, locally designed curriculum. The *Curriculum Framework* delineates in greater specificity the minimum content that all teachers should teach and all students should learn. Teachers are encouraged to go beyond the standards as well as to select instructional strategies and assessment methods appropriate for all students.

The *Curriculum Framework* also serves as a guide for Standards of Learning assessment development. Students are expected to continue to connect and apply knowledge and skills from Standards of Learning presented in previous grades as they deepen their mathematical understanding. Assessment items may not and should not be a verbatim reflection of the information presented in the *Curriculum Framework*.

Each topic in the 2016 *Mathematics Standards of Learning* *Curriculum Framework* is developed around the Standards of Learning. The format of the *Curriculum Framework* facilitates teacher planning by identifying the key concepts, knowledge, and skills that should be the focus of instruction for each standard. The *Curriculum Framework* is divided into two columns: Understanding the Standard and Essential Knowledge and Skills. The purpose of each column is explained below.

*Understanding the Standard*

This section includes mathematical content and key concepts that assist teachers in planning standards-focused instruction. The statements may provide definitions, explanations, examples, and information regarding connections within and between grade level(s)/course(s).

*Essential Knowledge and Skills*

This section provides a detailed expansion of the mathematics knowledge and skills that each student should know and be able to demonstrate. This is not meant to be an exhaustive list of student expectations.

**Mathematical Process Goals for Students**

The content of the mathematics standards is intended to support the following five process goals for students: becoming mathematical problem solvers, communicating mathematically, reasoning mathematically, making mathematical connections, and using mathematical representations to model and interpret practical situations. Practical situations include real-world problems and problems that model real-world situations.

**Mathematical Problem Solving**

Students will apply mathematical concepts and skills and the relationships among them to solve problem situations of varying complexities. Students also will recognize and create problems from real-world data and situations within and outside mathematics and then apply appropriate strategies to determine acceptable solutions. To accomplish this goal, students will need to develop a repertoire of skills and strategies for solving a variety of problems. A major goal of the mathematics program is to help students apply mathematics concepts and skills to become mathematical problem solvers.

**Mathematical Communication**

Students will communicate thinking and reasoning using the language of mathematics, including specialized vocabulary and symbolic notation, to express mathematical ideas with precision. Representing, discussing, justifying, conjecturing, reading, writing, presenting, and listening to mathematics will help students clarify their thinking and deepen their understanding of the mathematics being studied. Mathematical communication becomes visible where learning involves participation in mathematical discussions.

**Mathematical Reasoning**

Students will recognize reasoning and proof as fundamental aspects of mathematics. Students will learn and apply inductive and deductive reasoning skills to make, test, and evaluate mathematical statements and to justify steps in mathematical procedures. Students will use logical reasoning to analyze an argument and to determine whether conclusions are valid. In addition, students will use number sense to apply proportional and spatial reasoning and to reason from a variety of representations.

**Mathematical Connections**

Students will build upon prior knowledge to relate concepts and procedures from different topics within mathematics and see mathematics as an integrated field of study. Through the practical application of content and process skills, students will make connections among different areas of mathematics and between mathematics and other disciplines, and to real-world contexts. Science and mathematics teachers and curriculum writers are encouraged to develop mathematics and science curricula that support, apply, and reinforce each other.

**Mathematical Representations**

Students will represent and describe mathematical ideas, generalizations, and relationships using a variety of methods. Students will understand that representations of mathematical ideas are an essential part of learning, doing, and communicating mathematics. Students should make connections among different representations – physical, visual, symbolic, verbal, and contextual – and recognize that representation is both a process and a product.

**Instructional Technology**

The use of appropriate technology and the interpretation of the results from applying technology tools must be an integral part of teaching, learning, and assessment. However, facility in the use of technology shall not be regarded as a substitute for a student’s understanding of quantitative and algebraic concepts and relationships or for proficiency in basic computations. Students must learn to use a variety of methods and tools to compute, including paper and pencil, mental arithmetic, estimation, and calculators. In addition, graphing utilities, spreadsheets, calculators, dynamic applications, and other technological tools are now standard for mathematical problem solving and application in science, engineering, business and industry, government, and practical affairs.

Calculators and graphing utilities should be used by students for exploring and visualizing number patterns and mathematical relationships, facilitating reasoning and problem solving, and verifying solutions. However, according to the National Council of Teachers of Mathematics, “…the use of calculators does not supplant the need for students to develop proficiency with efficient, accurate methods of mental and pencil-and-paper calculation and in making reasonable estimations.” State and local assessments may restrict the use of calculators in measuring specific student objectives that focus on number sense and computation. On the grade three state assessment, all objectives are assessed without the use of a calculator. On the state assessments for grades four through seven, objectives that are assessed without the use of a calculator are indicated with an asterisk (\*).

**Computational Fluency**

Mathematics instruction must develop students’ conceptual understanding, computational fluency, and problem-solving skills. The development of related conceptual understanding and computational skills should be balanced and intertwined, each supporting the other and reinforcing learning.

Computational fluency refers to having flexible, efficient, and accurate methods for computing.  Students exhibit computational fluency when they demonstrate strategic thinking and flexibility in the computational methods they choose, understand, and can explain, and produce accurate answers efficiently.

The computational methods used by a student should be based on the mathematical ideas that the student understands, including the structure of the base-ten number system, number relationships, meaning of operations, and properties. Computational fluency with whole numbers is a goal of mathematics instruction in the elementary grades.  Students should be fluent with the basic number combinations for addition and subtraction to 20 by the end of grade two and those for multiplication and division by the end of grade four.   Students should be encouraged to use computational methods and tools that are appropriate for the context and purpose.

**Algebra Readiness**

The successful mastery of Algebra I is widely considered to be the gatekeeper to success in the study of upper-level mathematics. “Algebra readiness” describes the mastery of, and the ability to apply, the *Mathematics Standards of Learning*, including the Mathematical Process Goals for Students, for kindergarten through grade eight. The study of algebraic thinking begins in kindergarten and is progressively formalized prior to the study of the algebraic content found in the Algebra I Standards of Learning. Included in the progression of algebraic content is patterning, generalization of arithmetic concepts, proportional reasoning, and representing mathematical relationships using tables, symbols, and graphs. The K-8 *Mathematics Standards of Learning* form a progression of content knowledge and develop the reasoning necessary to be well-prepared for mathematics courses beyond Algebra I, including Geometry and Statistics.

**Equity**

**“**Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic, linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.”   
 – National Council of Teachers of Mathematics

Mathematics programs should have an expectation of equity by providing all students access to quality mathematics instruction and offerings that are responsive to and respectful of students’ prior experiences, talents, interests, and cultural perspectives. Successful mathematics programs challenge students to maximize their academic potential and provide consistent monitoring, support, and encouragement to ensure success for all. Individual students should be encouraged to choose mathematical programs of study that challenge, enhance, and extend their mathematical knowledge and future opportunities.

Student engagement is an essential component of equity in mathematics teaching and learning. Mathematics instructional strategies that require students to think critically, to reason, to develop problem-solving strategies, to communicate mathematically, and to use multiple representations engages students both mentally and physically. Student engagement increases with mathematical tasks that employ the use of relevant, applied contexts and provide an appropriate level of cognitive challenge. All students, including students with disabilities, gifted learners, and English language learners deserve high-quality mathematics instruction that addresses individual learning needs, maximizing the opportunity to learn.

Students in kindergarten through grade two have a natural curiosity about their world, which leads them to develop a sense of number. Young children are motivated to count everything around them and begin to develop an understanding of the size of numbers (magnitude), multiple ways of thinking about and representing numbers, strategies and words to compare numbers, and an understanding of the effects of simple operations on numbers. Building on their own intuitive mathematical knowledge, they also display a natural need to organize things by sorting, comparing, ordering, and labeling objects in a variety of collections.

Consequently, the focus of instruction in the number and number sense strand is to promote an understanding of counting, classification, whole numbers, place value, fractions, number relationships (“more than,” “less than,” and “equal to”), and the effects of single-step and multistep computations. These learning experiences should allow students to engage actively in a variety of problem-solving situations and to model numbers (compose and decompose), using a variety of manipulatives. Additionally, students at this level should have opportunities to observe, to develop an understanding of the relationship they see between numbers, and to develop the skills to communicate these relationships in precise, unambiguous terms.

| **1.1 The student will**  a) count forward orally by ones to 110, starting at any number between 0 and 110;   1. write the numerals 0 to 110 in sequence and out-of-sequence; 2. **count backward orally by ones when given any number between 1 and 30; and** 3. count forward orally by ones, twos, fives, and tens to determine the total number of objects to 110. | |
| --- | --- |
| Understanding the Standard | Essential Knowledge and Skills |
| * There are three developmental levels of counting: * rote sequence; * one-to-one correspondence; and * the cardinality of numbers. * Counting involves two separate skills: verbalizing the list (rote sequence counting) of standard number words in order (“one, two, three,…”) and connecting this sequence with the items in the set being counted, using one-to-one correspondence. Association of number words with collections of objects is achieved by moving, touching, or pointing to objects as the number words are spoken. * The last number stated represents the number of objects in the set. The total number of objects in the set is known as the cardinality of the set. After having a student count a collection of objects, the teacher may be able to assess whether the student has cardinality of number by asking the question, “How many are there?” Students who do not yet have cardinality of number are often unable to tell you how many objects there were without recounting them. * Rote counting is a prerequisite skill for the understanding of addition (one more), subtraction (one less), and the ten-to-one concept of place value. * Conservation of number is applied when students understand that a group of 10 objects is still 10 objects regardless of whether they are arranged in a cup, group, stack, etc. * Unitizing is the concept that a group of objects can be counted as one unit (e.g., 10 ones can be counted as 1 ten.) * Using objects and asking the questions such as “How many are in each group?” or “How many groups are there?” and” What is the total number you have?” supports students as they learn to skip count and helps to solidify their understanding of cardinality and assists in developing multiplicative reasoning. * The patterns developed as a result of skip counting are precursors for recognizing numeric patterns, functional relationships, and concepts underlying money, time, and multiplication.   Powerful models for developing these concepts include counters, number paths, and hundred charts.  Example of a Number Path   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |      * A number path is a counting model where each number is represented within a square and the squares can be clearly counted. * A number line is a length model with each number being represented by its length from zero. When young children use a number line as a counting tool, they often confuse what should be counted, the numbers or the spaces between the numbers. A number path is more appropriate for students at this age. * Counting backward by rote lays the foundation for subtraction. * Skip counting by twos supports the development of the concept of even numbers and the development of multiplication facts for two. * Skip counting by fives lays the foundation for telling time to the nearest five minutes, counting money, and developing the multiplication facts for five. * Skip counting by tens is a precursor for place value, addition (10 more), subtraction (10 less), counting money, and developing the multiplication facts for ten. * Manipulatives that can be physically connected and separated into groups of tens and leftover ones, such as connecting or snap cubes, beans on craft sticks, pennies in cups, bundle of sticks, or beads on pipe cleaners should be used. * Ten-to-one trading activities with manipulatives on place value mats and base ten blocks are more appropriate in grade two. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Count forward orally, by ones, from 0 to 110 starting at any number between 0 and 110. (a) * Use the oral counting sequence to tell how many objects are in a set. (a) * Write numerals 0-110 in sequence and out of sequence. (b) * Count backward orally by ones when given any number between 1 and 30. (c) * Count forward orally by ones, twos, fives, and tens to determine the total number of objects to 110. (d) |

| **1.2 The student, given up to 110 objects, will**  a) group a collection into tens and ones and write the corresponding numeral;  b) compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words *greater than, less than* or *equal to*;and  **c) order three or fewer sets from least to greatest and greatest to least.** | |
| --- | --- |
| Understanding the Standard | Essential Knowledge and Skills |
| * Conservation of number is applied when students understand that a group of 10 objects is still 10 objects regardless of whether they are arranged in a cup, group, stack, etc. * Unitizing is the concept that a group of objects can be counted as one unit (e.g., 10 ones can be counted as 1 ten). * The number system is based on a pattern of tens where each place has 10 times the value of the place to its right. This is known as the ten-to-one concept of place value. * Opportunities to experience the relationships among tens and ones through hands-on experiences with manipulatives are essential to developing the ten-to-one place value concept of our number system and to understanding the value of each digit in a two-digit number. * Models that clearly illustrate the relationships among tens and ones as physically proportional are most appropriate for this grade (e.g., the tens piece is 10 times larger than the ones piece). * Knowledge of place value is essential when comparing numbers. * Students are generally familiar with the concept of *more* and have limited experience with the term *less*. It is important to use the terms together to build understanding of their relationship. For example, when asking which group has more, follow by asking which group has fewer. * Recording the numeral when using physical and pictorial models leads to an understanding that the position of each digit in a numeral determines the size of the group it represents. * Manipulatives that can be physically connected and separated into groups of tens and leftover ones, such as connecting or snap cubes, beans on craft sticks, pennies in cups, bundle of sticks, or beads on pipe cleaners should be used. * Ten-to-one trading activities with manipulatives on place value mats and base-ten blocks are more appropriate in grade two. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Group a collection of up to 110 objects into sets of tens and ones. (a) * Write the numeral that corresponds to the total number of objects in a given collection of up to 110 objects that have been grouped into sets of tens and ones. (a) * Identify the place and value of each digit in a two-digit numeral (e.g., in the number 23, the 2 is in the tens place and the value of the 2 is 20). (a) * Identify the number of tens and ones that can be made from any number up to 100 (e.g., 47 is 47 ones or can also be grouped into 4 tens with 7 ones left over). (a) * Compare two numbers between 0 and 110 represented pictorially or with concrete objects, using the words *greater than, less than* or *equal to.* (b) * Order three or fewer sets, each set containing up to 110 objects, from least to greatest and greatest to least. (c) |

| **1.3 The student, given an ordered set of ten objects and/or pictures, will indicate the ordinal position of each object, first through tenth.** | |
| --- | --- |
| Understanding the Standard | Essential Knowledge and Skills |
| * An understanding of the cardinal and ordinal meanings of numbers is necessary to quantify, measure, and identify the order of objects. * An ordinal number is a number that names the place or position of an object in a sequence or set (e.g., first, third). *Ordered position*, *ordinal position*, and *ordinality* are terms that refer to the place or position of an object in a sequence or set. * The ordinal position is determined by where one starts in an ordered set of objects or sequence of objects. * The ordinal meaning of numbers is developed by identifying and verbalizing the place or position of objects in a set or sequence (e.g., the student’s position in line when students are lined up alphabetically by first name). * Students at this level do not need to recognize or read the written words for ordinal numbers (i.e., first, second, third, etc.). * Practical applications of ordinal numbers can be experienced through calendar and patterning activities. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Identify the ordinal positions first through tenth using ordered sets of 10 objects and/or pictures of such sets presented from: * left to right; * right to left; * top to bottom; and/or * bottom to top. |

| **1.4 The student will**  **a) represent and solve practical problems involving equal sharing with two or four sharers; and**  **b) represent and name fractions for halves and fourths, using models.** | |
| --- | --- |
| Understanding the Standard | Essential Knowledge and Skills |
| * Practical situations with fractions should involve real-life problems in which students themselves determine how to subdivide a whole into equal parts, test those parts to be sure they are equal, and use those parts to re-create the whole. * When working with fractions, the whole must be defined. * Fractions can have different meanings: part-whole, division, measurement, ratio, and operator. The focus of this grade level is to develop the idea of equal sharing (division) and part-whole relationships. Fraction notation will be introduced in grade two. * An equal sharing problem is an idea that young children understand intuitively because of their experiences sharing objects with siblings, friends, etc. Consider the following examples: * Two children sharing six sandwiches * Two children sharing one sandwich * Four children sharing one piece of paper * Four children sharing two brownies * Fraction models that can be continuously divided should be used at this grade (e.g., cookies, brownies). It is important to use models that can be continuously divided when there are remainders so those remainders can be cut into as many equal parts as needed. * Students should be encouraged to observe and state what happens as you add more sharers, noticing that when more sharers are added, the smaller the share will be for each person. * Equal parts may be different shapes but maintain the same value (e.g., a sandwich could be cut in two equal pieces vertically, horizontally, or diagonally to represent halves). * The words *denominator* and *numerator* are not required at this grade, but the concepts of part and whole are required for understanding of a fraction. * Students should use the vocabulary for halves and fourths, but should not be expected to use fraction notation at this level. Informal, integrated experiences with fractions at this level will help students develop a foundation for deeper learning at later grades. Understanding the language of fractions furthers this development. * Students should have opportunities to make connections and comparisons among fraction representations by connecting concrete or pictorial representations with spoken representations (e.g., “one-half,” “one out of two equal parts, ”or “one-half is more than one-fourth of the same whole”). | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Share a whole equally with two or four sharers, when given a practical situation. (a) * Represent fair shares pictorially, when given a practical situation. (a) * Describe shares as equal pieces or parts of the whole (e.g., halves, fourths), when given a practical situation. (a) * Represent halves and fourths of a whole, using a region/area model (e.g., pie pieces, pattern blocks, paper folding, and drawings). (b) * Name fractions represented by drawings or concrete materials for halves and fourths. (b) |

| **1.5 The student, given a familiar problem situation involving magnitude, will**  a) select a reasonable order of magnitude from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, 500); and  b) explain the reasonableness of the choice. | |
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| Understanding the Standard | Essential Knowledge and Skills |
| * Magnitude refers to the size of a set. * Exploring ways to estimate the number of objects in a set, based on appearance (e.g., clustering, grouping, comparing), enhances the development of number sense. * To estimate means to determine a number that is close to the exact amount. When asking for an estimate, teachers might ask, “A*bout* how much?” or “A*bout* how many?” or “Is this about 10 or about 50?” * Students should be provided opportunities to estimate a quantity, given a benchmark of 10 and/or 100 objects. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Select a reasonable order of magnitude for a given set from three given quantities: a one-digit numeral, a two-digit numeral, and a three-digit numeral (e.g., 5, 50, or 500 jelly beans in jars) in a familiar problem situation. (a) * Explain why a particular estimate was chosen as the most reasonable from three given quantities (a one‑digit numeral, a two‑digit numeral, and a three‑digit numeral), given a familiar problem situation. (b) |

A variety of contexts and problem types are necessary for children to develop an understanding of the meanings of the operations such as addition and subtraction. These contexts often arise from practical experiences in which they are simply joining sets, taking away or separating from a set, or comparing sets. These contexts might include conversations, such as “How many books do we have altogether?” or “How many cookies are left if I eat two?” or “I have three more candies than you do.” Although young children first compute using objects and manipulatives, they gradually shift to performing computations mentally or using paper and pencil to record their thinking. Therefore, computation and estimation instruction in the early grades revolves around modeling, discussing, and recording a variety of problem situations. This approach helps students transition from the concrete to the representation to the symbolic in order to develop meaning for the operations and how they relate to each other.

In kindergarten through grade two, computation and estimation instruction focuses on:

* relating the mathematical language and symbolism of operations to problem situations;
* understanding different meanings of addition and subtraction of whole numbers and the relation between the two operations;
* developing proficiency with basic addition, and subtraction within 20;
* gaining facility in manipulating whole numbers to add and subtract and in understanding the effects of the operations on whole numbers;
* developing and using strategies and algorithms to solve problems and choosing an appropriate method for the situation;
* choosing, from mental computation, estimation, paper and pencil, and calculators, an appropriate way to compute;
* recognizing whether numerical solutions are reasonable; and
* experiencing situations that lead to multiplication and division, such as skip counting and solving problems that involve equal groupings of objects as well as problems that involve sharing equally, the initial work with fractions.

| **1.6 The student will create and solve single-step story and picture problems using addition and subtraction within 20.** | |
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| Understanding the Standard | Essential Knowledge and Skills |
| * Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. * The problem-solving process is enhanced when students:   + create their own story problems;   + visualize the action in the story problem and draw a picture to show their thinking; and   + model the problem using manipulatives, representations, or number sentences/equations. * The least number of steps necessary to solve a single-step problem is one. * In problem solving, emphasis should be placed on thinking and reasoning rather than on key words.  Focusing on key words such as *in all, altogether, difference,* etc.,encourages students to perform a particular operation rather than make sense of the context of the problem.  A key-word focus prepares students to solve a limited set of problems and often leads to incorrect solutions as well as challenges in upcoming grades and courses. * Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the basic addition and subtraction facts include:   + counting on;   + counting back;   + “one more than,” “two more than”;   + “one less than,” “two less than”;   + “doubles” (e.g., 6 + 6 =\_\_);   + “near doubles” (e.g., 7 + 8 = (7 + 7) + 1 = or (8 + 8) – 1);   + “make ten” (e.g., 7 + 4 can be thought of as 7 + 3 + 1 in order to make a 10 );   + “think addition for subtraction” (e.g., for 9 – 5 = \_\_, think “5 and what number makes 9?”);   + use of the commutative property (e.g., 14 +3 is the same as 3 + 14);   + use of related facts (e.g., 14 + 3 = 17 , 3 + 14 = 17, 17 – 4 = 13, and 17 – 13 = 4);   + use of the additive identity property (e.g., 14 + 0 = 14); and   + use patterns to make sums (e.g., 0 + 15 = 15, 1 + 14 = 15, 2 + 13 = 15, etc.). * Students at this level are not expected to use the parentheses or to name the properties. * Students should develop fluency with facts to 10 and then use strategies and known facts to 10 to determine facts to 20. * Flexibility with facts to 10 should be applied to facts to 20 (e.g., when adding 4 + 7, it is appropriate to think of 4 as 3 + 1 in order to combine 3 and 7 to make a 10 whereas adding 4 + 8, it is appropriate to think of 4 as 2 + 2 in order to combine 8 and 2 to make a 10). * Extensive research has been undertaken over the last several decades regarding different problem types. Many of these studies have been published in professional mathematics education publications using different labels and terminology to describe the varied problem types. * Students should have exposure to a variety of problem types related to addition and subtraction. Examples are represented in the chart below. It is important to note that Join Problems (with start unknown), Separate Problems (with start unknown), Compare Problems (with larger unknown – using “fewer”) and Compare problems (with smaller unknown – using “more”) are the most difficult and should be mastered in grade two.  |  |  |  | | --- | --- | --- | | **GRADE 1: COMMON ADDITION AND SUBTRACTION PROBLEM TYPES** | | | | Join  (Result Unknown) | Join  (Change Unknown) | Join  (Start Unknown) | | Sue had 9 pencils. Alex gave her 5 more pencils. How many pencils does Sue have altogether? | Sue had 9 pencils. Alex gave her some more pencils. Now Sue has 14 pencils. How many did Alex give her? | Sue had some pencils. Alex gave her 5 more. Now Sue has 14 pencils. How many pencils did Sue have to start with? | | Separate  (Result Unknown) | Separate  (Change Unknown) | Separate  (Start Unknown) | | Brooke had 10 cookies. She gave 6 cookies to Joe. How many cookies does Brooke have now? | Brooke had 10 cookies. She gave some to Joe. She has 4 cookies left. How many cookies did Brooke give to Joe? | Brooke had some cookies. She gave 6 to Joe. Now she has 4 cookies left. How many cookies did Brooke start with? | | Part-Part-Whole  (Whole Unknown) | Part-Part-Whole  (One Part Unknown) | Part-Part-Whole  (Both Parts Unknown) | | Lisa has 4 red markers and 8 blue markers. How many markers does she have? | Lisa has 12 markers. Four of the markers are red, and the rest are blue. How many blue markers does Lisa have? | Lisa has a pack of red and blue markers. She has 12 markers in all. How many markers could be red? How many could be blue? | | Compare  (Difference Unknown) | Compare  (Bigger Unknown) | Compare  (Smaller Unknown) | | Ryan has 7 books and Chris has 2 books. How many more books does Ryan have than Chris?  Ryan has 7 books. Chris has 2 books. How many fewer books does Chris have than Ryan? | Chris has 2 books. Ryan has 5 more books than Chris. How many books does Ryan have?  Chris has 5 fewer books than Ryan. Chris has 2 books. How many books does Ryan have? | Ryan has 2 more books than Chris. Ryan has 7 books. How many books does Chris have?  Chris has 5 fewer books than Ryan. Ryan has 7 books. How many books does Chris have? | | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Create and solve single-step oral or written story and picture problems, using addition and subtraction within 20. * Identify a number sentence to solve an oral or written story and picture problem, selecting from among addition and/or subtraction equations (e.g., number sentences). * Combine parts contained in larger numbers up to 20 by using related combinations (e.g., 9 + 7 can be thought of as 9 broken up into 2 and 7; using doubles, 7 + 7 = 14; 14 + 2 = 16 or 7 broken up into 1 and 6; making a ten, 1 + 9 = 10; 10 + 6 = 16). * Explain strategies used to solve addition and subtraction problems within 20 using spoken words, objects, pictorial models, and number sentences. |

| **1.7 The student will**   * 1. **recognize and describe with fluency part-whole relationships for numbers up to 10; and**   2. **demonstrate fluency with addition and subtraction within 10.** | |
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| Understanding the Standard | Essential Knowledge and Skills |
| * Computational fluency is the ability to think flexibly in order to choose appropriate strategies to solve problems accurately and efficiently. * Flexibility requires knowledge of more than one approach to solving a particular kind of problem. Being flexible allows students to choose an appropriate strategy for the numbers involved. * Mathematically fluent students are not only able to provide correct answers quickly but also to use facts and computation strategies they know to efficiently determine answers that they do not know. * Composing and decomposing numbers flexibly forms a basis for understanding properties of the operations and later formal algebraic concepts and procedures. * Parts of numbers to 10 should be represented in different ways, such as five frames, ten frames, strings of beads, arrangements of tiles or tooth picks, dot cards, or beaded number frames.      * Dot patterns should be presented in both regular and irregular arrangements. This will help students to understand that numbers are made up of parts, and it will later assist them in combining parts as well as counting on.      * Accuracy is the ability to determine a correct answer using knowledge of number facts and other important number relationships. * Efficiency is the ability to carry out a strategy easily when solving a problem without getting bogged down in too many steps or losing track of the logic of the strategy being used. * Addition and subtraction should be taught concurrently in order to develop understanding of the inverse relationship. * Manipulatives should be used to develop an understanding of addition and subtraction facts. * Automaticity of facts can be achieved through meaningful practice which may include games, hands-on activities, dot cards, and ten frames. * Subtraction is the inverse of addition. Subtraction can be viewed as a process of taking away or separating, or as a process of comparing two sets to determine the difference between them. * Provide practice in the use and selection of strategies. Encourage students to develop efficient strategies. Examples of strategies for developing the addition and subtraction facts include:   + counting on;   + counting back;   + “one more than,” “two more than”;   + “one less than,” “two less than”;   + “doubles” (e.g., 2 + 2 =\_\_; 3 + 3 =\_\_);   + “near doubles” (e.g., 3 + 4 = (3 + 3) + 1 = \_\_);   + “make ten” (7 + 4 can be thought of as 7 + 3 + 1 in order to make a ten));   + “think addition for subtraction” (e.g., for 9 – 5 = \_\_, think “5 and what number makes 9?”);   + use of the commutative property (e.g., 4 + 3 is the same as 3 + 4);   + use of related facts (e.g., 4 + 3 = 7 , 3 + 4 = 7, 7 – 4 = 3, and 7 – 3 = 4);   + use of the additive identity property (e.g., 4 + 0 = 4); and   + use patterns to make sums (e.g., 0 + 5 = 5, 1 + 4 = 5, 2 + 3 = 5, etc.). * Students at this level are not expected to name the properties. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Recognize and describe with fluency part-whole relationships for numbers up to 10 in a variety of configurations. (a) * Identify + as a symbol for addition, − as a symbol for subtraction, and = as a symbol for equality. (b) * Demonstrate fluency with addition and subtraction within 10. (b) |

The exploration of measurement and geometry in the primary grades allows students to learn more about the world around them. Measurement is important because it helps to quantify the world around us and is useful in so many aspects of everyday life. Students in kindergarten through grade two encounter measurement in their daily lives, from their use of the calendar and science activities that often require students to measure objects or compare them directly, to situations in stories they are reading and to descriptions of how quickly they are growing.

Measurement instruction at the primary level focuses on developing the skills and tools needed to measure length, weight, capacity, time, temperature, and money. Measurement at this level lends itself especially well to the use of concrete materials. Children can see the usefulness of measurement if classroom experiences focus on estimating and measuring real objects. They gain a deep understanding of the concepts of measurement when handling the materials, making physical comparisons, and measuring with tools.

As students develop a sense of the attributes of measurement and the concept of a measurement unit, they also begin to recognize the differences between using nonstandard and standard units of measure. Learning should give them opportunities to apply several techniques, direct comparison, nonstandard units, and standard tools to determine measurements and to develop an understanding of the use of U.S. Customary units.

Teaching measurement offers the challenge to involve students actively and physically in learning and is an opportunity to tie together other aspects of the mathematical curriculum, such as fractions and geometry. It is also one of the major vehicles by which mathematics can make connections with other content areas, such as science, health, and physical education.

Children begin to develop geometric and spatial knowledge before beginning school, stimulated by the exploration of figures and structures in their environment. Geometric ideas help children systematically represent and describe their world as they learn to represent plane and solid figures through drawing, block constructions, dramatization, and verbal language.

The focus of instruction at this level is on

* observing, identifying, describing, comparing, contrasting, and investigating solid objects and their faces;
* sorting objects and ordering them directly by comparing them one to the other;
* describing, comparing, contrasting, sorting, and classifying figures; and
* exploring symmetry.

In the primary grades, children begin to develop basic vocabulary related to figures but do not develop precise meanings for many of the terms they use until they are thinking beyond Level 2 of the van Hiele theory (see below).

The van Hiele theory of geometric understanding describes how students learn geometry and provides a framework for structuring student experiences that should lead to conceptual growth and understanding.

* **Level 0: Pre-recognition.** Geometric figures are not recognized. For example, students cannot differentiate between three-sided and four-sided polygons.
* **Level 1: Visualization.** Geometric figures are recognized as entities, without any awareness of the parts of figures or relationships between components of a figure. Students should recognize and name figures and distinguish a given figure from others that look somewhat the same. (This is the expected level of student performance during kindergarten and grade one.)
* **Level 2: Analysis.** Properties are perceived but are isolated and unrelated. Students should recognize and name properties of geometric figures. (Students are expected to transition to this level during grades two and three).
* **Level 3: Abstraction.** Definitions are meaningful, with relationships being perceived between properties and between figures. Logical implications and class inclusions are understood, but the role and significance of deduction is not understood. (Students should transition to this level during grades five and six and fully attain it before taking algebra.)

| **1.8 The student will determine the value of a collection of like coins (pennies, nickels, or dimes) whose total value is 100 cents or less.** | |
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| Understanding the Standard | Essential Knowledge and Skills |
| * Many experiences with coins help students develop an understanding of money, such as * drawing pennies to show the value of a given coin (e.g., nickel, dime, or quarter); * playing store and purchasing classroom objects, using play money (pennies); * using skip counting to count a collection of like coins; * representing the value of coins using a variety of organizers, such as five/ten frames or hundreds charts, pictures; and * trading the equivalent value of pennies for a nickel, a dime, and a quarter, using play money. * Counting coins is an application of unitizing. * Unitizing is the concept that a group of objects can be counted as one unit (e.g., 10 pennies can be counted as 1 dime.) * Counting money helps students gain an awareness of consumer skills and the use of money in everyday life. * A variety of classroom experiences in which students manipulate physical models of money and count forward to determine the value of a collection of coins are important activities to develop competence with counting money. * The last number stated represents the value of a collection of coins being counted. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Count by ones to determine the value of a collection of pennies whose total value is 100 cents or less. * Group a collection of pennies by fives and tens as a way to determine the value. The total value of the collection is 100 cents or less. * Count by fives to determine the value of a collection of nickels whose total value is 100 cents or less. * Count by tens to determine the value of a collection of dimes whose total value is 100 cents or less. |

| **1.9 The student will investigate the passage of time and**   1. **tell time to the hour and half-hour, using analog and digital clocks; and** 2. **read and interpret a calendar.** | |
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| Understanding the Standard | Essential Knowledge and Skills |
| * Many experiences using clocks help students develop an understanding of the telling of time to the hour and half-hour, including: * identifying the parts of an analog clock (minute and hour hands); * demonstrating a given time to the hour and half-hour, using a model clock; * writing digital time to the hour and half-hour; * relating time on the hour and half-hour to daily routines and school schedules (e.g., bedtime, lunch time, recess time); and * connecting the hour and half-hour to fraction concepts. * Practical situations are appropriate to develop a sense of the interval of time between events (e.g., club meetings occur every week on Monday; there is a week between meetings). * The calendar is a way to represent units of time (e.g., days, weeks, months). * Using a calendar develops the concept of day as a 24-hour period rather than a period of time from sunrise to sunset. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Identify different types of clocks (analog and digital) as instruments to measure time. (a)   + - Tell time shown on an analog clock to the hour and half-hour. (a)     - Tell time shown on a digital clock to the hour and half-hour. (a)     - Match a written time (e.g., 1:00, 3:30, 11:00) to the time shown on a digital and analog clock to the hour and half-hour. (a) * Read a calendar to locate a given day or date (e.g., What day of the week is the 10th? What date is Saturday?). (b) * Determine the day/date before and after a given day/date (e.g., Today is the 30th, so yesterday must have been the \_\_?). (b) * Given a calendar, determine the number of any day of the week (e.g., How many Fridays are in the month of October?) (b) |

| **1.10 The student will use nonstandard units to measure and compare length, weight, and volume.** | |
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| Understanding the Standard | Essential Knowledge and Skills |
| * The process of measurement involves selecting a unit of measure, comparing the unit to the object to be measured, counting the number of times the unit is used to measure the object, and arriving at an approximate total number of units. * Measurement involves comparing an attribute of an object to the same attribute of the unit of measurement (e.g., the length of a cube measures the length of a book; the weight of the cube measures the weight of the book; the volume of the cube measures the volume of a book). * Premature use of instruments or formulas leaves children without the understanding necessary for solving measurement problems. * When children’s initial explorations of length, weight, and volume involve the use of nonstandard units, they develop some understanding about the need for standard measurement units for length, weight, and volume, especially when they communicate about these measures. * The level of difficulty in measuring volume can be increased by varying and mixing the sizes and/or shapes of the containers (e.g., using short, wide containers as well as tall, narrow containers). * Students develop conservation of measurement when they understand that the attributes do not change when the object is manipulated (e.g., a piece of string that is coiled maintains its length as it is straightened; the volume of water does not change when poured from a pitcher into a fish tank.) * Physically measuring the weights of objects, using a balance scale, helps students develop an intuitive idea of what it means to say something is “lighter,” “heavier,” or “the same.” * Balance scales are instruments used for comparing weight. A balance scale usually has a beam that is supported in the center. On each side of the beam are two identical trays. When the trays hold equal weights, the beam is level, and the scale is “balanced.” The tray containing less weight will rise and the tray containing more weight will fall. * Experience estimating the weights of two objects (one in each hand) using the terms “lighter,” “heavier,” or “the same” promotes an understanding of the concept of balance. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Measure the length of objects, using various nonstandard units (e.g., connecting cubes, paper clips, erasers). * Compare the length of two objects, using the terms *longer/shorter, taller/shorter,* or *same as*. * Measure the weight of objects, using a balance or pan scale with various nonstandard units (e.g., paper clips, bean bags, cubes). * Identify a balance scale or a pan scale as a tool for measuring weight. * Compare the weight of two objects, using the terms *lighter, heavier*, or *the same*, using a balance scale. * Measure the volume of objects, using various nonstandard units (e.g., connecting cubes, blocks, rice, water). * Compare the volumes of two containers to determine whether the volume of one is *more*, *less*, or *equivalent to* the other, using nonstandard units of measure (e.g., a spoonful or scoopful of rice, sand, jelly beans). * Compare the volumes of two containers to determine whether the volume of one is *more*, *less*, or *equivalent to* the other by pouring the contents of one container into the other. |

| **1.11 The student will**   1. **Identify, trace, describe, and sort plane figures (triangles, squares, rectangles, and circles) according to number of sides, vertices, and angles; and** 2. **identify and describe representations of circles, squares, rectangles, and triangles in different environments, regardless of orientation, and explain reasoning.** | |
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| Understanding the Standard | Essential Knowledge and Skills |
| * Early experiences with comparing, sorting, and subdividing figures assist students in analyzing the characteristics of plane figures. * A plane figure is any closed, two-dimensional shape. * A vertex is the point at which two or more lines, line segments, or rays meet to form an angle. The term *vertices* is the plural form of vertex. * An angle is formed by two rays that share a common endpoint called the vertex. Angles are found wherever lines or line segments intersect. * A polygon is a closed plane figure composed of at least three line segments that do not cross. * A triangle is a polygon with three sides. * A quadrilateral is a polygon with four sides. * A rectangle is a quadrilateral with four right angles. * A square is a quadrilateral with four congruent (equal length) sides and four right angles. At this level, students might describe a square as a special rectangle with four sides of equal length. * Students at this level do not need to use the terms *polygon*, *quadrilateral*, or *congruent*. * A right angle measures exactly 90 degrees. * A circle is the set of points in a plane that are the same distance from a point called the center. A circle is not a polygon, because it does not have straight sides. * Presentation of triangles, rectangles, and squares should be made in a variety of spatial orientations so that students do not develop the common misconception that triangles, rectangles, and squares must have one side parallel to the bottom of the page on which they are printed. * Representations of circles, squares, rectangles, and triangles can be found in the students’ environment at school and at home. Students should have opportunities to identify/classify things in the environment by the type of figure those things represent. * A common misconception students have when a figure such as a square is rotated is that they will frequently refer to the rotated square as a diamond. Clarification needs to be ongoing (e.g., a square is a square regardless of its location in space; there is no plane figure called a diamond). * Building geometric and spatial capabilities fosters enthusiasm for mathematics while providing a context to develop spatial sense. * Polygons can be constructed using other polygons (e.g., six equilateral triangles can be used to construct a hexagon; a triangle can be added to a rectangle to create a pentagon, etc.). * Early experiences with comparing, sorting, and subdividing figures or manipulatives (e.g., pattern blocks) assist students in analyzing the characteristics of plane geometric figures. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Identify the name of the plane figure when given information about the number of sides, vertices, and angles. (a) * Trace triangles, squares, rectangles, and circles. (a) * Describe a circle using terms such as *round* and *curved*. (a) * Describe triangles, squares, and rectangles by the number of sides, vertices, and angles. (a) * Recognize that rectangles and squares have special types of angles called right angles. (a) * Sort plane figures based on their characteristics (number of sides, vertices, angles, curved, etc.). (a) * Identify and describe representations of circles, squares, rectangles, and triangles, regardless of orientation, in different environments and explain reasoning. (b) |

Students in the primary grades have a natural curiosity about their world, which leads to questions about how things fit together or connect. They display their natural need to organize things by sorting and counting objects in a collection according to similarities and differences with respect to given criteria.

The focus of probability instruction at this level is to help students begin to develop an understanding of the concept of chance. In grade two, students experiment with spinners, two-colored counters, dice, tiles, coins, and other manipulatives to explore the possible outcomes of situations and predict results. They begin to describe the likelihood of events, using the terms *impossible, unlikely, equally likely, more likely,* and *certain.*

The focus of statistics instruction at this level is to help students develop methods of collecting, organizing, describing, displaying, and interpreting data to answer questions they have posed about themselves and their world.

| **1.12 The student will**   1. **collect, organize, and represent various forms of data using tables, picture graphs, and object graphs; and** 2. **read and interpret data displayed in tables, picture graphs, and object graphs, using the vocabulary *more, less, fewer, greater than, less than,* and *equal to*.** | |
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| Understanding the Standard | Essential Knowledge and Skills |
| * Data are pieces of information collected about people or things. The primary purpose of collecting data is to answer questions. The primary purpose of interpreting data is to inform decisions (e.g., which type of clothing to pack for a vacation based on a weather graph or which type of lunch to serve based upon class favorites). * After generating questions, students decide what information is needed and how it can be collected. * The collection of the data often leads to new questions to be investigated. * Data collection could involve voting, informal surveys, tallying, and charts (e.g., recording daily temperature, lunch count, attendance, favorite ice cream). * Surveys, which are data-collecting tools that list choices, should have a limited number of questions at the primary grades. * Tallying is a method for gathering information. Tally marks are used to show how often something happens or occurs. Each tally mark represents one occurrence. Tally marks are clustered into groups of five, with four vertical marks representing the first four occurrences and the fifth mark crossing the first four on a diagonal to represent the fifth occurrence. * Picture graphs are graphs that use pictures to represent and compare information. At this level, each picture should represent one data point. * Object graphs are graphs that use concrete materials to represent and compare the categorical data that are collected (e.g., cubes stacked by the month, with one cube representing the birthday month of each student). * Tables are an orderly arrangement of data in columns and rows in an essentially rectangular format. Tables may be used to display some type of numerical relationship or organized lists. * At this level, data gathered and displayed by students should be limited to 16 or fewer data points for no more than 4 categories. * Students should have opportunities to interpret graphs, created with the assistance of the teacher, that contain data points where their entire class is represented (e.g., tables that show who   brought their lunch and who will buy their lunch for any given day, picture graph showing how students traveled to school – bus, car, walk). | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Collect and organize data using various forms of data collection (e.g., counting and tallying, informal surveys, observations, voting). Data points, collected by students, should be limited to 16 or fewer for no more than four categories. (a) * Represent data in tables, picture graphs, and object graphs. (a) * Analyze information displayed in tables, picture graphs, and object graphs (horizontally or vertically represented): * Read the graph to determine the categories of data and the data as a whole (e.g., the total number of responses) and its parts (e.g., 15 people are wearing sneakers); and * Interpret the data that represents numerical relationships, to include using the words *more, less, fewer, greater than, less than,* and *equal to*. (b) |

Stimulated by the exploration of their environment, children begin to develop concepts related to patterns, functions, and algebra before beginning school. Recognition of patterns and comparisons are important components of children’s mathematical development.

Students in kindergarten through grade two develop the foundation for understanding various types of patterns and functional relationships through the following experiences:

* sorting, comparing, and classifying objects in a collection according to a variety of attributes and properties;
* identifying, analyzing, and extending patterns;
* creating repetitive patterns and communicating about these patterns in their own language;
* analyzing simple patterns and making predictions about them;
* recognizing the same pattern in different representations;
* describing how both repeating and growing patterns are generated; and
* repeating predictable sequences in rhymes and extending simple rhythmic patterns.

The focus of instruction at the primary level is to observe, recognize, create, extend, and describe a variety of patterns. Students will experience and recognize visual, kinesthetic, and auditory patterns and develop the language to describe them orally and in writing as a foundation to using symbols. They will use patterns to explore mathematical and geometric relationships and to solve problems, and their observations and discussions of how things change will eventually lead to the notion of functions and ultimately to algebra.

| **1.13 The student will sort and classify concrete objects according to one or two attributes.** | |
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| Understanding the Standard | Essential Knowledge and Skills |
| * Sorting, classifying, and ordering objects facilitates work with patterns, geometric shapes, and data. * The same set of objects can be sorted and classified in different ways. * To classify is to arrange or organize a set of materials according to a category or attribute (a quality or characteristic). * A Venn diagram can be a helpful tool when sorting by more than one attribute. * One way to explore attributes is to investigate non-examples (e.g., a triangle could be a non-example in a sort of rectangles and circles). * General similarities and differences among items are easily observed by primary students, who can begin to focus on more than one attribute at a time. During the primary grades, the teacher’s task is to move students toward a more sophisticated understanding of classification in which two or more attributes connect or differentiate sets, such as those found in nature (e.g., leaves with different colors and different shapes). | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Sort and classify concrete objects into appropriate subsets (categories) based on one or two attributes, such as size, shape, color, and/or thickness (e.g., sort a set of objects that are both red and thick). * Label attributes of a set of objects that has been sorted. * Name multiple ways to sort a set of objects. |

| **1.14 The student will identify, describe, extend, create, and transfer growing and repeating patterns.** | |
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| Understanding the Standard | Essential Knowledge and Skills |
| * Opportunities to identify, describe, extend, create, and transfer patterns are essential to the primary school experience and lay the foundation for thinking algebraically. * Patterning should include: * creating a given pattern, using objects, sounds, movements and pictures; * recording a pattern with pictures or symbols; * transferring a pattern into a different form or different representation (e.g., blue–blue–red–green to an AABC repeating pattern); and * analyzing patterns in practical situations (e.g., calendar, seasons, days of the week). * In a repeating pattern the part of the pattern that repeats is the core. * At this level students should have experiences extending patterns when given a complete repetition of a core (e.g., ABACABACABAC) as well as when the final repetition of the core is incomplete (e.g., AABBAABBAA …; Red, Blue, Green, Red, Blue, Green, Red, Blue....). * Examples of repeating patterns include: * AABCAABC; * ABACABAC; * ABBCABBC; * AABCAABC; and * ABACDABACD. * Growing patterns involve a progression from step to step which make them more difficult for students than repeating patterns. Students must determine what comes next and also begin the process of generalization, which leads to the foundation of algebraic reasoning. Students need experiences identifying what changes and what stays the same in a growing pattern. Growing patterns may be represented in various ways, including dot patterns, staircases, pictures, etc. * Examples of growing patterns include:        * **5, 10, 15, 20…** * Transferring a pattern is creating the pattern in a different form or representation. * Examples of pattern transfers include: * 1, 2, 3, 4... has the same structure as 10, 11, 12, 13…; * ABABAB… has the same structure as red, blue, red, blue, red, blue; and * Snap, clap, jump, clap, snap, clap, jump, clap has the same structure as ABCBABCB. | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Identify the pattern in a given rhythmic, color, geometric figure, or numerical sequence. * Describe the pattern in a given rhythmic, color, geometric figure, or numerical sequence in terms of the core (the part of the sequence that repeats). * Extend a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators. * Create a repeating or growing pattern, using manipulatives, geometric figures, numbers, or calculators (e.g., the growing patterns 2, 3, 2, 4, 2, 5, 2, 6, 2, …). * Transfer a pattern from one form to another. |

| **1.15 The student will demonstrate an understanding of equality through the use of the equal symbol.** | |
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| Understanding the Standard | Essential Knowledge and Skills |
| * At this level, students should represent equality using objects, words, and symbols through the use of the equal symbol while inequality should be communicated primarily through words such as *not equal*, *not equivalent*, etc. * Equality can be shown using a balance scale or a number balance. An equation, such as 3 + 5 = 6 + 2, can be represented using a balance scale, with equal amounts on each side. * An equation (number sentence) is a mathematical statement representing two expressions that are equivalent. It consists of two expressions, one on each side of an *equal* symbol  (e.g., 5 + 3 = 8, 8 = 5 + 3 and 4 + 3 = 9 −2). * A common misunderstanding is that the equal symbol always means “the answer comes next.” The equal symbol represents a balance between expressions. The equal symbol means “is the same as” or “another name for” or “equal in value.” * Inequalities such as 5 < 4 + 3 are not equations. Equations must have the equal symbol (e.g., 5 + 6 = 11). * Equations should be routinely modeled in conjunction with story problems. Manipulatives such as connecting cubes and counters can be used to model equations. * An expression is a representation of a quantity. It contains numbers, variables, and/or computational operation symbols. It does not have an equal symbol (e.g., 5, 4 + 3, 8 − 2). * Students at this level are not expected to use the term *expression*. * Solving missing addend problems and stories helps with the understanding of equality and use of the equal symbol (e.g., There are four red birds in the tree. Some black birds fly to the tree. Now there are six birds in the tree. How many black birds flew to the tree? 4 +\_\_ = 6) | The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to   * Describe the concept of equality. * Identify equivalent values and represent equalities through the use of objects, words, and the equal (=) symbol. * Identify and describe expressions that are not equal (e.g., 4 + 3 is not equal to 3 + 5). * Recognize that equations can be used to represent the relationship between two expressions of equal value (e.g., 4 + 2 = 2 + 4 and 6 + 1= 4 + 3). * Model an equation that represents the relationship of two expressions of equal value. |