Rich Mathematical Task – Grade *7* – *Expressions for Gardening*

Task Overview/Description/Purpose:

- The purpose of this task is to explore the concept of evaluating algebraic expressions with given replacement values for variables.
- In this task, students will evaluate expressions to determine the total area of a garden which is split into four sections.

Standards Alignment: Strand – *Number and Number Sense*

Primary SOL: 7.11 The student will evaluate algebraic expressions for given replacement values of the variables.

Related SOL (within or across grade levels/courses): 5.19a, 5.19c, 7.12

Learning Intention(s):

- **Content** - I am learning how to evaluate algebraic expressions when given rational replacement values for the variables.
- **Language** - I am learning to use appropriate mathematical vocabulary when describing my methods for determining the total area of a garden, as well as evaluating algebraic expressions provided for the same garden (algebraic expression, length, width, area, sum, product, variable, term, constant, evaluate, substitute)**.**
- **Social** - I am learning to actively listen to classmates and communicate my own reasoning effectively so that my small group can accurately evaluate algebraic expressions to determine their validity.

Success Criteria (Evidence of Student Learning):

- I can accurately evaluate an algebraic expression when given rational replacement values for the variables.
- I can describe my method for determining the total area of a garden using appropriate mathematical vocabulary.
- I can use appropriate mathematical vocabulary when evaluating algebraic expressions provided and determining their validity.
- I can actively listen to classmates as they describe their method for determining total area or evaluating an algebraic expression provided.
- I can effectively communicate my thinking to peers as we work to determine the validity of algebraic expressions provided.

Mathematics Process Goals

Problem Solving	• Students will evaluate expressions to determine whether they produce accurate values for the total area of a garden.
Communication and Reasoning	• Students will communicate their method for determining the total area of a garden with peers. • Students will apply logical reasoning to determine whether algebraic expressions provide an accurate representation for the total area of a garden which is split into four sections. • Students will communicate their logical reasoning with other classmates.

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Standards Alignment: Strand – *Number and Number Sense*	
Connections and Representations	• Students will connect algebraic expressions with geometric representations of a garden. • Students will represent the total area of a rectangular garden with more than one algebraic expression.

Task Pre-Planning

Approximate Length/Time Frame: 45 minutes

Grouping of Students: Students may have enough background knowledge for this task to be administered as an introductory task. Using it as an introductory task allows results to be used formatively to assist with future differentiation. The task begins with a whole class launch. Students then work individually on Part 1 of the task. After working individually, the students will participate in a whole class discussion before being assigned to small groups of 3 or 4 to work on Part 2. Students who finish part 2 may be given opportunity to complete the Sequel, or extension, to the task. Once all of the groups have had an opportunity to finish Part 2 of the task, the teacher will return to a whole class setting to share several group responses and debrief.

Materials and Technology:

- Virtual Implementation Google Slides
- Copies of the task, printed as three separate pages
- Access to the Desmos scientific calculator or a handheld scientific calculator
- Graph paper
- Printed versions of Algebra Tiles (included in the Possible Graphic Organizers)

Vocabulary:

- algebraic expression
- length
- width
- area
- sum
- product
- variable
- term
- constant
- evaluate
- substitute

Anticipate Responses: See the Planning for Mathematical Discourse Chart (columns 1-3).

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Task Implementation (Before)

Task Launch

- Share the learning intentions and success criteria for the task.
- Display the image and questions listed below to invoke discussion about gardening, fertilizer, measurement, area of rectangular regions, etc.

What do you notice? What do you wonder?

- After this brainstorm session, the teacher should distribute Part 1 of the task and follow a three read protocol.
  - The first read is for context. Ensure that students understand the layout of Farmer Brown's garden and his need to determine the total area.
  - The second read is for mathematics. Students should highlight or circle any math vocabulary that they might need assistance with prior to beginning the task.
  - The third read allows students a chance to ask any other questions about the scenario itself.

Task Implementation (During)

Directions for Supporting Implementation of the Task

- Monitor – The teacher will observe students as they work independently on the first portion of the task and in small groups on the second part of the task. The teacher will engage with students by asking assessing or advancing questions as necessary (see attached *Planning for Mathematical Discourse Chart.).*
- Select – In Part 1, the teacher will select students to share their strategy for determining the total area of the garden. If student work supports this, the teacher can select students who have used the methods similar to those provided in Part 2 without recording the exact expressions presented in Part 2, along with one potential misconception. In Part 2 of the task, the teacher can select groups who explain Daughter Brown's expression, Son Brown's expression, and/or both sibling's strategies.
- Sequence – For each debriefing, the teacher will select 2-3 student strategies to share with the whole group. One suggestion is to look for one common misconception and two correct responses to share.
- Connect – The teacher will consider ways to facilitate connections between different student representations.

Suggestions For Additional Student Support

- Some students with visual-motor weaknesses may benefit from graph paper to help them organize their work and/or create sketches of gardens.
- Students with challenges in memory and language could benefit from word walls or graphic organizers to activate prior knowledge about algebra vocabulary and strategies for calculating the area of a rectangle.
- Provide manipulatives such as non-traditional algebra tiles. Samples are provided in the Possible Graphic Organizers section of this template.

- Post visual cues such as copies of Farmer Brown's garden design for students who need support with memory.
- For students who need support in justifying their thinking, you may choose to provide them with the sentence frames below.
    - What I know about the problem is…
    - My method for determining the total area was…
    - The first thing I did was …
    - I think Daughter Brown's algebraic expression is accurate because …
    - I think Son Brown's algebraic expression is accurate because …
    - I think both children's algebraic expressions are accurate because …

Task Implementation (After) *20 minutes*

Connecting Student Responses (From Anticipating Student Response Chart) and Closure of the Task:

- Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion.
- Connect different students' responses and connect the responses to the key mathematical ideas to bring closure to the task.
- Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion. For instance, provide the students whose work was not selected as part of the sequence of student work that will be shown the opportunity to validate and/or question what they see.

Teacher Reflection About Student Learning:

- Student understanding of the content through the use of the process goals will be assessed with the Rich Mathematical Task Rubric.
- Students have a variety of ways to enter this task. Teachers will need to anticipate these different entry points but also be prepared for others. The key here is allowing the students to communicate their reasoning as they are working through the task.
- This task can be used to introduce the concept of evaluating expressions and provides teachers with a common experience to reference as students continue their study and evaluate expressions involving both positive and negative numbers.

Planning for Mathematical Discourse

Mathematical Task: Expressions for Gardening Content Standard(s): 7.11

Teacher Completes Prior to Task Implementation			**Teacher Completes During Task Implementation**	
Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
Anticipated Student Response A (Part 1): **Non-starter**	• Is there anything that you need me to clarify about the task? • What shape are each of the sections in Farmer Brown's garden?	• Do you see any similarities between a section of Farmer Brown's garden and our task launch?		
Anticipated Student Response B (Part 1): **Student finds individual areas for each of the four sections numerically and adds the products together (with or without errors)**	• Can you explain your work? • Why do you decide to multiply ____ and ____? • Why did you add the four products together?	• Did you double-check each of your calculations? • Can you think of another way to find the total area for Farmer Brown's garden? • Could you write an algebraic expression that summarizes your work? (possibly offer algebra tiles provided)		
Anticipated Student Response C (Part 1):	• Can you explain your thinking?	• Did you double-check each of your calculations?		

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Student writes algebraic expressions for each of the four sections, evaluates each, and finds the sum of the four (with or without errors)	• What do each of these four algebraic expressions represent? • Can you explain why you decided to find the sum of the four values at the end?	• How could you write an algebraic expression to combine all of this work into one expression? • Can you think of another way to find the total area for Farmer Brown's garden?		
Anticipated Student Response D (Part 1): **Student writes an algebraic expression showing the four individual areas added together (Daughter Brown's strategy) (with or without errors)**	• Will you explain how you came up with your algebraic expression? • Tell me how you used that expression to determine the total area of Farmer Brown's garden.	• Did you double-check each of your calculations? • Can you think of another way to find the total area of Farmer Brown's garden?		
Anticipated Student Response E (Part 1): **Student finds the total area by adding the numerical values for x and y together and the numerical values of x and m**	• Where did these numbers come from? (referring to a sum representing $x + y$ or $x + m$) • Can you explain why you decided to find those sums? • What was your next step?	• Did you double-check each of your calculations? • Could you write an algebraic expression that summarizes your work?		

Teacher Completes Prior to Task Implementation			**Teacher Completes During Task Implementation**	
Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
together to determine values for the length and width of the garden. They multiply these two sums together but does not write an algebraic expression. (with or without errors)	• Why did you find that product?	• Can you think of another way to find the total area of Farmer Brown's garden?		
Anticipated Student Response F (Part 1): **Student writes algebraic expressions for both the length and width of the total garden, evaluates each, and multiplies these number together (with or without errors)**	• Can you explain why you wrote these algebraic expressions, $x + y$ and $x + m$? • After you evaluated each expression, why did you multiply the two values?	• Did you double-check each of your calculations? • How could you write an algebraic expression to combine all of this work into one expression? • Can you think of another way to find the total area of Farmer Brown's garden?		
Anticipated Student Response G (Part 1): **Student writes an algebraic expression that determines the total area of the garden by calculating the product of the length and width (Son Brown's**	• Will you explain how you came up with your algebraic expression? • Tell me how you used that expression to determine the total area of Farmer Brown's garden.	• Did you double-check each of your calculations? • Can you think of another way to find the total area of Farmer Brown's garden?		

Teacher Completes Prior to Task Implementation			**Teacher Completes During Task Implementation**	
Anticipated Student Response/Strategy *Provide examples of possible correct student responses along with examples of student errors/misconceptions*	**Assessing Questions** *Teacher questioning that allows student to explain and clarify thinking*	**Advancing Questions** *Teacher questioning that moves thinking forward*	**List of Students Providing Response** *Who? Which students used this strategy?*	**Discussion Order - sequencing student responses** • *Based on the actual student responses, sequence and select particular students to present their mathematical work during class discussion* • *Connect different students' responses and connect the responses to the key mathematical ideas* • *Consider ways to ensure that each student will have an equitable opportunity to share his/her thinking during task discussion*
strategy) (with or without errors)				
Anticipated Student Response A (Part 2): **Student argues that only Daughter Brown's expression is accurate (with or without errors)**	• Whose algebraic expression do you think is correct? • Why do you agree with Daughter Brown's algebraic expression? • Did you check Son Brown's expression to see if it is accurate?	• Did you double-check each of your calculations? • What do you think Son Brown's reasoning was when he wrote his algebraic expression?		
Anticipated Student Response B (Part 2): **Student argues that only Son Brown's expression is accurate (with or without errors)**	• Whose algebraic expression do you think is correct? • Why do you agree with Son Brown's algebraic expression? • Did you check Daughter Brown's expression to see if it is accurate?	• Did you double-check each of your calculations? • What do you think Daughter Brown's reasoning was when she wrote her algebraic expression?		
Anticipated Student Response C (Part 2): **Student argues that both siblings expressions are accurate**	• Whose algebraic expression do think is correct? • How did you determine that both siblings were correct?	• Are you convinced that these two algebraic expressions will prove accurate regardless of the values for m, x, and y?		

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	• Why do you think both algebraic expressions work for determining the total area?	• Whose algebraic expression do you think is more efficient?		
Anticipated Student Response A (Sequel): **Non-starter**	• Is there anything that you need me to clarify about the sequel? • Have you thought about what fruits and/or vegetables you want to plant in your garden?	• Do you think Daughter Brown's logic or Son Brown's logic might help you more to get started on this sequel? • Can you think of a potential length and width for a garden with a total area of 122.5 ft^2?		
Anticipated Student Response B (Sequel): **Student is successfully working towards a garden with a total area of 122.5 ft^2.**	• What dimensions are you using for your garden? • How did you determine each of those values for the variables? • Why did you decide to plant _____ in this section of the garden?	• Is there another way to create a garden with the same total area but different values for the variables?		

Name_____ Date_____

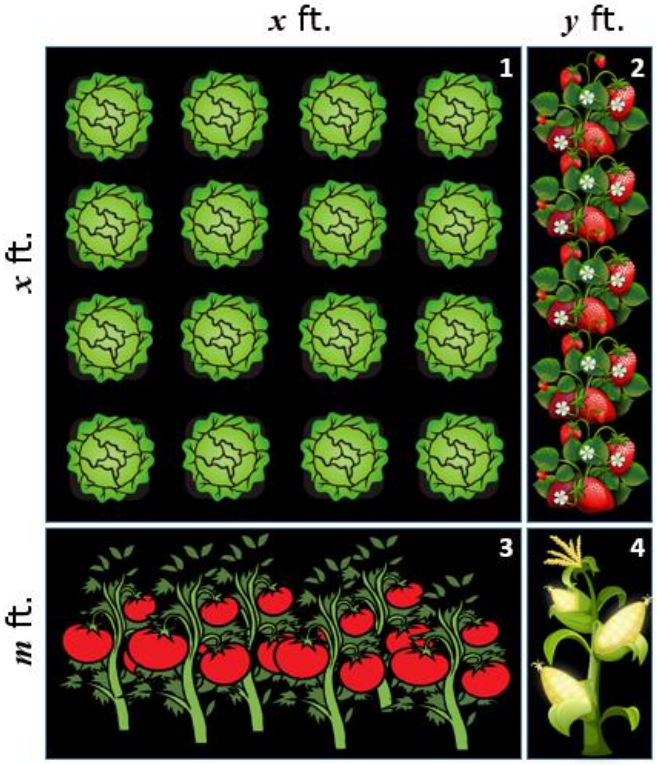
Expressions for Gardening

Part 1:

Farmer Brown planted a vegetable garden. It was split into four sections.

- In section 1, the farmer planted lettuce.
- In section 2, he planted strawberries.
- In section 3, Farmer Brown planted some tomatoes.
- In section 4, he planted corn.

The measurements for each section are given as variable terms.



Farmer Brown needs to cover the entire garden with fertilizer.

If $m = 3\frac{3}{5}$, $x = 10$, and $y = 2\frac{1}{2}$, what is the total area of the garden? Explain how you arrived at your solution.

Part 2:

Farmer Brown frequently plants gardens like this, with four sections that all need to be fertilized.

He knows there must be an algebraic expression that could be used to calculate the total area for any of his gardens.

He promises his two children that he will build them a treehouse in his spare time if they can help him by making one of these time-saving expressions.

Each child came up with their own possibility.

Daughter Brown	Son Brown
$x^2 + xy + mx + my$	$(x + m)(x + y)$

Decide which of Farmer Brown's children is correct, and explain Daughter Brown and/or Son Brown's reasoning for why their expression should work.

Sequel:

Farmer Brown is offering to come plant one of his gardens for you with one criteria: the total area must be 122.5 ft^2.

Determine values for m, x, and y that will create a garden to match Farmer Brown's criteria.

Explain how you found these values and why you are confident that they will produce the required total area.

Also, let Farmer Brown know what you plan to plant in each section and why you made each of those selections.

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Rich Mathematical Task Rubric

	Advanced	Proficient	Developing	Emerging
Mathematical Understanding	Proficient Plus: • Uses relationships among mathematical concepts or makes mathematical generalizations	• Demonstrates an understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to a valid and correct solution	• Demonstrates a partial understanding of concepts and skills associated with task • Applies mathematical concepts and skills which lead to an incomplete or incorrect solution	• Demonstrates no understanding of concepts and skills associated with task • Applies limited mathematical concepts and skills in an attempt to find a solution or provides no solution
Problem Solving	Proficient Plus: • Problem solving strategy is well developed or efficient	• Problem solving strategy displays an understanding of the underlying mathematical concept • Produces a solution relevant to the problem and confirms the reasonableness of the solution	• Problem solving strategy displays a limited understanding of the underlying mathematical concept • Produces a solution relevant to the problem but does not confirm the reasonableness of the solution	• A problem solving strategy is not evident • Does not produce a solution that is relevant to the problem
Communication and Reasoning	Proficient Plus: • Reasoning or justification is comprehensive • Consistently uses precise mathematical language to communicate thinking	• Demonstrates reasoning and/or justifies solution steps • Supports arguments and claims with evidence • Uses mathematical language to communicate thinking	• Reasoning or justification of solution steps is limited or contains misconceptions • Provides limited or inconsistent evidence to support arguments and claims • Uses limited mathematical language to partially communicate thinking	• Provides no correct reasoning or justification • Does not provide evidence to support arguments and claims • Uses no mathematical language to communicate thinking
Representations and Connections	Proficient Plus: • Uses representations to analyze relationships and extend thinking • Uses mathematical connections to extend the solution to other mathematics or to deepen understanding	• Uses a representation or multiple representations, with accurate labels, to explore and model the problem • Makes a mathematical connection that is relevant to the context of the problem	• Uses an incomplete or limited representation to model the problem • Makes a partial mathematical connection or the connection is not relevant to the context of the problem	• Uses no representation or uses a representation that does not model the problem • Makes no mathematical connections

Possible Graphic Organizers
Algebra Tiles