

Just In Time Quick Check
Standard of Learning (SOL) AII.11b

Strand: Statistics

Standard of Learning (SOL) AII.11b

The student will interpret and compare z-scores for normally distributed data.

Grade Level Skills:

- Solve problems involving the relationship of the mean, standard deviation, and z-score of a normally distributed data set.
- Compare two sets of normally distributed data using a standard normal distribution and z-scores, given the mean and standard deviation.
- Use a graphing utility to investigate, represent, and determine relationships between a normally distributed data set and its descriptive statistics.

Just in Time Quick Check

Just in Time Quick Check Teacher Notes

Supporting Resources:

- VDOE Mathematics Instructional Plans (MIPS)
 - o [AII.11abc Normal Distribution](#) (Word) / [PDF Version](#)
- VDOE Word Wall Cards: Algebra II ([Word](#)) | ([PDF](#))
 - o Statistics Notation
 - o Mean, Median, Mode, Variance
 - o Standard Deviation (Definition),
 - o Standard Deviation (Graphic)
 - o z-Score (Definition), z-Score (Graphic)
 - o Empirical Rule, and Elements within One Standard Deviation (σ) of the Mean (μ) (Graphic)

Supporting and Prerequisite SOL: N/A

SOL All.11b - Just in Time Quick Check

1. Data representing the grades of all the students on a recent test in Ann's class have a normal distribution. The z-score of Ann's test grade is 0.75. Explain what this z-score represents in relation to the mean of the class grades.
2. The mean of a normally distributed set of data is 125. A data value of 100 has a z-score of -1.40. Determine the standard deviation of this distribution. Show your work.
3. The standard deviation of a normally distributed set of data is 5. If a data value of 27 has a corresponding z-score of 1.22, what is the mean of this distribution? Justify your answer.
4. Among the students in a local school division, ACT scores were normally distributed this year with a mean score of 20.7 and a standard deviation of 5.9. SAT scores were normally distributed with a mean score of 1075 and a standard deviation of 168. Who scored better on their standardized test with respect to the other test takers in the school division, Jamal who took the ACT and scored 29 or Shamar who took the SAT and scored 1240? Explain your choice.
5. A data set of values has a mean of 45 and standard deviation of 5. The z-score for a point A is 0. The z-score for a point B is 0.2. State the values of point A and point B. Explain your answers.
6. Data on the grades of a morning math class and an afternoon math class were collected.
The grades in the morning math class were normally distributed with a mean of 82 and a standard deviation of 3.
 - Student A had a grade of 80.
 - Student B has a grade of 78.The grades in the afternoon math class were normally distributed with a mean of 80 and a standard deviation of 5.
 - Student C had a grade of 78.
 - Student D had a grade of 76.

Which student was in the lower 16% of their class?

SOL All.11b - Just in Time Quick Check Teacher Notes

Common Errors/Misconceptions and their Possible Indications

1. Data representing the grades of all the students on a recent test in Ann's class have a normal distribution. The z-score of Ann's test grade is 0.75. Explain what this z-score represents in relation to the mean of the class grades.

A common misconception that some students may have is to assume that the z-score represents a percentage. This may indicate that the student does not understand how the z-score is related to the normalized distribution of class grades. A potential teaching strategy could be to use the Vocabulary Word Wall card for Algebra II to review the Empirical rule and how the number of standard deviations from the mean represent the z-score.

2. The mean of a normally distributed set of data is 125. A data value of 100 has a z-score of -1.40. Determine the standard deviation of this distribution. Show your work.

A common error that some students may make is to multiply instead of divide the z-score by the difference between the mean and the given value to determine the standard deviation, resulting in a positive value but much larger than accurate. This may indicate that students might not recognize that a z-score derived from a given data value indicates how many standard deviations the data value falls above or below the mean. Teachers may wish to have students sketch a normal curve and label the x-axis with the mean and the given value. Then ask students to estimate what a unit of one standard deviation to the left of the mean might equal (for example, if $\sigma = 25$ then 100 would only be 1 standard deviation to the left of the mean, but the z-score tells us 100 is 1.4 standard deviations to the left of the mean, so σ must be smaller than 25). Students can then be more certain that their solution is reasonable and has been accurately calculated using the z-score formula.

3. The standard deviation of a normally distributed set of data is 5. If a data value of 27 has a corresponding z-score of 1.22, what is the mean of this distribution? Justify your answer.

A common error that some students may make is to use $(\bar{x} - x)$ versus $(x - \bar{x})$ in the numerator when using the z-score formula to solve for the value of the mean, resulting in a mean value that is larger than accurate. This may indicate that students do not recognize the meaning of the variables represented in the z-score formula. Teachers may wish to emphasize the relationship between the data values in a normally distributed data set and how they are related to the values of the mean and standard deviation. Consider referring students to the VDOE Vocabulary Word Wall Cards for Algebra II to review the cards related to z-scores.

4. Among the students in a local school division, ACT scores were normally distributed this year with a mean score of 20.7 and a standard deviation of 5.9. SAT scores were normally distributed with a mean score of 1075 and a standard deviation of 168. Who scored better on their standardized test with respect to the other test takers in the school division, Jamal who took the ACT and scored 29 or Shamar who took the SAT and scored 1240? Explain your choice.

A common error that some students may make is to not normalize the test scores before comparing them. This may indicate that students do not recognize that a standard normal distribution is a set of z-scores of a data set and allows for comparisons of unlike normal data.

5. A data set of values has a mean of 45 and standard deviation of 5. The z-score for a point A is 0. The z-score for a point B is 0.2. State the values of point A and point B. Explain your answers.

A common misconception is for students to misinterpret the meaning of the z-score. A potential teaching strategy would be for students to draw this normal distribution and label values that are integer multiples of the standard deviation above and below the mean. (45 in the center and then counting by 5s in each direction label where 30, 35, 40, 45, 50, 55, and 60 occur.) Then have students label the z-score for 30 ($z = -3$), 35 ($z = -2$), etc. on the same drawing. Finally, ask students to make predictions about the value of A and B to discuss what is reasonable before using the z-score formula.

6. Data on the grades of a morning math class and an afternoon math class were collected.
- The grades in the morning math class were normally distributed with a mean of 82 and a standard deviation of 3.
- Student A had a grade of 80.
 - Student B has a grade of 78.

The grades in the afternoon math class were normally distributed with a mean of 80 and a standard deviation of 5.

- Student C had a grade of 78.
- Student D had a grade of 76.

Which student was in the lower 16% of their class?

A common error that some students may make is to choose student D. This may indicate that a student is basing their decision on which student had the lowest grade rather than comparing the data value and its related z-score to its relative position to the mean. Teachers may wish to have students sketch a normal curve for each class and label the horizontal axis with the mean, each standard variation value, along with the student data value. Encourage students to use a visual representations when determining the position of a data value in relation to the mean and standard deviation of a normally distributed set of data. Consider referring students to the VDOE Vocabulary Word Wall Cards for Algebra II.