How Many Ways?

Strand: Probability and Statistics

Topic: Constructing sample spaces and determining the probability of outcomes

Primary SOL: 5.15 The student will determine the probability of an outcome by constructing a sample space or using the Fundamental (Basic) Counting Principle.

Related SOL: 4.13

Materials

- Probability Cards (Set 1 and 2) activity sheet (attached)
- Using a Tree Diagram to Determine How Many Outfits activity sheet (attached)
- So Now I Have More Choices Work Space activity sheet (attached)

Vocabulary

chance, experiment, experimental probability, Fundamental Counting Principle, more/less/equally likely, outcome, probability, sample space, theoretical probability, tree diagram

Student/Teacher Actions: What should students be doing? What should teachers be doing?

1. Ask, *"Thumbs-up if you have heard the term probability before, and thumbs-down if you do not think you have. What is probability?"* Use ideas the students share to help them develop the definition: Probability is the chance or likelihood of an event occurring.
    a. Have students brainstorm terms related to probability and their meanings, including *chance, data, more/less/equally likely, certain, sample space,* and *outcomes*. As students name a term, use Mathematics Vocabulary Word Wall Cards at the VDOE Office of Science, Technology, Engineering & Mathematics website to build a word wall. If students share an idea but do not recall the term, provide the term and post the vocabulary card. Other terms will come up throughout the lesson, and those cards can be posted at that time.
    b. Ask students to find partners and, using the think-pair-share strategy, brainstorm a list of situations that involve probability. List these ideas on the board, and if the context allows, list the probability it will happen. For instance, if all of the student's names were put into a hat, what is the probability a boy's name will be chosen? A girl's name? Use words like *impossible, unlikely, equally likely, likely* and *certain*.
    c. Emphasize that the chance or likelihood of an event occurring can range from zero (impossible) to 1 (certain). Draw a number line on the board with the numbers 0 and 1. Draw a tic mark in the middle of the line. Under 0, write" impossible." Under 1, write "certain." In the middle, write "equally likely" and the fraction $\frac{1}{2}$. Between 0 and the middle, write "unlikely." Between the middle and 1, write "likely."

2. Let students know they are going to conduct an experiment so they can develop a sample space that shows all possible combinations of shirts, pants, and shoes.
    a. Distribute the Probability Cards (Set 1) and review with the class what cards are in the set; two colors of pants, two colors of shirts, and two colors of shoes. Clarify what it means to wear a different outfit each day. Then ask students to find all of the different outfits they can wear with one shirt, one pair of pants and one pair of socks. Discuss how organization is important to avoid duplicates and to make sure all possible outfits have been made. Tell students to make sure they do not repeat an outfit. Allow students time to work on this with a partner. Circulate around the room and notice what students are saying and doing. If students are stuck, pose questions to help them think about the situation.
    b. Ask students about the sample space they have created: *"How many different outfits can be made?" "How do you know you have all the possible combinations?" "How do you know that a particular combination was not included twice?" "Could anyone look at your organization and know what all the possible combination of clothes or outfits would be?"*
    c. Let students know there is another way to organize information when looking for combinations, and the organizing tool is a *tree diagram.* Provide students the Using a Tree Diagram to Determine How Many Outfits activity sheet to take notes as you model for the students how to create a tree diagram. First, answer the questions to prepare for later introduction of the Fundamental Counting Principle. Then model and explain a tree diagram using tan and green pants, orange and yellow shirts, and black-and-white shoes. This activity has two goals: 1) learning how to make a tree diagram, and 2) building the foundation for understanding the Fundamental Counting Principle. With this in mind, ask purposeful questions along the way and help students make explicit connections between the number of branches needed and why each time a new category or item of clothing is introduced.
    d. Ask students to find where all possible outcomes of the outfits are located on the tree diagram. Using the tree diagram or the list they made, ask students how many outfits have a red shirt. Using $\frac{total\ number\ of\ favorable\ outcomes}{total\ number\ of\ outcomes}$, model for students how to write the total number of outcomes, $\frac{4}{8}$, where four out of eight outfits will have a red shirt. Ask students where this would fall on the number line of probability on the board.
    e. Ask, *"If we have two different colors of shirts, two colors of pants, and different colors of shoes, how can we get eight possible outfits?"* Revisit the questions the students answered before creating the tree diagram and facilitate discussions for students to discover 2 x 2 x 2 = 8. First, students need to find out how many possible outcomes there are for shirts and pants only, and then find all of the possible outcomes including shoes. Tell students this is called the *Fundamental Counting Principle*.

3. Distribute to individuals or pairs of students the Probability Cards (Set 2), red shirt and blue pants, to include with the first set. Distribute the Now I Have More Choices Work Space activity sheet, and ask students to make a list or table of all possible outcomes. Next, make a tree diagram and use the Fundamental Counting Principle to verify their answer for all possible outcomes. Last, ask students to generate a probability question to ask the class about the new outfits. Circulate and facilitate as needed. Make note of who is struggling and who you may want to call on to share.

    a. Create a checking station by posting a premade tree diagram in an out-of-the-way but observable space. As students finish, have them take their paper (no pencils or pens) to the checking station to determine whether they are finished. If their tree diagram is not correct, they can return to their seat to make revisions.

    b. Once students have completed the Now I Have More Choices Work Space activity sheet, have individual or pairs find another individual or pair, ask their probability question, and decide where it falls on the probability number line. Ask students for a question where the answer would be impossible and certain.

4. Give students two coins and have them find all possible outcomes if they tossed the coin three times. Have them make a list or table, tree diagram, use the Fundamental Counting Principle, and create a probability question. Create a checking station. Continue to circulate and facilitate as needed.

    a. As you circulate and students say they are finished, have them take their paper, (no pencils or pens) to the checking station to determine whether they are finished. If their tree diagram is incorrect, they can return to their seat to make revisions.

    b. Use the information in the tree diagram and list to introduce the ideas of theoretical and experimental probability. Discuss with students the idea that the more times they repeat an experiment, the closer the experimental probability becomes theoretical probability.

Assessment

- **Questions**
    - Why use a tree diagram instead of a list of outcomes?
    - What is the Fundamental Counting Principle?
    - What is probability? Explain in your own words.
- **Journal/writing prompts**
    - Create the sample space or choices for a lunch menu where there would be 12 possible combinations or outcomes. Show all possible outcomes using a list or table and a tree diagram.
    - Explain when it would be best to use a tree diagram and when it would be best to make a list or table.
    - Explain how the Fundamental Counting Principle works.

- Write a situation where the probability of the event occurring is $\frac{3}{5}$.

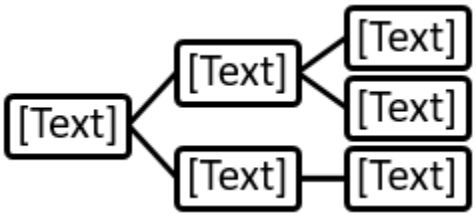
- **Other Assessments**
  - There are 7 blue squares and 3 orange squares, all the same size, in a bag. What is the probability of drawing a blue square? An orange triangle? Where do both fall on the Probability Number Line?
  - Why would you want to repeat an experiment?
  - Explain some ways you can decide whether a game or experiment is fair.
  - Describe a situation that involves probability. Define the sample space, and identify the chances of each outcome.

Extensions and Connections (for all students)

- Provide students with two fair number cubes, each side numbered from 1 to 6. In addition, ask them to create the sample space for all the possible sums. Then ask them to determine which of the following would be good games for the Fall Festival and to justify their answer. You win if you roll the following: a 4, a number other than 6, an odd number, a fraction, or a multiple of 5.
- Have students determine the theoretical probability of getting rock, paper, or scissors. If the students play the game eight times, what would be all possible outcomes? Have students prove their answer. Ask them to choose rock, paper, or scissors, and give the probability of getting it.
- Provide students with various spinners. Have them construct the sample space of possible outcomes and conduct experiments with the spinners to determine experimental probability. Students can compare the theoretical probability (expected outcomes) to the experimental probability (their results from the experiment).

Strategies for Differentiation

- Color-code the Probability Cards.
- Provide students with sets of cards to build the tree diagrams. Eights sets of Set 1 will be needed to create the complete tree diagram. Build a tree diagram with cards before drawing it on their papers.
- Use an electronic graphic organizer to create the tree diagram.



The following pages are intended for classroom use for students as a visual aid to learning.

Probability Cards (Set 1)

Orange shirt	**Yellow shirt**
Tan pants	**Green pants**
Black shoes	**White shoes**

(Set 2)

Red shirt	**Blue pants**

Using a Tree Diagram to Determine How Many Outfits

Answer the following questions before starting to create the tree diagram.

1. How many different categories or types of clothing make up an outfit? ____

2. List the different categories and the number of different color items in each category.

3. We are going to work together as a class to develop a tree diagram. Use the space below to take notes.

So Now I Have More Choices Work Space

Use the space below to identify all possible outfits when there are three colors of pants, three colors of shirts, and two colors of shoes.

1. Make a list/table.
2. Create a tree diagram.
3. Verify all possible outcomes using the Fundamental Counting Principle.
4. Create a probability question for the class to answer.