## The Standard Normal Curve

## Strand: Data Analysis

Topic: Understanding normal distribution
Primary SOL: AFDA. 7 The student will
a) identify and describe properties of a normal distribution; and
c) apply properties of normal distributions to determine probabilities associated with areas under the standard normal curve.

Related SOL:
AFDA.7b

## Materials

- Application of the Normal Distribution activity sheet (attached)
- Graphing utility


## Vocabulary

data table, empirical rule, mean, median, mode, normal curve, probability, scatterplot, standard deviation, z-score

## Student/Teacher Actions: What should students be doing? What should teachers be doing?

 Time: 90 minutes1. Create a data table like the one shown below, where each student in the class will input their height (inches). The data should have a minimum sample size of 30 . If there are fewer than 30 students in the class, ask some of the students in the class to include the height of their friends in the same age group to complete the data set.

| Student | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Height |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |


| Student | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Height |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

2. Ask students to create a scatterplot using the data from the table.
3. Have students input the data on a spreadsheet or a graphing utility so they can compute the mean and standard deviation.
4. Using the mean and standard deviation, ask the students to label the curve below with the height that corresponds to 1 standard deviation above the mean, 2 standard deviations above the mean, and so on.

5. Discuss the characteristics of the normal curve and what it means when a data set is normally distributed.
6. Using the data from the table, ask students to complete the chart below.

| How many students have heights that <br> fall ... | Number of <br> students | Percent of students |
| :--- | :--- | :--- |
| within 1 standard deviation of the mean |  |  |
| within 2 standard deviations of the mean |  |  |
| within 3 standard deviations of the mean |  |  |

7. Discuss the empirical rule. Then pose the following questions:
a) How do the results from the chart above compare with the empirical rule?
b) Do you think the result will be the same if the sample size is doubled?
c) Do you think the result will be the same if this experiment will be conducted using middle school boys and girls? Why?
d) If a male student is 6 feet tall, what percent of the students is shorter than him?
8. Review how to convert the heights into z-scores, and explain why is it necessary to convert raw scores into a z-score. Ask, "What does it mean when the height is equivalent to a $z$-score of 1.5 ?"
9. Ask students to convert their height into a z-score. Have students write their z-score on an index card and have them explain what the $z$-score equivalent of their height means in the context of the data.
10. Let the students stand up and pair up. Each pair will take turns explaining to each other the $z$-score equivalent of their height and what it means in relation to the height of all other students in the class.
11. Explain how to solve problems like the sample problem below using the z-table or a graphing utility.

GoPod, an MP3 player made by Mango Corporation, has an average battery life of 400 hours. Battery life for the GoPod is normally distributed, with a standard deviation of 25 hours.

PeaPod, an MP3 player made by Pineapple Inc., has an average battery life of 390 hours. The distribution for its battery life is also normally distributed, with a standard deviation of 30 hours.
a. Find the $z$-scores for each battery with lives of $250,350,410$, and 450 hours.
b. What percent of the GoPod batteries last between 375 and 410 hours?
c. What percent of PeaPod batteries last more than 370 hours?
12. Have students work in pairs to complete the Application of the Normal Distribution activity sheet.

## Assessment

- Questions

The data below shows the temperature for the past 28 days.

| 61 | 70 | 36 | 52 | 49 | 77 | 62 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 37 | 55 | 32 | 44 | 82 | 96 | 73 |
| 55 | 61 | 43 | 38 | 37 | 56 | 59 |
| 62 | 32 | 46 | 58 | 63 | 72 | 84 |

- What is the mean and standard deviation for the data set?
- Is the data normally distributed? How do you know?
- Draw a normal distribution curve indicating the mean and standard deviations and label the curve.
- Describe the data distribution.
- Journal/Writing prompts
- How do you know that a data set is normally distributed?
- Create a video explaining how use the empirical rule in determining the probability, given a data set that is normally distributed.
- Other Assessments
- "Empirical Rule," Normal Distributions, Khan Academy website
- "Normal Distribution: Area Above or Below a Point," Normal Distribution Calculators, Khan Academy website
- "Normal Distribution: Area Between Two Points," Normal Distribution Calculations, Khan Academy website
- "Normal Calculations in Reverse," Normal Distribution Calculations, Khan Academy website


## Extensions and Connections

- Assign a reading assignment and video lesson to watch. "Normal Distributions," CK-12 Foundation website
- Video resources
- "Standard Normal Distribution," American Public University System and CK-12
- "Calculating Probability Using a Table in a Normal Distribution," American Public University System and CK-12
- "Practice Questions: Normal Distribution," CK-12


## Strategies for Differentiation

- Create a choice board and have students choose one to complete to enhance their understanding of the topic.
- Use vocabulary cards for related vocabulary listed above.
- Use a color-coded standard normal curve by the standard deviations. Cut the sections apart to allow students to remove and add the areas under the curve.
- Assign a reading assignment about the normal curve ("Applications of Normal Distributions")

Note: The following pages are intended for classroom use for students as a visual aid to learning.

## Application of the Normal Distribution

Work with a partner and answer the questions pertaining to the given problem.
The life span of a particular machine gasket is normally distributed with a mean of 750 hours and a standard deviation of 25 hours. Label the graph of the distribution below using raw scores and z -scores:


Answer the following questions:
a) What percent of the machine gasket will have a life span between 725 and 775 hours?
b) What percent of the machine gasket will have a life span between 700 to 800 hours?
c) What percent of the machine gasket will have a life span lower than 700 hours?

Suppose the manufacturer makes 1,000 machine gasket.
a) How many machine gaskets will have a life span greater than 800 hours?
b) How many machine gaskets will have a life span lower than 675 hours?

Using the same machine gasket example, shade the region under the curve representing the given probability and then determine the numerical probability using a table or a calculator.
a) The probability that the gasket will last more than 730 hours.

$$
P(x>730)
$$


b) The probability that the gasket lasts less than 785 hours. $\mathrm{P}(\mathrm{x}<785)$


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c) The probability the gasket lasts between 730 and 785 hours.
$\mathrm{P}(730<x<785)$

d) The probability that the gasket lasts more than 780 hours or less than 720 hours. $P(780<x$ or $x<725)$


