Introduction to Logarithmic Functions

Strand: Algebra and Functions

Topic: An introduction to logarithmic functions as the inverse of an exponential.

Primary SOL: AFDA.1 The student will investigate and analyze linear, quadratic exponential and logarithmic function families and their characteristics. Key concepts include

a) domain and range; and

g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs.

Related SOL: AFDA.2, AFDA.3

Materials

- Switch It Up activity sheet (attached)
- Patty paper (or any thin tracing paper)
- Graph paper
- Colored pencils

Vocabulary

asymptote, domain, equation, exponential, horizontal, input, inverse, logarithmic, output, range, table, vertical

Student/Teacher Actions: What should students be doing? What should teachers be doing?
Time: 60 minutes

1. Start by introducing the concept of inverse. Ask students to define the word "inverse." Give students some practical examples of possible inverse operations (e.g., driving to a destination and back from it, putting on your shoes and taking them off again).
2. Distribute the Switch It Up activity sheet. Ask students what the inverse of some basic mathematical operations might be.
   a. What would be the inverse of $f(x)=x+5$?
   b. What about $g(x)=2x$?
   c. What about the function $h(x)=x^3$? What does the function do? What would the inverse do?
3. Have the students complete the table for the function $h(x)=x^3$ by hand or by using a graphing utility.
4. Have the students complete the table for $h^{-1}(x)$ on their activity sheets. Ask students what they notice about the tables: "*What happened to x and y in the inverse function?*" Be sure to use domain and range vocabulary in this discussion, as well as during discussions about the values in the table.

5. Have students graph $h(x)$ and $h^{-1}(x)$ on the same graph, using two colored pencils, on their activity sheets. Graphing the functions in different colors may help the students see the connections. Ask: "*What do you notice about the graphs?"* Ask students about the symmetry of the graphs: *"Could I fold my graph so that one function would line up with the other? Where would the fold have to be?"* Students can sketch the graphs on patty paper and try folding if they are having trouble noticing the symmetry about the line $y = x$.
6. Next, investigate the function $k(x) = 3^x$ on the activity sheet. Have students complete the table, graph the function, and identify the domain and range.
7. Have students complete the table for the inverse on their activity sheets. Remind the students of the relationship between the domain and range of inverse functions that they noticed before. The students should graph the inverse function and identify the domain and range.
8. Discuss the input and output of the two functions, helping students connect the input/power of the exponential function with the output of the inverse function. Define the logarithmic function as the inverse of the exponential function.
9. Have the students complete the table of exponential and logarithmic equations on their activity sheets, working through the first example, if needed. Summarize the common and natural log at the end of the chart.

Assessment

- **Questions**
  - What logarithmic function is the inverse of the exponential function?
    $g(x) = \left(\frac{1}{3}\right)^x$?
  - Joe wants to graph the function, $f(x) = \log_2 x$. He knows he cannot graph it on his graphing utility directly but thinks he can use an exponential function. How could he use the exponential function to graph it? What exponential function should he use?
- **Journal/writing prompts**
  - Your teacher has given you the graph of a function. Describe how you would graph the inverse function.
  - Do all functions have inverses that are also functions?
  - Describe the process for finding the equation of an inverse function.
  - Explain why the x-value and y-value switch places in inverse functions.
- **Other Assessments**
  - Use individual whiteboards (or another display tool) for graphing functions and inverses so that students might share out their graphs.

Extensions and Connections

- Have students summarize the findings about inverses.

- Discuss the horizontal asymptotes of the exponential functions and the vertical asymptotes of logarithmic functions as you discuss the graphs.

Strategies for Differentiation

- Use a graphing utility to help students generate tables and graphs.
- Use a graph organizer to define logarithmic functions, comparing what a logarithm is and is not, and characteristics including domain and range.
- Create a chart of all parent functions, equations, graphs, and domain and range so students can see, compare, and contract all parent functions.
- Use vocabulary cards for related vocabulary listed above.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Switch It Up

What is the *inverse* of a function?

The inverse ___________________ or undoes the previous action. If $f(x)$ represents the original function, then $f^{-1}(x)$ is the inverse function.

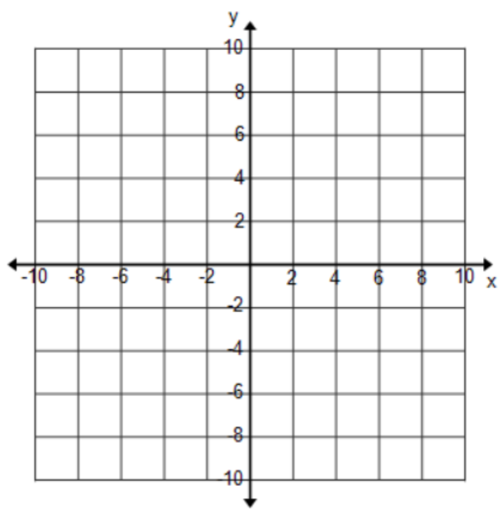
1. What would be the inverse of each function below?
   a) $f(x)=x+5$ b) $g(x)=2x$

2. Use the function, $h(x)=x^3$ to complete each below.
   a) What does the function do? ____
   b) What would the inverse function do? _______________________________________
   c) Complete the tables below for both $h(x)$ and $h^{-1}(x)$ below.

x	$h(x)$
-3	
-2	
-1	
0	
1	
2	
3	

x	$h^{-1}(x)$
-27	
-8	
-1	
0	
	1
	2
27	

   d) What do you notice about the domain and range of each function?
   e) Plot as many points as possible from the tables of each function and connect the points with a smooth curve. (You might want to graph your functions in different colors.)
   f) What do you notice about the graphs?



3. Use the function $k(x)=3^x$ to complete each of the following.

   a) Complete the table below for the exponential function $k(x)=3^x$.

x	-3	-2	-1	0	1	2	3	4
$k(x)$								

   b) What is the domain of the function? What is the range of the function?

   c) Complete the table below for the inverse of the function $k(x)=3^x$.

x		$\frac{1}{9}$		1	3		27	
$k^{-1}(x)$	-3		-1			2		4

   d) What is the domain of $k^{-1}(x)$? What is the range of $k^{-1}(x)$?

This inverse function is called a ***logarithmic function***. Its equation is $k^{-1}(x)=\log_3 x$

The inverse of an exponential function is a logarithmic function.
If $m(x)=b^x$, its inverse is $m^{-1}(x)=\log_b x$.

Complete the table below, creating two of your own problems in the shaded rows.

Exponential Form	Logarithmic Form
$4^3=64$	$\log_4 64=$
$5^4=$	$\log_5 625=4$
$3^4=81$	
	$\log_2 128=7$
$6^3=$	
	$\log_2 64=$
$10^{-1}=\frac{1}{10}$	$\log\left(\frac{1}{10}\right)=$
$e^2\approx$	$\ln 7.39\approx 2$

Notice the last two are the only ones that are on your graphing utility because they are the most commonly used. These two common logarithms are: LOG, which is base 10, and LN, which is base e (an irrational number). When there is no base shown, then the log has a base of 10.